

# GOVT. Model HIGH SCHOOL 343 GB

Solved Exercise Important Theory +MCQ's

According To Smart Syllabus

## MATHEMATICS

(SCIENCE GROUP)

10

Fully Solved Homework +Class Work



Compiled By: Muhammad Amin  
WhatsApp at: +92332-7965065

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# ACCELERATED LEARNING PROGRAMME

## UNIT - 1: Quadratic Equations

**Class Work:** Exercise:1.1,Q:1(iii),Q:2(ii), Q:3(v,ix),Exercise:1.2,Q:1(iii)

Exercise:1.3,Q:10,12, Exercise:1.4, Q:1,9

**Home Work:** Exercise:1.1,Q:1(i,iv),Q:2(iv,v),

Q:3(i,v),Exercise:1.2,Q:1(i,vii,viii),Exercise:1.3,Q:2,7,9,10,14,

Exercise:1.4,Q:3,8,Miscellaneous Exercise:1, Q:1,Q:2

## UNIT - 2: Theory of Quadratic Equations

**Class Work:** Exercise:2.1, Q:1(ii),Q:2(i),Q:3 Exercise:2.2,Q:1-4,

Exercise:2.3,Q:1(i),Q:2(ii),Q:5(ii), Exercise:2.5, Q:1(f), Q:2(b),

Q:3(b),Exercise:2.6,Q:1,2,5, Exercise:2.8,Q:4,10

**Home Work:** Exercise:2.1, Q:1(iv),Q:2(iv),Q:4(iii),Q:10,

Exercise:2.2,Q:2(ii,viii),Exercise:2.3,Q:1(v,vi),Q:2(ii), Q:6(i),Exercise:2.5, Q:1(g,h),

Q:2(d,e), Exercise:2.7,Q:2,5,10,13,Exercise:2.8,Q:1,5,9,Miscellaneous Exercise:2,Q:1,

Q:2(i-vii)

## UNIT - 3: Variations

**Class Work:** Exercise:3.1, Q:4,9, Q:11(iv), Exercise:3.2, Q:1(iii), Q:8,11,

Exercise:3.3, Q:1(i),

Q:2(iv,vi),Q:3(i),Q:4(iii):Exercise:3.4,Q:1(i),Q:2(iv,vii),Exercise:3.5,Q:1,Exercise:3.6,

Q:1(iii),Q:2(ii),Exercise:3.7,Q:2,9

**Home Work:** Exercise:3.1, Q:1(iv,v), Q:5,7, Q:11(v), Exercise:3.2, Q:2(ii), Q:5,10,13,

Exercise:3.3, Q:1(iv,vi), Q:2(ii,iv,v,vi),Q:3(iv),Q:4(ii):Exercise:3.4,Q:1(v,viii),Q:2(ii,

,v),Exercise:3.5,Q: 3,5, Exercise:3.6,Q:1(ii,vi),Exercise:3.7,Q:3,9, Miscellaneous

Exercise:3,Q:1,Q:2

## UNIT - 4: Partial Fractions

**Class Work:** Exercise:4.1,Q:8, Exercise:4.2,Q:2,Exercise:4.3, Q:8,

**Home Work:** Exercise:4.1,Q:2,4,7, Exercise:4.2,Q:1,6,8, Exercise:4.3, Q:1,6,

Exercise:4.4, Q: 3,6, Miscellaneous Exercise:4, Q:1,Q:2(i-v)

## UNIT - 5: Sets and Functions

**Class Work:** Exercise:5.1,Q:1(i),Q:3(i,vi),Q:4(i),Q:6(i), Exercise:5.2, Q:1(v), Q:2(iv),

Exercise:5.3, Q:1(i), Q:2(iii), Q:4(iii), Exercise:5.4, Q:3(iii), Exercise:5.5, Q:3(i), Q:5(ii)

**Home Work:** Exercise:5.1,Q:1(ii, iii, iv),Q:3(ii, iii,iv,v),Q:4(iii),Q:6(ii), Exercise:5.2,

Q:1(vi-viii),Q:3, Q:4(ii), Exercise:5.3, Q:1(iii,v), Q:2(ii), Q:4(v), Exercise:5.4, Q:5(ii),

Exercise:5.5, Q:3(ii,iii), Q:5(iii), Miscellaneous Exercise:5, Q:1,Q:2

## UNIT - 6: Basic Statistics

**Class Work:** Exercise:6.1,Q:1, Exercise:6.2, Q:3,7, Exercise:6.3, Q:5(ii)

**Home Work:** Exercise:6.1,Q:3, Exercise:6.2, Q:11,12, Exercise:6.3, Q:4, Q:7,

Miscellaneous Exercise:6,Q:1,Q:2

**UNIT - 7: Introduction to Trigonometry**

**Class Work:** Exercise#:7.1, Q:1(vii),Q:2(ii), Q:3(v), Q:4(viii), Q:5(iii), Exercise:7.2, Q:1(ii),,Exercise:7.3,Q:1(ii),Q:2(i),Q:3(iv),Q:4(vi),Q:9, Q:12(viii) Exercise:7.4, Q:7,20, Exercise:7.5:Q: 1,9,

**Home Work:** Exercise#:7.1, Q:1(ii,iii), Q:3(ii ,vi), Q:4(ii,iii,v),Q:5(vii,viii), Exercise:7.2,Q:3(i), Q:5,6,Exercise:7.3,Q:1(iii),Q:2(ii),Q:3(iii),Q:4(ii),Q:8, Q:12(i,v,xi) Exercise:7.4, Q: 10,11,16,24, Exercise:7.5,Q: 3,4,8, Miscellaneous Exercise:7,Q:1,Q:2

**UNIT - 8: Projection of a Side of a Triangle**

**Class Work:** Theorem:2, Miscellaneous Exercise:8,Q:3,5,8 (Exercises are excluded)

**Home Work:** Theorem:2, Miscellaneous Exercise:8,Q:3,5,8 (Exercises are excluded)

**UNIT - 9: Chords of a Circle**

**Class Work:** Theorem:2,4, (Exercises are excluded)

**Home Work:** Theorem:2,4, Miscellaneous Exercise:9,Q:1(v-xiv), (Exercises are excluded)

**UNIT - 10: Tangent to a Circle**

**Class Work:** Theorem:1,3, (Exercises are excluded)

**Home Work:** Theorem:1,3, Miscellaneous Exercise:10,Q:1(v-xi), (Exercises are excluded)

**UNIT - 11: Chords and Arcs:**

**Class Work:** Theorem:1,4, (Exercise is excluded)

**Home Work:** Theorem:1,4, Miscellaneous Exercise:11,Q:1, (Exercise is excluded)

**UNIT - 12: Angle in a Segment of a Circle**

**Class Work:** Theorem:1,2 (Exercise is excluded)

**Home Work:** Theorem:1,2 (Exercise is excluded)

**UNIT - 13: Practical Geometry-Circles**

**Class Work:** Exercise: 13.1,Q:1,Q:4, Exercise 13.2: Q:4,Q:5,Exercise:13.3,Q:6,9,11

**Home Work:** Exercise: 13.1,Q:1,Q:4, Exercise 13.2: Q:4,Q:5,Exercise:13.3,Q: 6,9,11, Miscellaneous Exercise:13,Q:1

# Unit-1

## [ QUADRATIC EQUATIONS]

### Define Quadratic Equation

An equation which contains the square of the unknown (variable) quantity, but no higher power is called a quadratic equation. Or an equation of the second degree.

For example:

$ax^2 + bx + c = 0$  is called Quadratic equation.

### Exercise 1.1

#### Home Work:

**Question No.1** write the following quadratic equations in the standard form and point out pure quadratic equations.

(i)  $(x+7)(x-3) = -7$

Solution:  $(x+7)(x-3) = -7$

$$x^2 - 3x + 7x - 21 = -7$$

$$x^2 + 4x - 21 + 7 = 0$$

$$x^2 + 4x - 14 = 0$$

The standard form of Quadratic equation is:

$$x^2 + 4x - 14 = 0$$

#### Class work:

**Question No.1** write the following quadratic equations in the standard form and point out pure quadratic equations.

(iii)  $\frac{x}{x+1} + \frac{x+1}{x} = 6$

Solution:

$$\frac{x}{x+1} + \frac{x+1}{x} = 6$$

Solution:  $\frac{x}{x+1} + \frac{x+1}{x} = 6$

$$\frac{x^2 + (x+1)^2}{x(x+1)} = 6$$

$$x^2 + x^2 + 1 + 2x = 6x(x+1)$$

$$2x^2 + 2x + 1 = 6x^2 + 6x$$

$$0 = 6x^2 + 6x - 2x^2 - 2x - 1$$

$$0 = 4x^2 + 4x - 1$$

$$4x^2 + 4x - 1 = 0$$

The standard form of Quadratic equation is

$$4x^2 + 4x - 1 = 0$$

(v)  $\frac{x+3}{x+4} - \frac{x-5}{x} = 1$

Solution:  $\frac{x+3}{x+4} - \frac{x-5}{x} = 1$

$$\frac{x(x+3) - (x+4)(x-5)}{x(x+4)} = 1$$

$$x^2 + 3x - (x^2 - 5x + 4x - 20) = 1x(x+4)$$

$$x^2 + 3x - (x^2 - 1x - 20) = x^2 + 4x$$

$$x^2 + 3x - x^2 + 1x + 20 = x^2 + 4x$$

$$3x + 1x + 20 = x^2 + 4x$$

$$4x + 20 = x^2 + 4x$$

$$x^2 + 4x - 4x - 20 = 0$$

$$x^2 - 20 = 0$$

$x^2 - 20 = 0$  is pure quadratic equation.

**Question No.2** Solve by factorization

Class work: (ii)  $y^2 = y(y-5)$

Solution.

$$3y^2 = y(y-5)$$

$$3y^2 = y^2 - 5y$$

$$3y^2 - y^2 + 5y = 0$$

$$2y^2 + 5y = 0$$

$$y(2y+5) = 0$$

$$y = 0 \text{ or } 2y + 5 = 0$$

$$y = 0 \text{ or } 2y = -5$$

$$y = 0 \text{ or } y = -\frac{5}{2}$$

$$\text{Solution set} = \left\{-\frac{5}{2}, 0\right\}$$

Home work:

(iv).  $x^2 - 11x = 152$

Solution.

$$x^2 - 11x = 152$$

$$x^2 - 11x - 152 = 0$$

$$x^2 - 19x + 8x - 152 = 0$$

$$x(x-19) + 8(x-19) = 0$$

$$(x-19)(x+8) = 0$$

$$(x-19) = 0 \text{ or } (x+8) = 0$$

$$x = 19 \text{ or } x = -8$$

$$\text{Solution set} = \{-8, 19\}$$

Home Work:

(v).  $\frac{x+1}{x} + \frac{x}{x+1} = \frac{25}{12}$

Solution.

$$\frac{x}{x+1} + \frac{x+1}{x} = \frac{25}{12}$$

multiplying by LCM  $12x(x+1)$  on both sides

$$12x(x+1) \times \frac{x}{x+1} + 12x(x+1) \times \frac{x+1}{x} = \frac{25}{12} \times 12x(x+1)$$

$$= 12x(x+1) \times \frac{25}{12}$$

$$12x^2 + 12(x+1)(x+1) = 25x(x+1)$$

$$12x^2 + 12(x^2 + x + x + 1) = 25x^2 + 25x$$

$$12x^2 + 12x^2 + 24x + 12 - 25x^2 - 25x = 0$$

$$-x^2 - x + 12 = 0$$

**Question No.3** Solve the following equations by completing square:

**Home Work:**

**(i).**  $7x^2 + 2x - 1 = 0$

**Solution.**

$$7x^2 + 2x - 1 = 0$$

Dividing each term by 7

$$\frac{7}{7}x^2 + \frac{2}{7}x - \frac{1}{7}$$

$$x^2 + 2(x)\left(\frac{1}{7}\right) = \frac{1}{7}$$

Adding  $\left(\frac{1}{7}\right)^2$  on both sides

$$(x)^2 + 2(x)\left(\frac{1}{7}\right) + \left(\frac{1}{7}\right)^2 = \frac{1}{7} + \left(\frac{1}{7}\right)^2$$

$$\left(x + \frac{1}{7}\right)^2 = \frac{1}{7} + \frac{1}{49}$$

$$\left(x + \frac{1}{7}\right)^2 = \frac{7+1}{49}$$

$$\left(x + \frac{1}{7}\right)^2 = \frac{8}{49}$$

Taking square root on both sides

$$\sqrt{\left(x + \frac{1}{7}\right)^2} = \pm \sqrt{\frac{8}{49}}$$

$$x + \frac{1}{7} = \pm \sqrt{\frac{4 \times 2}{49}}$$

$$x + \frac{1}{7} = \pm \frac{2\sqrt{2}}{7}$$

$$x = -\frac{1}{7} \pm \frac{2\sqrt{2}}{7}$$

$$x = \frac{-1 \pm 2\sqrt{2}}{7}$$

$$\text{Solution Set} = \left\{ \frac{-1 \pm 2\sqrt{2}}{7} \right\}$$

**(v).**  $3x^2 + 7x = 0$

**Solution.**

$$3x^2 + 7x = 0$$

Dividing each term by 3

$$\frac{3}{3}x^2 + \frac{7}{3}x = 0$$

$$x^2 + \frac{7}{3}x = 0$$

$$x^2 + 2(x)\left(\frac{7}{6}\right) = 0$$

Adding  $\left(\frac{7}{6}\right)^2$  on both sides

$$x^2 + 2(x)\left(\frac{7}{6}\right) + \left(\frac{7}{6}\right)^2 = 0 + \left(\frac{7}{6}\right)^2$$

$$\left(x + \frac{7}{6}\right)^2 = \frac{49}{36}$$

Taking square root on both sides

$$\sqrt{\left(x + \frac{7}{6}\right)^2} = \pm \sqrt{\frac{49}{36}}$$

$$\left(x + \frac{7}{6}\right) = \pm \frac{7}{6}$$

$$x = -\frac{7}{6} \pm \frac{7}{6}$$

$$x = -\frac{7}{6} + \frac{7}{6} \quad \text{or} \quad x = -\frac{7}{6} - \frac{7}{6}$$

$$x = 0 \quad \text{or} \quad x = -\frac{-7-7}{6}$$

$$x = 0 \quad \text{or} \quad x = \frac{-14}{6}$$

$$x = 0 \quad \text{or} \quad x = \frac{-7}{3}$$

$$\text{Solution Set} = \left\{ -\frac{7}{3}, 0 \right\}$$

**Home Work:**

**(ix).**  $4 - \frac{8}{3x+1} = \frac{3x^2+5}{3x+1}$

**Solution.**

$$4 - \frac{8}{3x+1} = \frac{3x^2+5}{3x+1}$$

$$4 = \frac{3x^2+5}{3x+1} + \frac{8}{3x+1}$$

$$4 = \frac{3x^2+5+8}{3x+1}$$

$$4(3x+1) = 3x^2+13$$

$$3x^2+13 = 12x+4$$

$$3x^2-12x+13-4=0$$

$$3x^2-12x+9=0$$

Dividing both sides by 3

$$\frac{3}{3}x^2 - \frac{12}{3}x + \frac{9}{3} = 0$$

$$x^2 - 4x + 3 = 0$$

$$x^2 - 4x = -3$$

$$(x)^2 - 2(x)(2) = -3$$

Adding  $(2)^2$  on both sides

$$(x)^2 - 2(x)(2) + (2)^2 = -3 + (2)^2$$

$$(x-2)^2 = -3+4$$

$$(x-2)^2 = 1$$

Taking square root on both sides

$$\sqrt{(x-2)^2} = \pm \sqrt{1}$$

$$(x-2) = \pm 1$$

$$x = 2 \pm 1$$

$$x = 2+1 \quad \text{or} \quad x = 2-1$$

$$x = 3 \quad \text{or} \quad x = 1$$

$$\text{Solution Set} = \{1, 3\}$$

Using Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Exercise 1.2**

Question No.1 Solve the following equation using quadratic formula.

**Home work:**

(i)  $2 - x^2 = 7x$

Solution

$$x^2 + 7x - 2 = 0$$

here  $a = 1$   $b = 7$  and  $c = -2$ 

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-7 \pm \sqrt{49 + 8}}{2}$$

$$x = \frac{-7 \pm \sqrt{57}}{2} \text{ ANSWER}$$

**Class work:**

iii)  $\sqrt{3}x^2 + x = 4\sqrt{3}$

solution

$$\sqrt{3}x^2 + x - 4\sqrt{3} = 0$$

here  $a = \sqrt{3}$   $b = 1$  and  $c = -4\sqrt{3}$ 

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(\sqrt{3})(-4\sqrt{3})}}{2(\sqrt{3})}$$

$$x = \frac{-1 \pm \sqrt{1 + 16(\sqrt{3})^2}}{2(\sqrt{3})}$$

$$x = \frac{-1 \pm \sqrt{1 + 48}}{2(\sqrt{3})}$$

$$x = \frac{-1 \pm \sqrt{49}}{2\sqrt{3}}$$

$$x = \frac{-1 \pm 7}{2\sqrt{3}}$$

$$x = \frac{-1 + 7}{2\sqrt{3}}, \frac{-1 - 7}{2\sqrt{3}}$$

$$x = \frac{6}{2\sqrt{3}}, -\frac{8}{2\sqrt{3}}$$

$$x = \frac{3}{\sqrt{3}}, -\frac{4}{\sqrt{3}}$$

$$x = \sqrt{3}, -\frac{4}{\sqrt{3}}$$

$$S.S = \{\sqrt{3}, -\frac{4}{\sqrt{3}}\}$$

**Home Work:**

vii)  $\frac{3}{x-6} - \frac{4}{x-5} = 1$

solution

multiplying by LCM

$$(x-5)(x-6) \times \frac{3}{x-6} - (x-5)(x-6) \times \frac{4}{x-5} = 1 \times (x-5)(x-6)$$

$$3(x-5) - 4(x-6) = (x^2 - 6x - 5x + 30)$$

$$3x - 15 - 4x + 24 = x^2 - 11x + 30$$

$$x^2 - 11x + 30 - 3x + 4x - 9 = 0$$

$$x^2 - 14x + 21 = 0$$

here  $a = 1$   $b = -14$  and  $c = 21$ 

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{14 \pm \sqrt{(-14)^2 - 4(1)(21)}}{2(1)}$$

$$x = \frac{14 \pm \sqrt{196 - 84}}{2}$$

$$x = \frac{14 \pm \sqrt{112}}{2}$$

$$x = \frac{14 \pm \sqrt{2 \times 2 \times 2 \times 2 \times 7}}{2}$$

$$x = \frac{14 \pm 4\sqrt{7}}{2}$$

$$x = \frac{2(7 \pm 2\sqrt{7})}{2}$$

$$x = 7 \pm 2\sqrt{7}$$

viii)  $\frac{x+2}{x-1} - \frac{4-x}{2x} = 2\frac{1}{3}$

$$\frac{x+2}{x-1} - \frac{4-x}{2x} = \frac{7}{3}$$

multiplying by LCM

$$6x(x-1) \times \frac{x+2}{x-1} - 6x(x-1) \times \frac{4-x}{2x} = 6x(x-1) \times \frac{7}{3}$$

$$6x(x+2) - 3(x-1)(4-x) = 14x(x-1)$$

$$6x^2 + 12x - 3(4x - x^2 - 4 + x) = 14x^2 - 14x$$

$$6x^2 + 12x - 12x + 3x^2 + 12 - 3x - 14x^2 + 14x = 0$$

$$-5x^2 + 11x + 12 = 0$$

$$5x^2 - 11x - 12 = 0$$

here  $a = 5$   $b = -11$  and  $c = -12$ 

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{11 \pm \sqrt{(-11)^2 - 4(5)(-12)}}{2(5)}$$

$$x = \frac{11 \pm \sqrt{121 + 240}}{10}$$

$$x = \frac{11 \pm \sqrt{361}}{10}$$

$$x = \frac{11 \pm 19}{10}$$

$$x = \frac{11 + 19}{10}, \frac{11 - 19}{10}$$

$$x = \frac{30}{10}, -\frac{8}{10}$$

$$x = 3, -\frac{4}{5}$$

**Equation reducible to quadratic form:**

**Type 1.**

The equation of the type  $ax^4 + bx^2 + c = 0$

Replace  $x^2 = y$  in equation  $ax^4 + bx^2 + c = 0$

**Type 2.**

The equation of the type  $ap(x) + \frac{b}{p(x)} + c$

**Type 3.**

Reciprocal equation of the type:

$$a\left(x^2 + \frac{1}{x^2}\right) + b\left(x + \frac{1}{x}\right) + c = 0 \text{ or}$$

$$ax^4 + bx^3 + cx^2 + bx + a = 0$$

An equation is said to be a reciprocal equation, if it remains unchanged, when  $x$  is replaced by  $\frac{1}{x}$

**Type 4.**

**Exponential Equation:**

In exponential equation, variable occurs in exponent.

**Type 5.**

The equation of the type:

$$(x+a)(x+b)(x+c)(x+d) = k \text{ where}$$

$$a+b = c+d$$

### Exercise 1.3

**Home Work:**

**Question No.2**

$$2x^4 = 9x^2 - 4$$

**Solution:**

$$2x^4 = 9x^2 - 4$$

$$2x^4 - 9x^2 + 4 = 0 \dots\dots(1)$$

$$\text{Let } x^2 = y \dots\dots(2)$$

Taking square on both sides

$$(x^2)^2 = y^2$$

$$x^4 = y^2$$

Put  $x^2 = y$  and  $x^4 = y^2$  in eq(1)

$$2y^2 - 9y + 4 = 0$$

$$2y^2 - 8y - 1y + 4 = 0$$

$$2y(y-4) - 1(y-4) = 0$$

$$(y-4)(2y-1) = 0$$

$$y-4=0 \quad 2y-1=0$$

$$y=4 \quad 2y=1$$

$$y=4 \quad y=\frac{1}{2}$$

$$\text{Put } y = x^2$$

$$x^2 = 4 \quad x^2 = \frac{1}{2}$$

$$\sqrt{x^2} = \pm\sqrt{4} \quad \sqrt{x^2} = \sqrt{\frac{1}{2}}$$

$$x = \pm 2 \quad x = \pm \frac{1}{\sqrt{2}}$$

$$\text{Solution set is } \left\{ \pm 2, \pm \frac{1}{\sqrt{2}} \right\}$$

**Home Work:**

**Question No.7**

$$\frac{x}{x-3} + 4\left(\frac{x-3}{x}\right) = 4$$

**Solution:**

$$\frac{x}{x-3} + 4\left(\frac{x-3}{x}\right) = 4$$

$$\text{Let } \frac{x}{x-3} = y \Rightarrow \frac{x-3}{x} = \frac{1}{y}$$

Equation become

$$y + 4\left(\frac{1}{y}\right) = 4$$

Multiplying both side by "y"

$$y^2 + 4 = 4y$$

$$y^2 - 4y + 4 = 0$$

$$y^2 - 2y - 2y + 4 = 0$$

$$y(y-2) - 2(y-2) = 0$$

$$(y-2)(y-2) = 0$$

$$y-2=0 \Rightarrow y=2$$

Put the value of y

$$\frac{x}{x-3} = 2 \Rightarrow x = 2(x-3)$$

$$x = 2x - 6 \Rightarrow 6 = 2x - x$$

$$x = 6$$

$$S.S \{6\}$$



**Home Work:****Question No.9**

$$\frac{x-a}{x+a} - \frac{x+a}{x-a} = \frac{7}{12}$$

**Solution:**

$$\text{Let } \frac{x-a}{x+a} = y \text{ or } \frac{x+a}{x-a} = \frac{1}{y}$$

$$y - \frac{1}{y} = \frac{7}{12}$$

$$12(y^2 - 1) = 7y$$

$$12y^2 - 12 = 7y \Rightarrow 12y^2 - 7y - 12 = 0$$

$$12y^2 - 16y + 9y - 12 = 0$$

$$4y(3y - 4) + 3(3y - 4)$$

$$(3y - 4)(4y + 3)$$

$$3y - 4 = 0 \quad 4y + 3 = 0$$

$$3y = 4 \quad 4y = -3$$

$$y = \frac{4}{3} \quad y = -\frac{3}{4}$$

$$\frac{x-a}{x+a} = \frac{4}{3} \quad \frac{x+a}{x-a} = -\frac{3}{4}$$

$$3(x-a) = 4(x+a)$$

$$3x - 3a = 4x + 4a$$

$$-3a - 4a = 4x - 3x$$

$$-7a = x$$

$$x = -7a$$

$$\frac{x+a}{x-a} = -\frac{3}{4}$$

$$4(x+a) = -3(x-a)$$

$$4x + 4a = -3x + 3a$$

$$4x + 3x = 3a - 4a$$

$$7x = -a$$

$$x = \frac{a}{7}$$

$$S.S \left\{ -7a, \frac{a}{7} \right\}$$

**Class Work+ Home Work:****Question No.10**

$$x^2 - 2x^3 - 2x^2 + 2x + 1 = 0$$

**Solution:**

$$x^4 - 2x^3 - 2x^2 + 2x + 1 = 0$$

Dividing both sides by " $x^2$ "

$$\frac{x^4}{x^2} - \frac{2x^3}{x^2} - \frac{2x^2}{x^2} + \frac{2x}{x^2} + \frac{1}{x^2} = \frac{0}{x^2}$$

$$x^2 - 2x - 2 + \frac{2}{x} + \frac{1}{x^2} = 0$$

$$\left(x^2 + \frac{1}{x^2}\right) - 2x + \frac{2}{x} - 2 = 0$$

$$\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) - 2 = 0 \rightarrow (1)$$

$$\text{Let } x - \frac{1}{x} = y \rightarrow (2)$$

Taking square on both sides

$$\left(x - \frac{1}{x}\right)^2 = (y)^2$$

$$x^2 + \frac{1}{x^2} - 2 = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 + 2$$

Putting values in eq.(1)

$$y^2 + 2 - 2(y) - 2 = 0$$

$$y^2 - 2y = 0$$

$$y(y - 2) = 0$$

$$y = 0 \quad y - 2 = 0$$

$$y = 2$$

Put  $y = x - \frac{1}{x}$  from eq.2

$$x - \frac{1}{x} = 0 \quad x - \frac{1}{x} = 2$$

$$\frac{x^2 - 1}{x} = 0 \quad \frac{x^2 - 1}{x} = 2$$

$$x^2 - 1 = 0 \quad x^2 - 1 = 2x$$

$$x^2 = 1 \quad x^2 - 2x - 1 = 0$$

$$\sqrt{x^2} = \pm\sqrt{1} \quad 1x^2 - 2x - 1 = 0$$

$$x = \pm 1$$

Solving  $1x^2 - 2x - 1 = 0$  by quadratic formula

$$a = 1 \quad b = -2 \quad c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{+2 \pm \sqrt{4+4}}{2}$$

$$x = \frac{+2 \pm \sqrt{8}}{2}$$

$$x = \frac{+2 \pm \sqrt{4 \times 2}}{2}$$

$$x = \frac{+2 \pm 2\sqrt{2}}{2}$$

$$x = \frac{2(1 \pm \sqrt{2})}{2}$$

$$x = 1 \pm \sqrt{2}$$

$$S.S \{ \pm 1, 1 \pm \sqrt{2} \}$$

## Class Work:

### Question No.12

$$4 \cdot 2^{2x+1} - 9 \cdot 2^x + 1 = 0$$

#### Solution:

$$4 \cdot 2^{2x+1} - 9 \cdot 2^x + 1 = 0$$

$$4 \cdot 2^{2x} \cdot 2^1 - 9 \cdot 2^x + 1 = 0$$

$$8 \cdot (2^x)^2 - 9 \cdot 2^x + 1 = 0$$

$$\text{Let } 2^x = y \quad (2^x)^2 = y^2$$

$$8y^2 - 9y + 1 = 0$$

$$8y^2 - 8y - y + 1 = 0$$

$$8y(y-1) - 1(y-1) = 0$$

$$(8y-1)(y-1) = 0$$

$$8y-1=0 \quad y-1=0$$

$$8y=1 \quad y=1$$

$$y = \frac{1}{8} \quad y = 1$$

Put the value of y in above equation

$$2^x = y \quad 2^x = 1$$

$$2^x = \frac{1}{8} \quad 2^x = 2^0$$

$$2^x = \frac{1}{2^3} \quad 2^x = 2^0$$

$$2^x = 2^{-3} \quad x = 0$$

$$x = -3$$

$$S.S \{ -3, 0 \}$$

## Home Work:

### Question No.14

$$2^x + 64 \cdot 2^{-x} - 20 = 0$$

#### Solution:

$$2^x + 64 \cdot 2^{-x} - 20 = 0$$

$$2^x + \frac{64}{2^x} - 20 = 0 \quad \dots\dots(1)$$

$$\text{Let } 2^x = y \quad \dots\dots(2)$$

$$\text{Put } 2^x = y \text{ in eq.(1)}$$

$$y + \frac{64}{y} - 20 = 0$$

Multiply both sides by "y"

$$y^2 + 64 - 20y = 0$$

$$y^2 - 20y + 64 = 0$$

$$y^2 - 16y - 4y + 64 = 0$$

$$y(y-16) - 4(y-16) = 0$$

$$(y-16)(y-4) = 0$$

$$y-16=0 \quad y-4=0$$

$$y=16 \quad y=4$$

$$\text{Put } y = 2^x \text{ from eq.(2)}$$

$$2^x = 16 \quad 2^x = 4$$

$$2^x = 2^4 \quad 2^x = 2^2$$

$$x = 4 \quad x = 2$$

$$S.S \{ 2, 4 \}$$

## Radicle Equation:

An equation involving expression under the radicle sign is called a radical equation.

$$e.g \sqrt{x+3} = x+1$$

### 1. Equation of the type:

$$\sqrt{ax+b} = cx+d$$

### 2. Equation of the type $\sqrt{x+a} + \sqrt{x+b} = \sqrt{x+c}$

### 3. Equation of the type:

$$\sqrt{x^2+px+m} + \sqrt{x^2+px+n} = q$$

## Exercise 1.4

### Class Work:

Solve the following equations.

#### Question no1:

$$2x+5 = \sqrt{7x+16}$$

$$\text{Solution: } 2x+5 = \sqrt{7x+16} \quad \dots\dots(1)$$

Taking square on both side

$$(2x+5)^2 = (\sqrt{7x+16})^2$$

$$(2x)^2 + (5)^2 + 2(2x)(5) = 7x+16$$

$$4x^2 + 25 + 20x = 7x+16$$

$$4x^2 + 20x - 7x + 25 - 16 = 0$$

$$4x^2 + 13x + 9 = 0$$

$$2x+5=\sqrt{7x+16}$$

$$4x^2+4x+9x+9=0$$

$$4x(x+1)+9(x+1)=0$$

$$(x+1)(4x+9)=0$$

$$x+1=0 \quad , \quad 4x+9=0$$

$$x=-1 \quad , \quad 4x=-9$$

$$x=-1 \quad , \quad x=\frac{-9}{4}$$

Checking :

put  $x=-1$  in the equation

$$2x+5=\sqrt{7x+16}$$

$$2(-1)+5=\sqrt{7(-1)+16}$$

$$-2+5=\sqrt{-7+16}$$

$$3=\sqrt{9}$$

$$3=3 \text{ which is true}$$

$$x=\frac{-9}{4} \text{ in the equation}$$

$$2x+5=\sqrt{7x+16}$$

$$2\left(\frac{-9}{4}\right)+5=\sqrt{7\left(\frac{-9}{4}\right)+16}$$

$$\frac{-18}{4}+5=\sqrt{\frac{-63}{4}+16}$$

$$\frac{-18+20}{4}=\sqrt{\frac{-63+64}{4}}$$

$$\frac{2}{4}=\sqrt{\frac{1}{4}} \quad , \quad \frac{1}{2}=\frac{1}{2} \text{ which is true}$$

$$\text{Solution set } \left\{-1, \frac{-9}{4}\right\}$$

**Home Work:**

**3)  $4x = \sqrt{13x+14} - 3$**

**Solution:**

$$4x+3=\sqrt{13x+14}$$

taking square on B.S

$$(4x+3)^2 = (\sqrt{13x+14})^2$$

$$(4x)^2 + 3^2 + 2(4x)(3) = 13x + 14$$

$$16x^2 + 9 + 24x - 13x - 14 = 0$$

$$16x^2 + 11x - 5 = 0$$

$$16x^2 + 16x - 5x - 5 = 0$$

$$16x(x+1) - 5(x+1) = 0$$

$$(x+1)(16x-5) = 0$$

$$x+1=0; \quad 16x-5=0$$

$$x=-1; \quad x=\frac{5}{16}$$

$$S.S = \left\{-1, \frac{5}{16}\right\}$$

**Home work:**

**8)  $\sqrt{4a+x} - \sqrt{a-x} = \sqrt{a}$**

**solution:**

taking square on B.S

$$(\sqrt{4a+x} - \sqrt{a-x})^2 = (\sqrt{a})^2$$

$$(\sqrt{4a+x})^2 + (\sqrt{a-x})^2 - 2(\sqrt{4a+x})(\sqrt{a-x}) = a$$

$$4a+x+a-x-2(\sqrt{(4a-x)(a-x)}) = a$$

$$5a-a=2(\sqrt{4a^2-4ax-ax+x^2})$$

$$4a=2(\sqrt{x^2-5ax+4a^2})$$

$$2a=(\sqrt{x^2-5ax+4a^2})$$

Taking square on B.S

$$(2a)^2 = (\sqrt{x^2-5ax+4a^2})^2$$

$$4a^2 = (x^2-5ax+4a^2)$$

$$x^2-5ax+4a^2-4a^2=0$$

$$x^2-5ax=0$$

$$x(x-5a)=0$$

$$x=0; \quad x-5a=0$$

$$x=0; \quad x=5a$$

$$S.S = \{0, 5a\}$$

**Class work:**

**9)  $\sqrt{x^2+x+1} - \sqrt{x^2+x-1} = 1$**

**Solution:**

$$\sqrt{x^2+x+1} = 1 + \sqrt{x^2+x-1}$$

Taking square on B.S

$$(\sqrt{x^2+x+1})^2 = (1 + \sqrt{x^2+x-1})^2$$

$$x^2+x+1 = 1^2 + (\sqrt{x^2+x-1})^2$$

$$+ 2(1)(\sqrt{x^2+x-1})$$

$$x^2+x+1 = 1+x^2+x-1+2\sqrt{x^2+x-1}$$

$$x^2+x+1-1-x^2-x+1 = 2\sqrt{x^2+x-1}$$

$$1 = 2\sqrt{x^2+x-1}$$

Taking square on both sides

$$(1)^2 = (2\sqrt{x^2+x-1})^2$$

$$1 = 4(x^2+x-1)$$

$$1 = 4x^2+4x-4$$

$$4x^2+4x-4-1=0$$

$$4x^2+4x-5=0$$

Solve by quadratic formula

$$\text{here } a=4 \text{ } b=4 \text{ and } c=-5$$

$$\text{as } x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Putting values

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(4)(-5)}}{2(4)}$$

$$x = \frac{-4 \pm \sqrt{16 + 80}}{8}$$

$$x = \frac{-4 \pm \sqrt{96}}{8}$$

$$x = \frac{-4 \pm \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 3}}{8}$$

$$x = \frac{-4 \pm \sqrt{2^2 \times 2^2 \times 6}}{8}$$

$$x = \frac{-4 \pm 4\sqrt{6}}{8}$$

$$x = \frac{4(-1 \pm \sqrt{6})}{8}$$

$$x = \frac{-1 \pm \sqrt{6}}{2}$$

## Miscellaneous Exercise -3

### (Exercise +Additional)

#### Question1

##### Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

1. Standard form of quadratic equation is:

- (a)  $bx + c = 0, b \neq 0$
- (b)  $ax^2 + bx + c = 0, a \neq 0$
- (c)  $ax^2 = bx, a \neq 0$
- (d)  $ax^2 = 0, a \neq 0$

2. The number of terms in a standard quadratic equation  $ax^2+bx+c=0$  is:

- (a) 1
- (b) 2
- (c) 3
- (d) 4

3. The number of methods to solve a quadratic equation is:

- (a) 1
- (b) 2
- (c) 3
- (d) 4

4. The quadratic formula is:

- (a)  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- (b)  $\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$
- (c)  $\frac{-b \pm \sqrt{b^2 + 4ac}}{2a}$
- (d)  $\frac{b \pm \sqrt{b^2 + 4ac}}{2a}$

5. Two linear factors of  $x^2 - 15x + 56$  are:

- (a)  $(x-7)$  and  $(x+8)$
- (b)  $(x+7)$  and  $(x-8)$
- (c)  $(x-7)$  and  $(x-8)$
- (d)  $(x+7)$  and  $(x+8)$

6. An equation, which remains unchanged when  $x$  is replaced by  $\frac{1}{x}$  is called a/an:

10301087

- (a) Exponential equation

(b) Reciprocal equation

(c) Radical equation

(d) None of these

7. An equation of the type  $3^x + 3^{2-x} + 6 = 0$  is a/an:

(a) Exponential equation

(b) Reciprocal equation

(c) Radical equation

(d) None of these

8. The solution set of equation  $4x^2 - 16 = 0$  is:

(a)  $\{\pm 4\}$

(b)  $\{4\}$

(c)  $\{\pm 2\}$

(d)  $\pm 2$

9. An equation of the form (Board 2014)  $10301090$   $2x^4 - 3x^3 + 7x^2 - 3x + 2 = 0$  is called a/an

(a) Reciprocal equation

(b) Radical equation

(c) Exponential equation

(d) None of these

10. The solution set of  $25x^2 - 1 = 0$  is 10301091

(a)  $\left\{\pm \frac{1}{5}\right\}$

(b)  $\left\{-\frac{1}{5}\right\}$

(c)  $\left\{+\frac{1}{5}\right\}$

(d) None of these

11. An equation of the form  $2^{2x} - 3 \cdot 2^x + 5 = 0$  is called a / an \_\_\_\_\_ equation.

(a) Exponential

(b) Radical

(c) Reciprocal

(d) None of these

12. The solution set of the equation  $x^2 - 9 = 0$  is:

(a)  $\{\pm 3\}$

(b)  $\{3\}$

(c)  $\{-3\}$

(d)  $\{9\}$

13. An equation of type  $x^4 + x^3 + x^2 + x + 1 = 0$  is called a/ an .....equation.

(a) Radical

(b) Reciprocal

(c) Exponential

(d) None of these

14. Solve the equation  $5^{1+x} + 5^{1-x} = 26$

(a)  $\{1\}$

(b)  $\{\pm 1\}$  10301095

(c)  $\{2\}$

(d)  $\{\pm 2\}$

15. The solution set of equation  $2+9x=5x^2$  is:

(a)  $\left\{\frac{-1}{5}, 2\right\}$

(b)  $\left\{\frac{+1}{5}, 2\right\}$

(c)  $\left\{\frac{1}{5}, -2\right\}$

(d)  $\left\{\frac{-1}{5}, -2\right\}$

16. The solution set of equation  $5x^2 = 30x$  is:

(a)  $\{5, 30\}$

(b)  $\{0, 6\}$

(c)  $\{0, -6\}$

(d)  $\{5, 0\}$

17. The solution set of equation

$x^2 - x - 2 = 0$  is:

- (a)  $\{2, 1\}$  (b)  $\{-2, 1\}$   
 (c)  $\{2, -1\}$  (d)  $\{-2, -1\}$

18. The solution set of equation

$x^2 - 16 = 0$  is:

- (a)  $\{\pm 4\}$  (b)  $\{+4\}$   
 (c)  $\{-4\}$  (d) None of these

19. The solution set of equation

$x^2 - 7x + 6 = 0$  is:

- (a)  $\{1, 6\}$  (b)  $\{-1, -6\}$   
 (c)  $\{-1, 6\}$  (d)  $\{1, -6\}$

20. The solution set of equation

$3x^2 + 4x = 5$  is:

- (a)  $\left\{ \frac{-2 \pm \sqrt{19}}{3} \right\}$   
 (b)  $\left\{ \frac{2 \pm \sqrt{19}}{3} \right\}$   
 (c)  $\left\{ \frac{4 \pm \sqrt{19}}{3} \right\}$   
 (d) None of these

21. If
- $b=0$
- in a quadratic equation

$ax^2 + bx + c = 0$ , then it is called:

- (a) Pure quadratic equation  
 (b) Linear equation  
 (c) Quadratic equation  
 (d) Exponential equation

22. Sentences involving the sign.....

between two algebraic expressions are called equations.

- (a)  $<$  (b)  $\geq$   
 (c)  $=$  (d)  $< \text{ or } >$

The standard form of the quadratic equation is  $ax^2 + bx + c = 0$  where  $a, b, c$  are:

- (a) Irrational numbers  
 (b) Rational numbers  
 (c) Real numbers  
 (d) Whole numbers

24. If
- $a=0$
- , in
- $ax^2 + bx + c = 0$
- , then it reduces to:

- (a) pure quadratic equation  
 (b) linear equation  
 (c) quadratic equations  
 (d) exponential equation

25. How many linear factors a quadratic equation has?

- (a) 1 (b) 2  
 (c) 3 (d) 4

26. What is the degree of quadratic equation?

- (c) 3 (d) 4

27. The number of roots of a quadratic equation is:

- (a) 1 (b) 2  
 (c) 3 (d) 4

28. Cancellation of
- $x$
- on both sides of
- $5x^2 = 30x$
- means:

- (a) the loss of one root  
 (b) no loss of any root  
 (c) gain of one root  
 (d) undefined solution

29. What should be done to make the co-efficient of
- $x^2$
- equal to 1, in
- $7x^2 + 2x - 1 = 0$
- ?

- (a) multiply the equation by 7  
 (b) divide the equation by 7  
 (c) add 7 in both sides  
 (d) subtract 7 from both sides

30. What should be done to make the co-efficient of
- $x^2$
- equal to 1 in
- $3x^2 + 7x = 0$
- ?

- (a) multiply the equation by  $\frac{1}{3}$   
 (b) divide the equation by  $\frac{1}{3}$   
 (c) add  $\frac{1}{3}$  in both sides  
 (d) subtract  $\frac{1}{3}$  from both sides

(Answer key)

1.	b	2.	c	3.	c	4.	a	5.	c
6.	b	7.	a	8.	c	9.	a	10.	a
11.	a	12.	a	13.	b	14.	b	15.	a
16.	b	17.	c	18.	a	19.	a	20.	a
21.	a	22.	c	23.	c	24.	b	25.	b
26.	b	27.	b	28.	a	29.	b	30.	a

### Question No.2

Question No.2 Write short answers of the following questions.

i solve  $x^2 + 2x - 2 = 0$

Solution:

comparing with equation  $ax^2 + bx + c = 0$

Here

$$a = 1, b = 2, c = -2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-2)}}{2(1)} = \frac{-2 \pm \sqrt{4 + 8}}{2}$$

$$= \frac{-2 \pm \sqrt{12}}{2} = \frac{-2 \pm \sqrt{4 \times 3}}{2} = \frac{-2 \pm 2\sqrt{3}}{2}$$

$$= \frac{2(-1 \pm \sqrt{3})}{2}$$

$$x = -1 \pm \sqrt{3}$$

$$S.S = \{-1 \pm \sqrt{3}\}$$

ii Solve by factorization  $5x^2 = 15x$

Solution:

$$5x^2 = 15x$$

$$5x^2 - 15x = 0$$

$$5x(x - 3) = 0$$

$$5x = 0, \quad x - 3 = 0$$

$$x = 0, \quad x = 3$$

$$S.S = \{0, 3\}$$

iii Write in standard form  $\frac{1}{x+4} + \frac{1}{x-1} = 3$

Solution:

$$\frac{1}{x+4} + \frac{1}{x-1} = 3$$

$$\frac{1}{x+4} + \frac{1}{x-1} = 3$$

$$\frac{x-4+x+4}{(x+4)(x-4)} = 3$$

$$2x = 3(x+4)(x-4)$$

$$2x = 3(x^2 - 16)$$

$$2x = 3x^2 - 48$$

$$3x^2 - 2x - 48 = 0$$

iv Write the names of the method for solving a quadratic equations.

Solution: There are three methods.

- Factorization
- Completing square
- Quadratic formula

v Solve  $\left(2x - \frac{1}{2}\right)^2 = \frac{9}{4}$

Solution:

$$\left(2x - \frac{1}{2}\right)^2 = \frac{9}{4}$$

Squaring both sides

$$2x - \frac{1}{2} = \pm \sqrt{\frac{9}{4}}$$

$$2x - \frac{1}{2} = \pm \frac{3}{2}$$

$$2x - \frac{1}{2} = \frac{3}{2}$$

$$2x = \frac{3}{2} + \frac{1}{2}$$

$$2x = \frac{3+1}{2} = \frac{4}{2} = 2$$

$$2x = 2$$

$$x = \frac{2}{2}$$

$$x = 1$$

$$2x - \frac{1}{2} = -\frac{3}{2}$$

$$2x = -\frac{3}{2} + \frac{1}{2}$$

$$2x = \frac{-3+1}{2} = \frac{-2}{2}$$

$$2x = -1$$

$$x = 1 \quad \left| \quad x = -\frac{1}{2} \right.$$

$$S.S = \left\{1, -\frac{1}{2}\right\}$$

vi Solve  $\sqrt{3x+18} = x$

Solution:  $\sqrt{3x+18} = x$

Squaring both sides by

$$(\sqrt{3x+18})^2 = (x)^2$$

$$3x+18 = x^2$$

$$x^2 - 3x + 18 = 0$$

$$x^2 - 6x + 3x - 18 = 0$$

$$x(x-6) + 3(x-6) = 0$$

$$(x-6)(x+3) = 0$$

$$x-6 = 0; \quad x+3 = 0$$

$$x = 6; \quad x = -3$$

$$S.S = \{-3, 6\}$$

vii Define Quadratic Equation

An equation which contains the square of the unknown (variable) quantity, but no higher power is called a quadratic equation. Or an equation of the second degree.

viii Define Reciprocal Equation.

An equation is said to be reciprocal equation. If it remains unchanged, when  $x$  is replaced by  $\frac{1}{x}$

ix Define Exponential Equation

In exponential equations, variable occurs in exponent.

x Define radical Equation.

An equation involving expression under the radical sign is called a radical equations.

For example

$$\sqrt{x+3} = x+1$$

# Unit-2

## THEORY OF QUADRATIC EQUATIONS

### NATURE OF THE ROOTS OF A QUADRATIC EQUATION:

#### 2.1.1

Discriminant ( $b^2 - 4ac$ ) of the quadratic expression  $ax^2 + bx + c$

We know that two roots of the equation

$$ax^2 + bx + c = 0, a \neq 0 \rightarrow (i)$$

$$\text{Are } \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

The nature of these roots depends on the value of the expression  $b^2 - 4ac$  which is called the discriminant of the quadratic equation(i) or the quadratic expression  $ax^2 + bx + c$

#### 2.1.2

To find the discriminant of a given quadratic equation.

Example1:

Find the discriminant of the following equations.

$$2x^2 - 7x + 1 = 0$$

Solution:

$$\begin{aligned} 2x^2 - 7x + 1 &= 0 \\ \text{here } a &= 2, b = -7, c = 1 \\ \text{Disc.} &= b^2 - 4ac \\ &= (-7)^2 - 4(2)(1) \\ &= 49 - 8 = 41 \end{aligned}$$

#### 2.1.3

The roots of the quadratic equation  $ax^2 + bx + c$ ,

$$(a \neq 0) \text{ are } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

And its discernment is  $b^2 - 4ac$ .

When  $a, b$  and  $c$  are rational numbers.

- If  $b^2 - 4ac > 0$  and is a perfect square, then the roots are rational (real) and unequal.
- If  $b^2 - 4ac > 0$  and is not a perfect square, then the roots are irrational (real) and unequal.
- If  $b^2 - 4ac = 0$  then the roots are rational (real) and equal.
- If  $b^2 - 4ac < 0$  then the roots are imaginary (complex conjugates)

### Exercise 2.1

Find the discriminant of the following given Quadratic Equation.

Class-Work

Question No.1

$$(ii) 6x^2 - 8x + 3 = 0$$

Solution:

$$6x^2 - 8x + 3 = 0$$

Compare it with

$$ax^2 + bx + c = 0$$

$$\Rightarrow a = 6, b = -8, c = 3$$

$$\text{Disc.} = b^2 - 4ac$$

$$= (-8)^2 - 4(6)(3)$$

$$= 64 - 72$$

$$= -8$$

Home Work:

$$(i) 4x^2 - 7x - 2 = 0$$

Solution:

$$4x^2 - 7x - 2 = 0$$

Compare it with

$$ax^2 + bx + c = 0$$

$$\Rightarrow a = 4, b = -7, c = -2$$

$$\text{Disc.} = b^2 - 4ac$$

$$= (-7)^2 - 4(4)(-2)$$

$$= 49 + 32$$

$$= 81$$

Question No.2

Find the nature of the roots of the follow given quadratic and verify the result by solving equations:

Classwork:

$$(i) x^2 + 23x + 120 = 0$$

Solution:

$$x^2 + 23x + 120 = 0$$

Compare it with

$$ax^2 + bx + c = 0$$

$$\Rightarrow a = 1, b = -23, c = 120$$

$$\text{Disc.} = b^2 - 4ac$$

$$= (-23)^2 - 4(1)(120)$$

$$= 529 - 480$$

$$= 49$$

$$= (7)^2 > 0$$

As the disc.is possible and is perfect square. Therefore the roots are rational (real) and unequal, verification by solving the equation.

$$x^2 - 23x + 120 = 0$$

$$x^2 - 15x - 8x + 120 = 0$$

$$x(x - 15) - 8(x - 15) = 0$$

$$(x - 15)(x - 8) = 0$$

$$\text{Either } x - 8 = 0 \text{ or } x - 15 = 0$$

$$x = 8 \quad x = 15$$

Thus, the roots are rational (real) and unequal.

Home Work:

$$iv) 3x^2 + 7x - 13 = 0$$

Solution:

$$3x^2 + 7x - 13 = 0$$

Compare it with

$$ax^2 + bx + c = 0$$

$$\Rightarrow a = 3, b = 7, c = -13$$

$$\text{Disc.} = b^2 - 4ac$$

$$= (7)^2 - 4(3)(-13)$$



$$= 49 + 156$$

$$= 205 > 0$$

As the Disc. Is positive and is not perfect square.

Therefore the roots are irrational (real) and unequal.

Verification by solving the equation.

$$3x^2 + 7x - 13 = 0$$

Using quadratic formula

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-7 \pm \sqrt{(7)^2 - 4(3)(-13)}}{2(3)} \\ &= \frac{-7 \pm \sqrt{49 + 156}}{6} \\ &= \frac{-7 \pm \sqrt{205}}{6} \end{aligned}$$

Thus, the roots are irrational (real) and unequal.

### Question No.3 Class work:

For what value of A, the expression

$k^2x^2 + 2(k+1)x + 4$  is square.

Solution:

$$k^2x^2 + 2(k+1)x + 4 = 0$$

Compare it with

$$ax^2 + bx + c = 0$$

$$\Rightarrow a = k^2, b = 2(k+1), c = 4$$

$$\text{Disc.} = b^2 - 4ac$$

$$= (2(k+1))^2 - 4(k^2)(4)$$

$$= 4(k^2 + 2k + 1) - 16k^2$$

$$= 4k^2 + 8k + 4 - 16k^2$$

$$= -12k^2 + 8k + 4 = 0$$

As the disc. Of the given expression is a perfect square.

Therefore the roots are rational and equal.

So,  $\text{Disc.} = 0$

$$-12k^2 + 8k + 4 = 0$$

$$-(12k^2 + 8k + 4) = 0$$

$$\Rightarrow 12k^2 + 8k + 4 = 0$$

$$12k^2 + 12k + 4k + 4 = 0$$

$$12k(k+1) + 4(k+1) = 0$$

$$(12k+4)(k+1) = 0$$

$$\text{Either } 12k+4 = 0 \text{ or } k+1 = 0$$

$$12k = -4 \text{ or } k = -1$$

$$k = -\frac{4}{12}$$

$$k = -\frac{1}{3}$$

### Home Work:

#### Question No.4

Find the value of k, if the roots of the following equations are equal.

$$(iii) (3k+2)x^2 - 5(k+1)x + (2k+3) = 0$$

Solution:

$$(3k+2)x^2 - 5(k+1)x + (2k+3) = 0$$

$$\Rightarrow a = 3k+2, b = -5(k+1), c = (2k+3)$$

As the roots are equal, So

$$\text{Disc.} = 0$$

$$b^2 - 4ac = 0$$

$$[-5(k+1)]^2 - 4(3k+2)(2k+3) = 0$$

$$25(k^2 + 2k + 1) - 4(6k^2 + 13k + 6) = 0$$

$$25k^2 + 50k + 25 - 24k^2 - 52k - 24 = 0$$

$$k^2 - 2k + 1 = 0$$

$$(k-1)^2 = 0$$

### Home Work:

#### Question No.10

Show that the roots of the equation.

$$(b-c)x^2 + (c-a)x + (a-b)^2 = 0$$

Solution:

$$\Rightarrow a = (b-c), b = (c-a), c = (a-b)$$

$$\text{Disc.} = b^2 - 4ac$$

$$= (c-a)^2 - 4(b-c)(a-b)$$

$$= (c^2 - 2ac + a^2) - 4(ab - b^2 - ac + bc)$$

$$= a^2 + 4b^2 + c^2 - 4ab - 4bc + 2ac$$

$$= (a)^2 + (-2b)^2 + (c)^2 + 2(a)(-2b) + 2(-2b)(c) + 2(a)(c)$$

$$= (a-2b+c)^2 > 0$$

hence the roots of the equation are real.

### 2.2 Cube Roots Of Unity And Their Properties

#### 2.2.1 The cube roots of unity:

let a number  $x$  be the cube root of unity.

$$\text{i.e. } x = (1)^{\frac{1}{3}}$$

$$\text{or } x^3 = 1$$

$$x^3 - 1 = 0$$

$$(x^3) - (1)^3 = 0$$

$$(x-1)(x^2+x+1) = 0$$

$$[\text{using } a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$$

$$\text{Either } x-1 = 0 \text{ or } x^2+x+1 = 0$$

$$\Rightarrow x = 1 \text{ or } x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

$\therefore$  three cube roots of unity are

$$1, \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}$$

#### 2.2.2 Recognize complex cube roots of unity as $\omega$ and $\omega^2$

$\omega$  pronoun as omega

#### 2.2.3 Properties of cube roots of unity

(a) Prove that each of the complex cube roots of unity is the square of the other.

Proof:

The complex cube roots of unity are

$$\frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}$$

We prove that



$$\left(\frac{-1+i\sqrt{3}}{2}\right)^2 = \frac{-1-\sqrt{3}}{2}$$

$$\begin{aligned}\left(\frac{-1+i\sqrt{3}}{2}\right)^2 &= \frac{1+(-3)-2\sqrt{-3}}{4} \\ &= \frac{-2-2\sqrt{3}}{4} \\ &= \frac{2(-1-\sqrt{3})}{4} \\ &= \frac{-1-\sqrt{3}}{2}\end{aligned}$$

And

The complex cube roots of unity are

$$\frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$$

We prove that

$$\left(\frac{-1-i\sqrt{3}}{2}\right)^2 = \frac{-1+\sqrt{3}}{2}$$

$$\begin{aligned}\left(\frac{-1-i\sqrt{3}}{2}\right)^2 &= \frac{1+(-3)+2\sqrt{-3}}{4} \\ &= \frac{-2+2\sqrt{3}}{4} \\ &= \frac{2(-1+\sqrt{3})}{4} \\ &= \frac{-1+\sqrt{3}}{2}\end{aligned}$$

Thus, each of the complex cube root of unity is square of the other, that is

$$\text{if } \omega = \frac{-1+i\sqrt{3}}{2} \text{ then } \omega^2 = \frac{-1-\sqrt{3}}{2}$$

$$\text{And if } \omega = \frac{-1-\sqrt{3}}{2} \text{ then } \omega^2 = \frac{-1+\sqrt{3}}{2}$$

(b) Prove that the product of three cube roots of unity is one.

**Proof:**

Three cube roots of unity are

$$1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$$

The product of cube roots of unity

$$\begin{aligned}1 \times \frac{-1+i\sqrt{3}}{2} \times \frac{-1-i\sqrt{3}}{2} \\ = \frac{(-1)^2 - (\sqrt{-3})^2}{4} = \frac{1-(-3)}{4} = \frac{1+3}{4} = \frac{4}{4} = 1 \\ \text{i.e. } (1)(\omega)(\omega^2) = 1 \text{ or } \omega^3 = 1\end{aligned}$$

**Remember that**

$$\omega^4 = \omega^3 \cdot \omega = 1 \cdot \omega = \omega$$

(c) Prove that each complex cube root of unity is reciprocal of the other.

**Proof:**

We know that  $\omega^3 = 1 \Rightarrow \omega\omega^2 = 1$  so.

$$\omega = \frac{1}{\omega^2} \text{ or } \omega^2 = \frac{1}{\omega}$$

(d) Prove that the sum of all the cube roots of unity is zero.

$$\text{i.e. } 1 + \omega + \omega^2 = 0$$

**Proof:**

Three cube roots of unity are

$$1, \frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$$

The product of cube roots of unity

$$\begin{aligned}&= 1 + \frac{-1+i\sqrt{3}}{2} + \frac{-1-i\sqrt{3}}{2} \\ &= \frac{2-1+i\sqrt{3}-1-i\sqrt{3}}{2} \\ &= \frac{0}{2} = 0\end{aligned}$$

$$\text{Thus, } 1 + \omega + \omega^2 = 0$$

**Remember that**

$$(i) \quad 1 + \omega^2 = -\omega$$

$$(ii) \quad 1 + \omega = -\omega^2$$

$$(iii) \quad \omega + \omega^2 = -1$$

## 2.2.4 Uses of properties of cube roots of unity to solve appropriate problems:

We can reduce the higher powers of  $\omega$  into 1

$\omega$  and  $\omega^2$

e.g

$$\omega^7 = (\omega^3)^2 \cdot \omega = (1)^2 \cdot \omega = \omega$$

$$\omega^{23} = (\omega^3)^7 \cdot \omega^2 = (1)^7 \cdot \omega^2 = \omega^2$$

$$\omega^{63} = (\omega^3)^{21} = (1)^{21} = 1$$

$$\omega^{-5} = \frac{1}{\omega^5} = \frac{1}{\omega^2 \cdot \omega^3} = \frac{1}{\omega^2} = \frac{\omega^3}{\omega^2} = \omega$$

$$\omega^{-16} = \frac{1}{\omega^{16}} = \frac{1}{(\omega^3)^5 \cdot \omega} = \frac{1}{1^5 \cdot \omega} = \frac{\omega^3}{\omega} = \omega^2$$

$$\omega^{-27} = \frac{1}{\omega^{27}} = \frac{1}{(\omega^3)^9} = \frac{1}{(1)^9} = 1$$

### Exercise 2.2

#### Class Work:

Q.1 Find the cube root of  $-1, 8, -27, 64$ .

(i) Cube roots of  $-1$

$$\text{Solution: Let } x = (-1)^{\frac{1}{3}}$$

$$x^3 = -1$$

$$x^3 + 1 = 0$$

$$x^3 + (1)^3 = 0$$

$$\therefore (a^3 + b^3) = (a+b)(a^2 - ab + b^2)$$

$$(x+1)(x^2 - (x)(1) + 1^2) = 0$$

$$(x+1)(x^2 - x + 1) = 0$$

$$x+1=0 \quad (x^2-x+1)=0$$

$$x=-1 \quad x^2-x+1=0$$

Then we solve  $x^2-x+1=0$  by formula

$$ax^2+bx+c=0$$

$$a=1, b=-1, c=1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2 \times 1}$$

$$x = \frac{1 \pm \sqrt{1-4}}{2}$$

$$x = \frac{1 \pm \sqrt{-3}}{2}$$

Cube roots of  $-1$

$$1, \frac{1-\sqrt{-3}}{2}, \frac{1+\sqrt{-3}}{2}$$

$$1, -\left(\frac{-1+\sqrt{-3}}{2}\right), -1\left(\frac{-1-\sqrt{-3}}{2}\right)$$

$$1, -\omega \quad 1, -\omega^2$$

$$x = -1\omega \quad x = -1(\omega)^2$$

$$x = -\omega \quad x = -\omega^2$$

Cube roots of  $-1$  are  $-1, -\omega, -\omega^2$

(ii) Cube roots of 8

$$\text{Solution : Let } x = (8)^{\frac{1}{3}}$$

$$x^3 = 8$$

$$x^3 - 8 = 0$$

$$x^3 - 2^3 = 0$$

$$\therefore (a^3 - b^3) = (a-b)(a^2 + ab + b^2)$$

$$(x-2)(x^2 + (x)(2) + 2^2) = 0$$

$$(x-2)(x^2 + 2x + 2^2) = 0$$

$$(x-2)(x^2 + 2x + 4) = 0$$

$$x-2=0 \quad x^2 + 2x + 4 = 0$$

$$x=2 \quad x^2 + 2x + 4 = 0$$

Then we solve  $x^2 + 2x + 4 = 0$  by formula

$$a=1, b=2, c=4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4-16}}{2}$$

$$x = \frac{-2 \pm \sqrt{-12}}{2}$$

$$x = \frac{-2 \pm \sqrt{4 \times (-3)}}{2}$$

$$x = \frac{-2 \pm \sqrt{4} \sqrt{-3}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{-3}}{2}$$

$$x = \frac{2(-1 \pm \sqrt{-3})}{2}$$

$$x = 2\left(\frac{-1 \pm \sqrt{-3}}{2}\right)$$

$$x = 2\left(\frac{-1 + \sqrt{-3}}{2}\right) \quad x = 2\left(\frac{-1 - \sqrt{-3}}{2}\right)$$

$$x = 2\omega \quad x = 2\omega^2$$

Cube roots of 8 are  $2, 2\omega, 2\omega^2$

(iii) Cube roots of  $-27$

$$\text{Solution : Let } x = (-27)^{\frac{1}{3}}$$

$$x^3 = -27$$

$$x^3 + 27 = 0$$

$$x^3 + 3^3 = 0$$

$$\therefore (a^3 + b^3) = (a+b)(a^2 - ab + b^2)$$

$$(x+3)(x^2 - (x)(3) + 3^2)$$

$$(x+3)(x^2 - 3x + 9) = 0$$

$$x+3=0 \quad x^2 - 3x + 9 = 0$$

$$x=-3 \quad x^2 - 3x + 9 = 0$$

Then we solve  $x^2 - 3x + 9 = 0$  by formula

$$a=1, b=-3, c=9$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{9-36}}{2}$$

$$x = \frac{3 \pm \sqrt{-27}}{2}$$

$$x = \frac{3 \pm \sqrt{9 \times (-3)}}{2}$$

$$x = \frac{3 \pm \sqrt{9} \sqrt{-3}}{2}$$

$$x = \frac{3 \pm 3\sqrt{-3}}{2}$$

$$x = \frac{-3(-1 \pm \sqrt{-3})}{2}$$

$$x = -3 \left( \frac{-1 \pm \sqrt{-3}}{2} \right)$$

$$x = -3 \left( \frac{-1 + \sqrt{-3}}{2} \right) \quad x = -3 \left( \frac{-1 - \sqrt{-3}}{2} \right)$$

$$x = -3\omega \quad x = -3\omega^2$$

Cube roots of  $-27$  are  $-3, -3\omega, -3\omega^2$

(ii) Cube roots of 64

Solution: Let  $x = (64)^{\frac{1}{3}}$

$$x^3 = 64$$

$$x^3 - 64 = 0$$

$$x^3 - 4^3 = 0$$

$$\therefore (a^3 - b^3) = (a-b)(a^2 + ab + b^2)$$

$$(x-4)(x^2 + (x)(4) + 4^2) = 0$$

$$(x-4)(x^2 + 4x + 16) = 0$$

$$x-4=0 \quad x^2 + 4x + 16 = 0$$

$$x=4 \quad x^2 + 4x + 16 = 0$$

Then we solve  $x^2 + 4x + 16 = 0$  by formula

$$a=1, \quad b=4, \quad c=16$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(16)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 - 64}}{2}$$

$$x = \frac{-4 \pm \sqrt{-48}}{2}$$

$$x = \frac{-4 \pm \sqrt{16 \times (-3)}}{2}$$

$$x = \frac{-4 \pm 4\sqrt{-3}}{2}$$

$$x = \frac{4(-1 \pm \sqrt{-3})}{2}$$

$$x = 4 \left( \frac{-1 \pm \sqrt{-3}}{2} \right)$$

$$x = 4 \left( \frac{-1 + \sqrt{-3}}{2} \right) \quad x = 4 \left( \frac{-1 - \sqrt{-3}}{2} \right)$$

$$x = 4\omega \quad x = 4\omega^2$$

Cube roots of 64 are  $4, 4\omega, 4\omega^2$

**Question No.2 Class Work:**

Q.2 Evaluate

$$(i) (1 - \omega - \omega^2)^7$$

$$\text{Solution: } (1 - \omega - \omega^2)^7$$

$$= [1 - (\omega - \omega^2)]^7$$

$$= [1 - (-1)]^7$$

$$= (1+1)^7$$

$$= 2^7 = 128$$

**Home Work:**

$$(ii) (1 - 3\omega - 3\omega^2)^5$$

$$\text{Solution: } (1 - 3\omega - 3\omega^2)^5$$

$$= [1 - 3(\omega + \omega^2)]^5$$

$$= [1 - 3(-1)]^5$$

$$= (1+3)^5$$

$$= 4^5 = 1024$$

$$(iii) (9 + 4\omega + 4\omega^2)^3$$

$$\text{Solution: } (9 + 4\omega + 4\omega^2)^3$$

$$= [9 + 4(\omega + \omega^2)]^3$$

$$= [9 + 4(-1)]^3 \quad (\because \omega + \omega^2 = -1)$$

$$= (9-4)^3$$

$$= 5^3 = 125$$

$$(iv) (2 + 2\omega + 2\omega^2)(3 - 3\omega + 3\omega^2)$$

$$\begin{aligned} \text{Solution: } & (2 + 2\omega + 2\omega^2)(3 - 3\omega + 3\omega^2) \\ &= (2(1 + \omega) - 2\omega^2)(3 + 3\omega^2 - 3\omega) \\ &= [2(1 + \omega) - 2\omega^2][3(1 + \omega^2) - 3\omega] \\ &\quad \{ \because 1 + \omega + \omega^2 = 0 \} \\ &\quad \{ 1 + \omega = -\omega^2 \quad 1 + \omega^2 = -\omega \} \\ &= [2(-\omega)^2 - 2\omega^2][3(-\omega) - 3\omega] \\ &= (-2\omega^2 - 2\omega^2)(-3\omega - 3\omega) \\ &= (-4\omega^2)(-6\omega) \\ &= 24\omega^3 = 24(1) = 24 \end{aligned}$$

$$(v) (-1 + \sqrt{-3})^6 + (-1 - \sqrt{-3})^6$$

$$\text{Solution: } (-1 + \sqrt{-3})^6 + (-1 - \sqrt{-3})^6$$

$$\begin{aligned} \text{As } \frac{-1 + \sqrt{-3}}{2} = \omega \quad \frac{-1 - \sqrt{-3}}{2} = \omega^2 \\ -1 + \sqrt{-3} = 2\omega \quad -1 - \sqrt{-3} = 2\omega^2 \end{aligned}$$

Then it becomes

$$\begin{aligned} &= (2\omega)^6 + (2\omega^2)^6 \\ &= 2^6 \omega^6 + 2^6 \omega^{12} \\ &= 2^6 [(\omega^3)^2 + (\omega^3)^4] \\ &= 2^6 [(1)^2 + (1)^4] \\ &= 64(1 + 1) \\ &= 64(2) = 128 \end{aligned}$$

$$(vi) \left( \frac{-1 + \sqrt{-3}}{2} \right)^9 + \left( \frac{-1 - \sqrt{-3}}{2} \right)^9$$

$$\text{Solution: } \left( \frac{-1 + \sqrt{-3}}{2} \right)^9 + \left( \frac{-1 - \sqrt{-3}}{2} \right)^9$$

$$\text{As } \frac{-1 + \sqrt{-3}}{2} = \omega \quad \frac{-1 - \sqrt{-3}}{2} = \omega^2$$

Then it becomes

$$\begin{aligned} &= (\omega)^9 + (\omega^2)^9 \\ &= \omega^9 + \omega^{18} \\ &= (\omega^3)^3 + (\omega^3)^6 \\ &= (1)^3 + (1)^6 \end{aligned}$$

$$(vii) \omega^{37} + \omega^{38} - 5$$

$$\begin{aligned} \text{Solution: } & \omega^{37} + \omega^{38} - 5 \\ &= \omega^{36} \omega + \omega^{36} \omega^2 - 5 \\ &= (\omega^3)^{12} \omega + (\omega^3)^{12} \omega^2 - 5 \\ &= (1)^{12} \omega + (1)^{12} \omega^2 - 5 \\ &= 1\omega + 1\omega^2 - 5 \\ &= (\omega + \omega^2) - 5 \\ &= (-1) - 5 \\ &= -1 - 5 = -6 \end{aligned}$$

**Home Work:**

$$(viii) \omega^{-13} + \omega^{-17}$$

$$\begin{aligned} \text{Solution: } & \omega^{-13} + \omega^{-17} \\ &= \frac{1}{\omega^{13}} + \frac{1}{\omega^{17}} \\ &= \frac{1}{\omega^{12} \omega} + \frac{1}{\omega^{15} \omega^2} \\ &= \frac{1}{(\omega^3)^4 \omega} + \frac{1}{(\omega^3)^5 \omega^2} \\ &= \frac{1}{(1)^4 \omega} + \frac{1}{(1)^5 \omega^2} \\ &= \frac{1}{\omega} + \frac{1}{\omega^2} \\ &= \frac{\omega^2 + \omega}{(\omega)(\omega^2)} = \frac{-1}{\omega^3} \\ &= -\frac{1}{1} = -1 \end{aligned}$$

**Q.3 Prove that Class Work:**

$$x^3 + y^3 = (x + y)(x + \omega y)(x + \omega^2 y)$$

*Solution: Let,*

$$\begin{aligned} R.H.S &= (x + y)(x + \omega y)(x + \omega^2 y) \\ &= (x + y)(x^2 + \omega^2 xy + \omega xy + \omega^3 y^2) \\ &= (x + y)[x^2 + (\omega^2 + \omega)xy + \omega^3 y^2] \\ &\quad \because 1 + \omega + \omega^2 = 0, \quad \omega + \omega^2 = -1, \quad \omega^3 = 1 \\ &= (x + y)[x^2 + (-1)xy + 1y^2] \\ &= (x + y)(x^2 - xy + y^2) \end{aligned}$$

$$\begin{aligned} \text{As } (a^3 + b^3) &= (a + b)(a^2 - ab + b^2) \text{ so,} \\ &= x^3 + y^3 = L.H.S \end{aligned}$$

**Question.4 Prove that Class Work:**

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x + \omega y + \omega^2 z)(x + \omega^2 y + \omega z)$$

*Solution: Let R.H.S*

$$\begin{aligned} &= (x + y + z)(x + \omega y + \omega^2 z)(x + \omega^2 y + \omega z) \\ &= (x + y + z)(x^2 + \omega^2 xy + \omega xz + \omega yx + \omega^3 y^2 + \omega^2 yz + \omega^2 xz + \omega^4 zy + \omega^3 z^2) \\ &= (x + y + z)[(x^2 + \omega^3 y^2 + \omega^3 z^2 + (\omega^2 + \omega)xy + (\omega^2 + \omega^4)yz + (\omega + \omega^2)zx)] \\ &= (x + y + z)[x^2 + 1y^2 + 1z^2 + (-1)xy + (\omega^2 + \omega^3\omega)yz + (-1)zx] \\ &= (x + y + z)[x^2 + 1y^2 + 1z^2 + (-1)xy + (\omega^2 + 1\omega)yz + (-1)zx] \\ &= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) \\ &= x^3 + y^3 + z^3 - 3xyz = L.H.S \\ \text{Using formula:} \\ &= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc \end{aligned}$$

**3 Roots and co-efficient of a quadratic equation:****2.3.1) Relationship between roots and co-efficient of a quadratic equation:**

**Remember that**

$$S = -\frac{b}{a} = -\frac{\text{co-efficient of } x}{\text{Co-efficient of } x^2}$$

$$P = \frac{c}{a} = \frac{\text{constant term}}{\text{Co-efficient of } x^2}$$

**2.3.2) The sum and the product of the roots of a given quadratic equation without solving it.**

**Example:**

**Without solving find the sum and product of the roots of the equations.**

**Solution:**

$$(a) 3x^2 - 5x + 7 = 0$$

$$\text{Sum of roots} = \alpha + \beta = -\frac{b}{a} = -\left(\frac{-5}{3}\right) = \frac{5}{3}$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a} = \frac{7}{3}$$

**2.3.3) To find unknown values involved in a given quadratic equation.**

- (a) Sum of the roots is equal to a multiple of the product of the roots.
- (b) Sum of the squares of the roots is equal to a given number.
- (c) Two roots differ by a given number.
- (d) Roots satisfy a given relation.
- (e) Both sum and product of the roots are equal to a given number.

**Exercise 2.3**

**Question.1 Without solving, find the sum and product of the roots of following quadratic equations:**

**Class Work:**

$$(i) x^2 - 5x + 3 = 0$$

$$\text{Solution: } x^2 - 5x + 3 = 0$$

$$ax^2 + bx + c = 0$$

$$a = 1, b = -5, c = 3$$

$$\text{Sum of roots} = S = \frac{-b}{a} = -\left(\frac{-5}{1}\right) = 5$$

$$\text{Product of roots} = P = \frac{c}{a} = \frac{3}{1} = 3$$

**Home Work:**

$$(vi) 7x^2 - 5mx + 9n = 0$$

**Solution:**

$$(l + m)x^2 + (m + n)x + n - l = 0$$

$$ax^2 + bx + c = 0$$

$$a = (l + m), b = (m + n), c = n - l$$

$$\text{Sum of roots} = S = \frac{-b}{a} = \frac{-(m + n)}{l + m}$$

$$\text{Product of roots} = P = \frac{c}{a} = \frac{n - l}{l + m}$$

**Home Work:**

$$(vi) 7x^2 - 5mx + 9n = 0$$

$$\text{Solution: } 7x^2 - 5mx + 9n = 0$$

$$ax^2 + bx + c = 0$$

$$a = 7, b = -5m, c = 9n$$

$$\text{Sum of roots} = S = \frac{-b}{a} = -\left(\frac{-5m}{7}\right) = \frac{5m}{7}$$

$$\text{Product of roots} = P = \frac{c}{a} = \frac{9n}{7}$$

**Question.2 Find the value of k if.**

**Class work + Home Work.**

(ii) Sum of the roots of the equation

$$x^2 + (3k - 7)x + 5k = 0 \text{ is } \frac{3}{2} \text{ times the}$$

products of roots.

$$\text{Solution: } 1x^2 + (3k - 7)x + 5k = 0$$

$$ax^2 + bx + c = 0$$

$$a = 1, b = (3k - 7), c = 5k$$

Let  $\alpha, \beta$  be the roots of equation

$$\text{Sum of roots} = \alpha + \beta = \frac{-b}{a} = \frac{-(3k - 7)}{1} = -3k + 7$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a} = \frac{5k}{1} = 5k$$

According to given conditions :

$$S = \frac{3}{2}P$$

$$-3k + 7 = \frac{3}{2}(5k)$$

$$2(-3k + 7) = 3(5k)$$

$$-6k + 14 = 15k$$

$$14 = 15k + 6k$$

$$14 = 21k$$

$$k = \frac{14}{21} = \frac{2}{3}$$

$$k = \frac{2}{3}$$

### Class Work:

#### Question No.5

(ii) Find m if the roots of the equation

$$x^2 - 7x + 3m - 5 = 0 \text{ satisfy the relation } 3\alpha - 2\beta = 4.$$

**Solution:**  $x^2 - 7x + 3m - 5 = 0$

Let  $\alpha, \beta$  be the roots of given equation

$$1x^2 + 7x + 3m - 5 = 0$$

$$ax^2 + bx + c = 0$$

$$a = 1, b = 7, c = 3m - 5$$

$$\text{Sum of roots} = S = \alpha + \beta = \frac{-b}{a}$$

$$= \frac{-7}{1} = -7 \dots(i)$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a} = \frac{3m-5}{1}$$

$$= 3m - 5 \dots(ii)$$

$$\text{Since } 3\alpha + 2\beta = 4 \dots(iii)$$

From equation (i)

$$\alpha + \beta = -7$$

$$\beta = -7 - \alpha$$

Put  $\beta = -7 - \alpha$  in equation (iii)

$$3\alpha - 2\beta = 4$$

$$3\alpha - 2(-7 - \alpha) = 4$$

$$3\alpha + 14 + 2\alpha = 4$$

$$5\alpha + 14 = 4$$

$$5\alpha = 4 - 14$$

$$5\alpha = -10$$

$$\alpha = \frac{-10}{5}$$

$$\alpha = -2$$

Put  $\alpha = -2$  in equation (i)

$$\alpha + \beta = -7$$

$$-2 + \beta = -7$$

$$\beta = -7 + 2 \Rightarrow \beta = -5$$

Put  $\alpha = -2$  and  $\beta = -5$  in equation (ii)

$$\alpha\beta = 3m - 5$$

$$(-2)(-5) = 3m - 5$$

$$10 = 3m - 5$$

$$10 + 5 = 3m$$

$$15 = 3m$$

$$\frac{15}{3} = m \Rightarrow m = 5$$

### Home Work:

Q.6 Find m if sum and product of the roots of the following equations is equal to given number  $\lambda$ .

(i)  $(2m+3)x^2 + (7m-5)x + (3m-10) = 0$

**Solution:**  $(2m+3)x^2 + (7m-5)x + (3m-10) = 0$

$$ax^2 + bx + c = 0$$

$$a = (2m+3), b = (7m-5), c = (3m-10)$$

Let  $\alpha, \beta$  are the roots of the equation, then

$$\text{Sum of roots} = S = \alpha + \beta = \frac{-b}{a}$$

$$= -\frac{(7m-5)}{2m+3} = \frac{5-7m}{2m+3}$$

$$\text{Product of roots} = P = \frac{c}{a} = \frac{3m-10}{2m+3}$$

As given that

$$\alpha + \beta = \lambda \dots(i) \quad \alpha\beta = \lambda \dots(ii)$$

From (i) and (ii)

$$\alpha + \beta = \alpha\beta$$

$$\frac{5-7m}{2m+3} = \frac{3m-10}{2m+3}$$

$$5-7m = \frac{3m-10}{2m+3} \times 2m+3$$

$$5-7m = 3m-10$$

$$5+10 = 3m+7m$$

$$15 = 10m$$

$$\frac{15}{10} = m \Rightarrow m = \frac{3}{2}$$

**Exercise 2.4 Exclude from Syllabus****2.5 Formation Of A Quadratic Equation.**

if  $\alpha$  and  $\beta$  are the roots of the quadratic equation.

Let  $x = \alpha$  and  $x = \beta$

i.e  $x - \alpha = 0$ ,  $x - \beta = 0$

And  $(x - \alpha)(x - \beta) = 0$

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Which is required quadratic equation in the standard form.

2.5.1) Find a quadratic equation from given roots and established the formula

$$x^2 - (\text{sum of the roots})x + \text{product of the roots} = 0$$

let  $\alpha, \beta$  be the roots of the quadratic equation

$$ax^2 + bx + c = 0, a \neq 0 \rightarrow (i)$$

Then  $\alpha + \beta = -\frac{b}{a}$  and  $\alpha\beta = \frac{c}{a}$

Rewrite eq. (i) as

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 - \left(-\frac{b}{a}\right)x + \frac{c}{a} = 0$$

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Or  $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$

That is  $x^2 - Sx + P = 0$  where  $S = \alpha + \beta$

And  $P = \text{product of the roots} = \alpha\beta$

2.5.2) From quadratic equation whose roots are of the type.

(i)  $2\alpha + 1, 2\beta + 1$

(ii)  $\alpha^2, \beta^2$

(iii)  $\frac{1}{\alpha}, \frac{1}{\beta}$

(iv)  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

(v)  $(\alpha + \beta), \frac{1}{\alpha} + \frac{1}{\beta}$

where  $\alpha, \beta$  are the roots of a given quadratic equation.

**Exercise 2.5****Class Work:**

**Question No.1** Write the quadratic equation having following roots.

a)  $-1, -7$

**Solution:**

Since -1 and -7 are the roots of the required quadratic equation, therefore

$$\text{Sum of roots} = S = (-1) + (-7) = -1 - 7 = -8$$

$$\text{product of roots} = P = -1 \times -7 = 7$$

As  $x^2 - Sx + P = 0$  so the required equation is

$$x^2 + 8x + 7 = 0$$

**Home Work:**

b)  $(1 + i, 1 - i)$

**Solution:**

Since  $1 + i$  and  $1 - i$  are the roots of the required quadratic equation, therefore

$$\text{Sum of roots} = S = 1 + i + 1 - i = 2$$

$$\text{product of roots} = P = (1 + i) \times (1 - i)$$

$$= P = (1)^2 - (i)^2$$

$$P = 1 - (-1)$$

$$P = 1 + 1 = 2$$

As  $x^2 - Sx + P = 0$  so the required equation is

$$x^2 - 2x + 2 = 0$$

**Home Work:**

c)  $3 + \sqrt{2}, 3 - \sqrt{2}$

**Solution:**

Since  $3 + \sqrt{2}$  and  $3 - \sqrt{2}$  are the roots of the required quadratic equation, therefore

$$\text{Sum of roots} = S = 3 + \sqrt{2} + 3 - \sqrt{2} = 6$$

$$\text{product of roots} = P = (3 + \sqrt{2})(3 - \sqrt{2})$$

$$P = (3)^2 - (\sqrt{2})^2$$

$$P = 9 - 2$$

$$P = 7$$

As  $x^2 - Sx + P = 0$  so the required equation is

$$x^2 - 6x + 7 = 0$$

**Question No.2** if  $\alpha, \beta$  are the roots of the equation  $x^2 - 3x + 6 = 0$  from equation whose roots are

**Solution:**

As  $\alpha, \beta$  are the roots of the equation

$$x^2 - 3x + 6 = 0$$

$$a = 1, b = -3, c = 6$$

Therefore

$$\alpha + \beta = -\frac{b}{a} = -\frac{-3}{1} = 3$$

$$\Rightarrow \alpha + \beta = 3$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6$$

$$\alpha\beta = 6$$

**Class Work:**

(b)  $\alpha^2, \beta^2$

**Solution:**

As  $\alpha, \beta$  are the roots of the equation  $x^2 - 3x + 6 = 0$

$$a = 1, b = -3, c = 6$$

Therefore

$$\alpha + \beta = -\frac{b}{a} = \frac{-3(-3)}{1} = 3$$

$$\alpha + \beta = 3$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6$$

$$\Rightarrow \alpha\beta = 6$$

$$\text{Sum of the roots} = S = \alpha^2 + \beta^2$$

$$S = (\alpha + \beta)^2 - 2\alpha\beta$$

$$S = (3)^2 - 2(6)$$

$$S = 9 - 12 = -3$$

$$S = -3$$

$$\text{Product of roots} = P = \alpha^2\beta^2$$

$$P = (\alpha\beta)^2$$

$$P = (6)^2 = 36$$

$$P = 36$$

Using  $x^2 - Sx + P = 0$ , we have

$$x^2 + 3x + 36 = 0$$

**Home Work:**

(d)  $\frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha}$

Solution:

As  $\alpha, \beta$  are the roots of the equation

$$x^2 - 3x + 6 = 0$$

$$a = 1, b = -3, c = 6$$

Therefore

$$\alpha + \beta = -\frac{b}{a} = -\frac{-3}{1} = 3$$

$$\Rightarrow \alpha + \beta = 3$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6$$

$$\alpha\beta = 6$$

Sum of the roots  $S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$$S = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$

$$S = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$S = \frac{(3)^2 - 2(6)}{6}$$

$$S = \frac{9 - 12}{6}$$

$$S = -\frac{3}{6}$$

$$S = -\frac{1}{2}$$

Product of roots  $= P = \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right) = 1$

using  $x^2 - Sx + P = 0$  we have  $x^2 + \frac{1}{2}x + 1 = 0$

we have

$$2x^2 + x + 2 = 0$$

**Home Work:**

(e)  $\alpha + \beta, \frac{1}{\alpha} + \frac{1}{\beta}$

Solution:

As  $\alpha, \beta$  are the roots of the equation

$$x^2 - 3x + 6 = 0$$

$$a = 1, b = -3, c = 6$$

Therefore

$$\alpha + \beta = -\frac{b}{a} = -\frac{-3}{1} = 3$$

$$\Rightarrow \alpha + \beta = 3$$

$$\alpha\beta = \frac{c}{a} = \frac{6}{1} = 6$$

$$\alpha\beta = 6$$

Sum of roots  $= S = (\alpha + \beta) + \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$

$$S = (\alpha + \beta) + \left(\frac{\beta + \alpha}{\alpha\beta}\right)$$

$$S = 3 + \frac{3}{6}$$

$$S = 3 + \frac{1}{2}$$

$$S = \frac{6 + 1}{2}$$

$$S = \frac{7}{2}$$

$$S = \frac{7}{2}$$

Product of roots  $P = (\alpha + \beta) \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$

$$(\alpha + \beta) \times \left(\frac{\beta + \alpha}{\alpha\beta}\right)$$

$$= 3 \left(\frac{3}{6}\right)$$

$$P = \frac{3}{2}$$

Using  $x^2 - Sx + P = 0$  we have

$$x^2 - \frac{7}{2}x + \frac{3}{2} = 0$$

Multiplying both sides by 2 we have

$$2x^2 - 7x + 3 = 0$$

**Class Work:****Question No.3**if  $\alpha, \beta$  are the roots of the equation $x^2 + px + q = 0$  from equation whose roots are**Solution:**Since  $\alpha, \beta$  are the roots of the equation

$$x^2 + px + q = 0$$

$$ax^2 + bx + c = 0$$

By comparing the coefficients of these equations, we have

$$a = 1, b = p, c = q$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{p}{1} = -p$$

$$\alpha + \beta = -p$$

$$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q$$

$$\alpha\beta = q$$

(b)  $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$

Solution:

Since  $\alpha, \beta$  are the roots of the equation

$$x^2 + px + q = 0$$

$$ax^2 + bx + c = 0$$

By comparing the coefficients of these equations, we have

$$a = 1, b = p, c = q$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{p}{1} = -p$$

$$\alpha + \beta = -p$$

$$\alpha\beta = \frac{c}{a} = \frac{q}{1} = q$$

$$\alpha\beta = q$$

Sum of roots  $= S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

$$S = \frac{\alpha^2 + \beta^2}{\alpha\beta}$$



$$S = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$S = \frac{(-P)^2 - 2(q)}{q}$$

$$S = \frac{p^2 - 2q}{q}$$

$$\text{Product of roots} = P = \left(\frac{\alpha}{\beta}\right)\left(\frac{\beta}{\alpha}\right) = 1$$

Using  $x^2 - Sx + P = 0$  we have

$$x^2 - Sx + P = 0 \text{ we have}$$

$$x^2 - \left(\frac{p^2 - 2q}{q}\right)x + 1 = 0$$

Multiplying by q

$$qx^2 - (p^2 - 2q)x + q = 0$$

### Synthetic Division:

Synthetic division is the process of finding the quotient and remainder, when a polynomial is divided by a linear polynomial. In fact synthetic division is simply a shortcut of long division method.

### Exercise 2.6 (Class Work)

#### Question No.1

Uses synthetic division to find the quotient and the remainder, when

(i)  $(x^2 + 7x - 1) \div (x + 1)$

Solution:

$$P(x) = x^2 + 7x - 1$$

$$x + 1 = x - (-1)$$

$$\Rightarrow a = -1$$

1	7	-1
-1	↓	
1	6	-7

$$\text{Quotient} = Q(x) = x + 6$$

$$\text{Remainder} = -7$$

(ii)  $(4x^3 - 5x + 15) \div (x - 3)$

Solution:

$$P(x) = 4x^3 - 5x + 15 \div (x + 3)$$

$$(x + 3) = x - (-3) \Rightarrow a = -3$$

4	0	-5	15
-3	↓	-12	36
1	-12	31	-78

$$\text{Quotient} = 4x^2 - 12x + 31$$

$$\text{Remainder} = -78$$

(iii)  $(x^3 + x^2 - 3x + 2) \div (x - 2)$

Solution:

$$P(x) = (x^3 + x^2 - 3x + 2)$$

$$x - 2 = x - (2) \Rightarrow a = 2$$

1	1	-3	2
2	↓	2	6

1	3	3	8
---	---	---	---

$$\text{Quotient} = x^2 + 3x + 3$$

$$\text{Remainder} = 8$$

#### Question NO.2

Find the value of h using synthetic division, if

(i) 3 is the zero of the polynomial  $2x^3 - 3hx^2 + 9$

Solution:

$$P(x) = 2x^3 - 3hx^2 + 9 \text{ and its root is } 3$$

2	-3h	0	9
3	↓	6	3(6 - 3h)
1	(6 - 3h)	3(6 - 3h)	9 + 9(6 - 3h)

$$\text{Quotient} = Q(x) = 2x^2 + (6 - 3h)x + 3(6 - 3h)$$

$$\text{Remainder} = 9 + 9(6 - 3h)$$

$$9 + 9(6 - 3h) = 0$$

$$9 + 9(6 - 3h) = 0$$

$$9 + 54 - 27h = 0$$

$$63 - 27h = 0$$

$$63 - 27h = 0$$

$$-27h = -63$$

$$h = \frac{63}{27}$$

$$h = \frac{7}{3}$$

(ii) 1 is the zero of the polynomial  $2x^3 - 2hx^2 + 11$

Solution:

$$P(x) = 2x^3 - 2hx^2 + 11 \text{ and its root is } 1$$

1	-2h	0	11
1	↓	1	(1 - 2h)
1	(1 - 2h)	(1 - 2h)	11 + (1 - 2h)

$$\text{Quotient} = Q(x) = x^2 + (1 - 2h)x + (1 - 2h)$$

$$\text{Remainder} = 11 + (1 - 2h)$$

$$11 + (1 - 2h) = 0$$

$$11 + 1 - 2h = 0$$

$$12 - 2h = 0$$

$$-2h = -12$$

$$h = 6$$

(iii) -1 is the zero of the polynomial  $2x^3 + 5hx - 23$

Solution:

$$P(x) = 2x^3 + 5hx - 23 \text{ and its root is } -1$$

2	0	5h	-23
-1	↓	-2	-(5h + 2)
2	-2	(5h + 2)	-23 - (5h + 2)

$$\text{Quotient} = Q(x) = 2x^2 - 2x + (5h + 2)$$

$$\begin{aligned}
 \text{Remainder} &= -23 - (5h + 2) \\
 -23 - (5h + 2) &= 0 \\
 -23 - 5h - 2 &= 0 \\
 -23 - 2 - 5h &= 0 \\
 -25 - 5h &= 0 \\
 -5h &= 25 \\
 h &= -5
 \end{aligned}$$

**Question No.5 Solve by using synthetic division.**

(i) 1 and 3 are the roots of the equation

$$x^4 - 10x^2 + 9 = 0$$

Solution:

$$x^4 - 10x^2 + 9 = 0$$

$$P(x) = x^4 - 10x^2 + 9 = 0$$

	1	0	-10	0	9
1	↓	1	1	-9	-9
	1	1	-9	-9	0
3	↓	3	12	9	
	1	4	3	0	

$$\text{Quotient} = Q(x) = x^2 + 4x + 3$$

$$\text{Remainder} = 0$$

the depressed equation is  $x^2 + 4x + 3 = 0$

$$x^2 + 4x + 3 = 0$$

$$x^2 + 3x + x + 3 = 0$$

$$x(x + 3) + 1(x + 3) = 0$$

$$(x + 1)(x + 3) = 0$$

$$(x + 3) = 0 \quad (x + 1) = 0$$

$$x = -3 \quad x = -1$$

hence 1, 3, -1, -3 are the roots of the given equation

(ii)

**3 and -4 are the roots of the equation**

$$x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$$

Solution:

$$x^4 + 2x^3 - 13x^2 - 14x + 24 = 0$$

$$P(x) = x^4 + 2x^3 - 13x^2 - 14x + 24$$

	1	2	-13	-14	24
3	↓	3	15	6	-24
	1	5	2	-8	0
-4	↓	-4	-4	8	
	1	1	-2	0	

$$\text{Quotient} = Q(x) = x^2 + x - 2$$

$$\text{Remainder} = 0$$

The depressed equation is  $x^2 + x - 2 = 0$

$$x^2 + x - 2 = 0$$

$$x^2 + 2x - x - 2 = 0$$

$$x(x + 2) - 1(x + 2) = 0$$

$$(x - 1)(x + 2) = 0$$

$$x - 1 = 0 \quad x + 2 = 0$$

$$x = 1 \quad x = -2$$

Hence 3, -4, 1, -2 are the roots of the given equation.

### Simultaneous equation:

An system of equation having a common solution is called a **system of simultaneous equations**.

The set of all ordered pairs  $(x, y)$  which satisfy the system of equations is called the **solution set of the system**.

### Exercise 2.7 (Home Work)

Solve the following simultaneous equations.

#### Question No.2

$$3x - 2y = 1$$

$$x^2 + xy - y^2 = 1$$

$$\text{Solution: } 3x - 2y = 1 \quad \dots (i)$$

$$x^2 + xy - y^2 = 1 \quad \dots (ii)$$

From eq. (i)

$$3x = 1 + 2y$$

$$x = \frac{1 + 2y}{3} \quad \dots (iii)$$

Put it in eq. (ii)

$$\left(\frac{1 + 2y}{3}\right)^2 + \left(\frac{1 + 2y}{3}\right)y - y^2 = 1$$

$$\frac{1 + 4y^2 + 4y}{9} + \frac{y + 2y^2}{3} - y^2 = 1$$

Multiplying by '9' on both sides

$$\frac{9(1 + 4y^2 + 4y)}{9} + \frac{9(y + 2y^2)}{3} - 9(y^2) = 1 \times 9$$

$$1 + 4y^2 + 4y + 3y + 6y^2 - 9y^2 = 9$$

$$y^2 + 7y - 8 = 0$$

$$y^2 + 8y - y - 8 = 0$$

$$y(y + 8) - 1(y + 8) = 0$$

$$(y + 8)(y - 1) = 0$$

$$y + 8 = 0 \quad \text{or} \quad y - 1 = 0$$

$$y = -8 \quad \text{or} \quad y = 1$$

Putting these values in eq. (iii)

$$y = -8 \quad y = 1$$

$$y = -8 \quad y = 1$$

$$x = \frac{1 + 2y}{3} \quad x = \frac{1 + 2y}{3}$$

$$x = \frac{1 + 2(-8)}{3} \quad x = \frac{1 + 2(1)}{3}$$

$$x = \frac{1 - 16}{3} \quad x = \frac{1 + 2}{3}$$

$$x = \frac{-15}{3} = \boxed{-5} \quad x = \frac{3}{3} = \boxed{1}$$

Solution set =  $\{(-5, -8), (1, 1)\}$

### Question No.5

$$x^2 + (y-1)^2 = 10$$

$$x^2 + y^2 + 4x = 1$$

**Solution:**  $x^2 + (y-1)^2 = 10$  .....(i)

$$x^2 + y^2 + 4x = 1 \quad \text{.....(ii)}$$

Subtracting eq. (ii) from (i)

$$x^2 + y^2 + 1 - 2y = 10$$

$$\pm x^2 \pm y^2 \quad \pm 4x = 1$$

$$1 - 2y - 4x = 9$$

$$-4x - 2y = 9 - 1$$

$$-4x - 2y = 8$$

$$-2(2x + y) = 8$$

$$2x + y = \frac{8}{-2}$$

$$2x + y = -4$$

$$y = -4 - 2x \quad \text{.....(iii)}$$

Put in eq. (ii)

$$x^2 + (-4 - 2x)^2 + 4x = 1$$

$$x^2 + [-(4 + 2x)]^2 + 4x = 1$$

$$x^2 + [16 + 4x^2 + 16x] + 4x = 1$$

$$5x^2 + 20x + 16 - 1 = 0$$

$$5x^2 + 20x + 15 = 0$$

$$5(x^2 + 4x + 3) = 0$$

$$x^2 + 4x + 3 = 0 \quad (\because 5 \neq 0)$$

$$x^2 + 3x + x + 3 = 0$$

$$x(x + 3) + 1(x + 3) = 0$$

$$(x + 3)(x + 1) = 0$$

$$x + 3 = 0 \quad \text{or} \quad x + 1 = 0$$

$$x = -3 \quad \text{or} \quad x = -1$$

Putting the values of x in eq. (iii)

$$x = -3 \quad x = -1$$

$$y = -4 - 2x \quad y = -4 - 2x$$

$$y = -4 - 2(-3) \quad y = -4 - 2(-1)$$

$$y = -4 + 6 \quad y = -4 + 2$$

$$\boxed{y = 2} \quad \boxed{y = -2}$$

Solution set is

$$\{(-3, 2), (-1, -2)\}$$

### Question No.10

$$x^2 + 2y^2 = 3$$

$$x^2 + 4xy - 5y^2 = 0$$

**Solution:**  $x^2 + 2y^2 = 3$  .....(i)

$$x^2 + 4xy - 5y^2 = 0 \quad \text{.....(ii)}$$

Factorizing eq. (ii)

$$x^2 + 4xy - 5y^2 = 0$$

$$x^2 + 5xy - xy - 5y^2 = 0$$

$$x(x + 5y) - y(x + 5y) = 0$$

$$(x + 5y)(x - y) = 0$$

$$x + 5y = 0 \quad x - y = 0$$

$$x = -5y \quad \text{.....(iii)} \quad x = y \quad \text{.....(iv)}$$

Putting value of x in eq. (i)

$$x = -5y \quad x = y$$

$$(-5y)^2 + 2y^2 = 3 \quad (y)^2 + 2y^2 = 3$$

$$25y^2 + 2y^2 = 3 \quad y^2 + 2y^2 = 3$$

$$27y^2 = 3 \quad 3y^2 = 3$$

$$y^2 = \frac{3}{27} \quad y^2 = \frac{3}{3}$$

$$y^2 = \frac{1}{9} \quad y^2 = 1$$

$$y = \pm \sqrt{\frac{1}{9}} \quad y = \pm 1$$

$$y = \pm \frac{1}{3} \quad y = 1 \quad \text{or} \quad y = -1$$

$$y = \frac{1}{3} \quad \text{or} \quad y = -\frac{1}{3}$$

Putting values of  $y = \pm \frac{1}{3}$  in eq. (iii)

$$y = \frac{1}{3} \quad y = -\frac{1}{3}$$

$$x = -5y \quad x = -5y$$

$$x = -5\left(\frac{1}{3}\right) \quad x = -5\left(-\frac{1}{3}\right)$$

$$\boxed{x = -\frac{5}{3}} \quad \boxed{x = \frac{5}{3}}$$

Now putting values of  $y = \pm 1$  in eq. (iv)

$$y = 1 \quad y = -1$$

$$x = y \quad x = y$$

$$\boxed{x = 1} \quad \boxed{x = -1}$$

**Question No.13**

$$x^2 - 2xy = 7$$

$$xy + 3y^2 = 2$$

**Solution:**  $x^2 - 2xy = 7$  .....(i)

$$xy + 3y^2 = 2$$
 .....(ii)

Multiplying eq. (i) by 2 and eq. (ii) by 7

$$2x^2 - 4xy = 14$$
 .....(iii)

$$7xy + 21y^2 = 14$$
 .....(iv)

Subtract eq. (iv) from eq. (iii)

$$2x^2 - 4xy = 14$$

$$\pm 7xy \pm 21y^2 = -14$$

$$2x^2 - 11xy - 21y^2 = 0$$

$$2x^2 - 14xy + 3xy - 21y^2 = 0$$

$$2x(x - 7y) + 3y(x - 7y) = 0$$

$$(x - 7y)(2x + 3y) = 0$$

$$x - 7y = 0 \quad 2x + 3y = 0$$

$$x = 7y \quad 2x = -3y$$

$$x = 7y \quad \dots(v) \quad x = \frac{-3y}{2} \quad \dots(vi)$$

Put  $x=7y$  in eq. (i)

$$(7y)^2 - 2(7y)y = 7$$

$$49y^2 - 14y^2 = 7$$

$$35y^2 = 7 \Rightarrow y^2 = \frac{7}{35}$$

$$y^2 = \frac{1}{5} \Rightarrow y = \pm \frac{1}{\sqrt{5}}$$

$$y = \frac{1}{\sqrt{5}} \quad y = -\frac{1}{\sqrt{5}}$$

Putting values of y in eq. (v)

$$x = 7y \quad x = 7y$$

$$y = \frac{1}{\sqrt{5}} \quad y = -\frac{1}{\sqrt{5}}$$

$$x = 7\left(\frac{1}{\sqrt{5}}\right) \quad x = 7\left(-\frac{1}{\sqrt{5}}\right)$$

$$x = \frac{7}{\sqrt{5}} \quad x = \frac{-7}{\sqrt{5}}$$

$$(x, y) = \left(\frac{7}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right) \quad (x, y) = \left(\frac{-7}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right)$$

From eq. (vi) put value of x in eq. (i)

$$\left(\frac{-3}{2}y\right)^2 - 2\left(\frac{-3}{2}y\right)y = 7$$

$$\frac{9}{4}y^2 + 3y^2 = 7$$

$$9y^2 + 12y^2 = 28$$

$$21y^2 = 28$$

$$y^2 = \frac{28}{21} \quad y^2 = \frac{4}{3}$$

$$\sqrt{y^2} = \sqrt{\frac{4}{3}} \quad y = \pm \frac{2}{\sqrt{3}}$$

$$y = \frac{2}{\sqrt{3}} \quad y = -\frac{2}{\sqrt{3}}$$

Putting values of y in eq. (vi)

$$y = \frac{2}{\sqrt{3}} \quad y = -\frac{2}{\sqrt{3}}$$

$$x = \frac{-3}{2}\left(\frac{2}{\sqrt{3}}\right) \quad x = \frac{-3}{2}\left(\frac{-2}{\sqrt{3}}\right)$$

$$x = -\sqrt{3} \quad x = \sqrt{3}$$

$$(x, y) = \left(-\sqrt{3}, \frac{2}{\sqrt{3}}\right) \quad (x, y) = \left(\sqrt{3}, \frac{-2}{\sqrt{3}}\right)$$

Solution set is

$$\left\{\left(\frac{7}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right), \left(\frac{-7}{\sqrt{5}}, \frac{-1}{\sqrt{5}}\right), \left(-\sqrt{3}, \frac{2}{\sqrt{3}}\right), \left(\sqrt{3}, \frac{-2}{\sqrt{3}}\right)\right\}$$

**Exercise 2.8****Home Work:****Question No.1**

The product of two positive consecutive numbers is 182. Find the numbers.

**Solution:**

Suppose first positive number = x

Second positive number = x+1

According to given condition:

$$x(x+1) = 182$$

$$x^2 + x - 182 = 0$$

$$x^2 + 14x - 13x - 182 = 0$$

$$x(x+14) - 13(x+14) = 0$$

$$(x+14)(x-13) = 0$$

$$x+14 = 0 \quad \text{or} \quad x-13 = 0$$

$$x = -14 \quad \text{or} \quad x = 13$$

As x is positive number therefore we neglect the negative value, So  $x=13$

Then first positive number =  $x=13$

Second positive number =  $x+1$

$$=13+1=14$$

So, 13 and 14 are two required consecutive positive numbers.

### Class Work:

#### Question No.4

The product of five less than three times a certain number and one less than four times the number is 7. Find the number.

**Solution:** Let required number= $x$

Five less than three times the number= $3x-5$

One less than four times the number= $4x-1$

According to given condition:

$$(3x-5)(4x-1)=7$$

$$12x^2-3x-20x+5-7=0$$

$$12x^2-23x-2=0$$

$$12x^2-24x+x-2=0$$

$$12x(x-2)+1(x-2)=0$$

$$(x-2)(12x+1)=0$$

$$x-2=0 \quad \text{or} \quad 12x+1=0$$

$$\boxed{x=2} \quad \text{or} \quad 12x=-1$$

$$\boxed{x = \frac{-1}{12}}$$

So Required number is  $-2$  and  $-\frac{1}{2}$

### Home Work

#### Question No.5

The difference of a number and its reciprocal is  $\frac{15}{4}$ . Find the number.

**Solution:** Let required number= $x$

Reciprocal of the number= $\frac{1}{x}$

According to given condition:

$$x - \frac{1}{x} = \frac{15}{4}$$

$$\frac{x^2-1}{x} = \frac{15}{4}$$

$$4(x^2-1)=15x$$

$$4x^2-4-15x=0$$

$$4x^2-15x-4=0$$

$$4x^2-16x+1x-4=0$$

$$4x(x-4)+1(x-4)=0$$

$$(x-4)(4x+1)=0$$

$$x-4=0 \quad \text{or} \quad 4x+1=0$$

$$x=4 \quad \text{or} \quad 4x=-1$$

$$\boxed{x=4} \quad \text{or} \quad \boxed{x = \frac{-1}{4}}$$

So required is  $4$  and  $-\frac{1}{4}$

### Home Work:

#### Question No.9

Find two integers whose difference is 4 and whose squares differ by 72.

**Solution:** Let  $x$  and  $y$  are two integers

According to given condition:

$$x - y = 4 \quad \dots(i)$$

$$x^2 - y^2 = 72 \quad \dots(ii)$$

From eq. (i)

$$x = 4 + y \quad \dots(iii)$$

Putting the value of  $x$  in eq. (ii)

$$(4+y)^2 - y^2 = 72$$

$$[(4)^2 + (y)^2 + 2(4)(y)] - y^2 = 72$$

$$16 + y^2 + 8y - y^2 = 72$$

$$16 + 8y = 72$$

$$8y = 72 - 16$$

$$8y = 56$$

$$y = \frac{56}{8} \Rightarrow \boxed{y=7}$$

Putting the value of  $y$  in eq. (iii)

$$x = 4 + y$$

$$x = 4 + 7 \Rightarrow \boxed{x=11}$$

So the required number is  $7$  and  $11$ .

### Class Work:

#### Question No.10

Find the dimensions of a rectangle, whose perimeter is 80cm and its area is  $375 \text{ cm}^2$

**Solution:**

Let width of a rectangle= $x$  cm

Length of rectangle= $y$  cm

Perimeter of rectangle= $80$ cm

Area of rectangle= $375 \text{ cm}^2$

$$\therefore 2(L+W)=P$$

$$2(x+y)=80$$

$$x+y = \frac{80}{2}$$

$$x+y=40 \quad \dots(i)$$

Area=Length  $\times$  Width

$$375 = x \times y$$

$$xy=375 \quad \dots(ii)$$

From eq. (i)

$$x+y=40$$

$$x=40-y$$

Put it in eq. (ii)

$$x(40-x) = 375$$

$$40x - x^2 = 375$$

$$0 = x^2 - 40x + 375$$

$$x^2 - 40x + 375 = 0$$

$$x^2 - 25x - 15x + 375 = 0$$

$$x(x-25) - 15(x-25) = 0$$

$$(x-15)(x-25) = 0$$

$$x-15=0 \quad \text{or} \quad x-25=0$$

$$x=15 \quad \text{or} \quad x=25$$

Putting the value of x in eq. (i)

$$x=15 \quad \text{or} \quad x=25$$

$$15+y=40 \quad \text{or} \quad 25+y=40$$

$$y=40-15 \quad \text{or} \quad y=40-25$$

$$\boxed{y=25} \quad \text{or} \quad \boxed{y=15}$$

If  $x=15$  then  $y=25$  and  $x=25$  then  $y=15$

So, dimensions of rectangle are either 25cm by 15cm or 15cm by 25cm.

## Miscellaneous Exercise-2

### Question No.1

#### Multiple Choice Question.(Exercise +Additional )

Four possible answers are given for the following questions. Tick (✓) the correct answer.

1. If  $\alpha, \beta$  are the roots of  $3x^2+5x-2=0$  then  $\alpha+\beta$

- (a)  $\frac{5}{3}$  (b)  $\frac{3}{5}$   
(c)  $\frac{-5}{3}$  (d)  $\frac{-2}{3}$

2. If  $\alpha, \beta$  are the roots of  $7x^2-x+4=0$  then  $\alpha\beta$  is:

- (a)  $\frac{-1}{7}$  (b)  $\frac{4}{7}$   
(c)  $\frac{7}{4}$  (d)  $\frac{-4}{7}$

3. Roots of the equation  $4x^2-5x+2=0$  are:

- (a) irrational (b) imaginary  
(c) rational (d) none of these

4. Cube roots of  $-1$  are:

- (a)  $-1, -\omega, -\omega^2$  (b)  $-1, \omega, -\omega^2$   
(c)  $-1, -\omega, \omega^2$  (d)  $1, -\omega, -\omega^2$

5. Sum of the cube roots of unity is:

- (a) 0 (b) 1  
(c)  $-1$  (d) 3

6. Product of cube roots of unity is:

- (a) 0 (b) 1

- (c)  $-1$  (d) 3

7. If  $b^2-4ac < 0$  then the roots of  $ax^2+bx+c=0$  are:

- (a) irrational (b) rational  
(c) imaginary (d) None of these

8. If  $b^2-4ac > 0$ , but not a perfect square then roots of  $ax^2+bx+c=0$  are:

- (a) imaginary (b) rational  
(c) irrational (d) None of these

9.  $\frac{1}{\alpha} + \frac{1}{\beta}$  is equal to:

- (a)  $\frac{1}{\alpha}$  (b)  $\frac{1}{\alpha} - \frac{1}{\beta}$   
(c)  $\frac{\alpha-\beta}{\alpha\beta}$  (d)  $\frac{\alpha+\beta}{\alpha\beta}$

10.  $\alpha^2 + \beta^2$  is equal to:

- (a)  $\alpha^2 - \beta^2$  (b)  $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$   
(c)  $(\alpha + \beta)^2 - 2\alpha\beta$  (d)  $\alpha + \beta$

11. Two square roots of unity are:

- (a)  $1, -1$  (b)  $1, \omega$   
(c)  $1, -\omega$  (d)  $\omega, \omega^2$

12. Roots of the equation  $4x^2-4x+1=0$  are:

- (a) real, equal (b) real, unequal  
(c) imaginary (d) irrational

13. If  $\alpha, \beta$  are the roots of  $px^2+qx+r=0$ , then sum of the roots  $2\alpha$  and  $2\beta$  is:

- (a)  $\frac{-q}{p}$  (b)  $\frac{r}{p}$   
(c)  $\frac{-2q}{p}$  (d)  $-\frac{q}{2p}$

14. If  $\alpha, \beta$  are the roots of  $x^2-x-1=0$ , then product of the roots  $2\alpha$  and  $2\beta$  is:

- (a)  $-2$  (b) 2  
(c) 4 (d)  $-4$

15. The nature of the roots of equation  $ax^2+bx+c=0$  is determined by:

- (a) Sum of the roots  
(b) Product of the roots  
(c) Synthetic division  
(d) Discriminant

16. The discriminant of  $ax^2+bx+c=0$  is:

- (a)  $b^2-4ac$  (b)  $b^2+4ac$   
(c)  $-b^2+4ac$  (d)  $-b^2-4ac$

17. If  $b^2-4ac > 0$  and is a perfect square, then roots of  $ax^2+bx+c=0$  are:

- (a) irrational, equal
- (b) rational, equal
- (c) rational, unequal
- (d) irrational, unequal

18. If  $b^2 - 4ac = 0$ , then roots of  $ax^2 + bx + c = 0$  are

- (a) irrational, equal
- (b) rational, equal
- (c) rational, unequal
- (d) irrational, unequal

19. Disc. of  $2x^2 - 7x + 1 = 0$  is:

- (a) 47 (b) 41
- (c) 40 (d) 51

20. Disc. of  $x^2 - 3x + 3 = 0$  is:

- (a) 6 (b) 12
- (c) 21 (d) -3

21. The roots of  $x^2 + 8x + 16 = 0$  are

- (a) imaginary (b) equal
- (c) unequal (d) irrational

22. If roots of a quadratic equation are equal, then Disc. is:

- (a) positive (b) negative
- (c) zero (d) irrational

23. If roots of a quadratic equation are imaginary, then Disc. is:

- (a) positive (b) negative
- (c) zero (d) irrational

24. If roots of a quadratic equation are real and distinct then Disc. is:

- (a) positive (b) negative
- (c) zero (d) imaginary

25. If roots of a quadratic equation are rational and distinct, then Disc. is:

- (a) perfect square
- (b) not perfect square
- (c) zero
- (d) negative

26. If roots of a quadratic equation are irrational and distinct, then Disc. is:

- (a) perfect square
- (b) not perfect square
- (c) zero
- (d) negative

27. If for a quadratic equation  $b^2 - 4ac = 49$ , then roots are real and:

- (a) equal (b) unequal
- (c) irrational (d) imaginary

28. If for a quadratic equation

$b^2 - 4ac = -47$ , then roots are:

- (a) real (b) rational
- (c) irrational (d) complex

29. If for a quadratic equation  $b^2 - 4ac = 0$ ,

then roots are:

- (a) complex (b) irrational
- (c) repeated (d) distinct

30. If for a quadratic equation

$b^2 - 4ac = 205$ , then roots are:

- (a) complex (b) irrational
- (c) rational (d) equal

31. Which of the following is true description of nature of roots of a quadratic equation?

- (a) real, irrational, equal
- (b) real, imaginary, unequal
- (c) real, irrational, unequal
- (d) complex, repeated, rational

32. If roots of a quadratic equation are real, rational and equal, then possible value of Disc. is:

- (a) 0 (b) 36
- (c) 40 (d) -49

33. If roots of a quadratic equation are real, rational and unequal then possible value of Disc. is:

- (a) 0 (b) 36
- (c) 40 (d) -25

34. If roots of a quadratic equation are real, irrational and unequal then possible value of Disc. is:

- (a) 0 (b) 9
- (c) 5 (d) -7

35. If roots of a quadratic equation are imaginary, and unequal, the possible value of Disc. is:

- (a) 0 (b) 9
- (c) 8 (d) -9

36. If  $\omega = \frac{-1 - \sqrt{-3}}{2}$ , then  $\omega^2 = \dots$

- (a)  $\frac{-1 \pm \sqrt{3}}{2}$  (b)  $\frac{-1 + \sqrt{3}}{2}$
- (c)  $\frac{-1 + \sqrt{-3}}{2}$  (d)  $\frac{-1 \pm \sqrt{-3}}{2}$

37. If  $\omega$  and  $\omega^2$  are complex cube root of unity, then  $\omega \cdot \omega^2 = \dots$

- (a) 1 (b) -1
- (c) 0 (d) 2

38.  $\omega^4 = \dots$

- (a)  $\omega^2$  (b)  $\omega$
- (c) 1 (d) 0

39. If 1,  $\omega$ ,  $\omega^2$  are cube root of unity, then  $1 + \omega + \omega^2 = \dots$

- (a) 0 (b)  $\omega^3$
- (c) 1 (d) -1

40. If 1,  $\omega$ ,  $\omega^2$  are cube root of unity, then  $1 + \omega =$



.....

- (a) 0 (b)  $\omega$   
(c)  $\omega^2$  (d)  $-\omega^2$

41. If 1,  $\omega$ ,  $\omega^2$  are cube root of unity, then  $1 + \omega^2 =$ 

.....

- (a)  $-\omega$  (b)  $\omega$   
(c)  $\omega^2$  (d)  $-\omega^2$

42. If 1,  $\omega$ ,  $\omega^2$  are cube root of unity, then  $\omega + \omega^2 =$ 

.....

- (a) 1 (b)  $-1$   
(c)  $\omega^3$  (d)  $2\omega^2$

43. If  $\omega$  is complex cube root of unity, then  $\omega^7 =$  .....

- (a)  $\omega$  (b)  $-\omega$   
(c)  $\omega^2$  (d)  $-\omega^2$

44. If  $\omega$  is complex cube root of unity, then  $\omega^{23} =$  .....

- (a)  $\omega$  (b)  $-\omega$   
(c)  $\omega^2$  (d)  $-\omega^2$

45. If  $\omega$  is complex cube root of unity, then  $\omega^{63} =$  .....

- (a)  $\omega$  (b) 1  
(c)  $-\omega$  (d)  $-\omega^2$

46. If  $\omega$  is complex cube root of unity, then  $\omega^{-5} =$  .....

- (a)  $\omega$  (b) 1  
(c)  $-\omega$  (d)  $-\omega^2$

47. If  $\omega$  is complex cube root of unity, then  $\omega^{-16} =$  .....

- (a)  $\omega$  (b)  $-\omega$   
(c)  $-\omega^2$  (d)  $\omega^2$

48. If  $\omega$  is complex cube root of unity, then  $\omega^{-27} =$  .....

- (a) 1 (b)  $-1$   
(c)  $\omega$  (d)  $\omega^2$

49.  $(-1 + \sqrt{-3})^3 =$  .....

- (a) 8 (b) 1  
(c)  $-4$  (d)  $-28$

50. Cube roots of 8 are:

- (a) 2,  $2\omega$ ,  $2\omega^2$  (b)  $-2$ ,  $-2\omega$ ,  $-2\omega^2$   
(c) 2,  $-2\omega$ ,  $-2\omega^2$  (d) 2,  $-2\omega$ ,  $2\omega^2$

51. Cube roots of  $-27$  are:

- (a) 3,  $-3\omega$ ,  $3\omega^2$  (b)  $-3$ ,  $-3\omega$ ,  $-3\omega^2$   
(c)  $-3$ ,  $3\omega$ ,  $3\omega^2$  (d) 3,  $3\omega$ ,  $-3\omega^2$

52. Cube root of 64 are:

- (a)  $-4$ ,  $-4\omega$ ,  $-4\omega^2$  (b) 4, 16,  $\omega$   
(c) 4,  $4\omega$ ,  $4\omega^2$  (d)  $(4)^3$

53.  $(1 - \omega - \omega^2)^5 =$  .....

- (a) 6 (b) 16  
(c) 32 (d) 64

54.  $(1 - 3\omega - 3\omega^2)^3 =$  .....

- (a) 12 (b) 16  
(c)  $-125$  (d) 64

55.  $(9 + 4\omega + 4\omega^2)^3 =$  .....

- (a) 15 (b) 25  
(c) 125 (d)  $(17)^3$

## (Answer key

1.	c	2.	b	3.	b	4.	a	5.	a
6.	b	7.	c	8.	c	9.	d	10.	c
11.	a	12.	a	13.	c	14.	d	15.	d
16.	a	17.	c	18.	b	19.	b	20.	d
21.	b	22.	c	23.	b	24.	a	25.	a
26.	b	27.	b	28.	d	29.	c	30.	b
31.	c	32.	a	33.	b	34.	c	35.	d
36.	c	37.	a	38.	b	39.	a	40.	d
41.	a	42.	b	43.	a	44.	c	45.	b
46.	a	47.	d	48.	a	49.	a	50.	a
51.	b	52.	c	53.	c	54.	d	55.	c

## Question No.2

Write short answers of the following questions.

(i) Discuss the nature of roots of the following equations.

(a)  $x^2 + 3x + 5 = 0$

Solution:

$$a = 1, b = 3, c = 5$$

Discriminant =  $b^2 - 4ac = (3)^2 - 4(1)(5)$

Dis. =  $9 - 20 = -11 < 0$

Roots are imaginary.

(b)  $2x^2 - 7x + 3 = 0$

Solution:

$$a = 2, b = -7, c = 3$$

Discriminant =  $b^2 - 4ac = (-7)^2 - 4(2)(3)$

$$= 49 - 24 = 25$$

$$Dis = 25 > 0$$

Roots are real( rational, unequal.

(c)  $x^2 + 6x - 1 = 0$

Solution:

$$a =$$

$$1, b = 6, c = -1$$



$$\begin{aligned}\text{Discriminant} &= b^2 - 4ac = (6)^2 - 4(1)(-1) \\ &= 36 + 4 = 38 \\ \text{Dis.} &= 38 > 0\end{aligned}$$

Roots are real( irrational, unequal).

(d)  $16x^2 - 8x + 1 = 0$

$$a = 16, b = -8, c = 1$$

$$\text{Discriminant} = b^2 - 4ac = (-8)^2 - 4(16)(1)$$

$$\text{Dis.} = 64 - 64 = 0$$

Roots are (real) rational and equal.

(ii) Find  $\omega^2$ , if  $\omega = \frac{-1 + \sqrt{-3}}{2}$

Solution:

$$\omega = \frac{-1 + \sqrt{-3}}{2}$$

Squaring both sides by

$$\begin{aligned}\omega^2 &= \left(\frac{-1 + \sqrt{-3}}{2}\right)^2 \\ \omega^2 &= \frac{(-1)^2 + (-3) + 2(-1)\sqrt{-3}}{4} \\ \omega^2 &= \frac{1 - 3 - 2\sqrt{-3}}{4} \\ \omega^2 &= \frac{-2 - 2\sqrt{-3}}{4} = \frac{2(-1 - \sqrt{-3})}{4} \\ &= \frac{-1 - \sqrt{-3}}{2}\end{aligned}$$

(iii) Prove that the sum of all the cube roots of unity is zero.

Solution:

We know that

$$\omega = \frac{-1 + \sqrt{-3}}{2} \text{ and } \omega^2 = \frac{-1 - \sqrt{-3}}{2}$$

$$\text{sum of the roots} = 1 + \omega + \omega^2$$

The sum of cube roots of unity

$$\begin{aligned}&= 1 + \frac{-1 + \sqrt{-3}}{2} + \frac{-1 - \sqrt{-3}}{2} \\ &= \frac{2 - 1 + i\sqrt{-3} - 1 - \sqrt{-3}}{2} \\ &= \frac{0}{2} = 0\end{aligned}$$

$$\text{Thus, } 1 + \omega + \omega^2 = 0$$

(iv) Find the product of complex cube roots of unity.

Proof:

Three cube roots of unity are

$$1, \frac{-1 + \sqrt{-3}}{2}, \frac{-1 - \sqrt{-3}}{2}$$

The product of cube roots of unity

$$\begin{aligned}&1 \times \frac{-1 + \sqrt{-3}}{2} \times \frac{-1 - \sqrt{-3}}{2} \\ &= \frac{(-1)^2 - (\sqrt{-3})^2}{4} = \frac{1 - (-3)}{4} = \frac{1 + 3}{4} = \frac{4}{4} = 1 \\ &\text{i.e. } (1)(\omega)(\omega^2) = 1 \text{ or } \omega^3 = 1\end{aligned}$$

(v) Show that  $x^3 + y^3 = (x + y)(x + \omega y)(x + \omega^2 y)$

Solution:

$$\begin{aligned}R.H.S &= (x + y)(x + \omega y)(x + \omega^2 y) \\ &= (x + y)(x^2 + x\omega^2 y + \omega xy + \omega^3 y^2) \\ &= (x + y)(x^2 + xy(\omega^2 + \omega) + \omega^3 y^2) \\ &= (x + y)(x^2 + xy(-1) + (1)y^2) \\ &= (x + y)(x^2 - xy + y^2) \\ L.H.S &= x^3 + y^3\end{aligned}$$

Hence proof:

$$(x + y)(x + \omega y)(x + \omega^2 y) = x^3 + y^3$$

(vi) Evaluate  $\omega^{37} + \omega^{38} + 1$

Solution:

$$\begin{aligned}\omega^{37} + \omega^{38} + 1 &= \omega^{36} \cdot \omega + \omega^2 \cdot \omega^{36} + 1 \\ &= (\omega^3)^{12} \cdot \omega + \omega^2 \cdot (\omega^3)^{12} + 1 \\ &= (1)^{12} \cdot \omega + \omega^2 \cdot (1)^{12} + 1 \\ &= \omega + \omega^2 + 1 \\ &= -1 + 1 = 0\end{aligned}$$

vii) Evaluate  $(1 - \omega + \omega^2)^6$

solution:

$$\begin{aligned}(1 - \omega + \omega^2)^6 &= (1 + \omega^2 - \omega)^6 \\ &= (1 - \omega - \omega)^6 \quad \because 1 + \omega^2 = -\omega \\ &= (-2\omega)^6 \\ &= (-2)^6 \omega^6 \\ &= 64(\omega^3)^2 \\ &= 64(1)^2 = 64\end{aligned}$$

# Unit-3

## Variations

### Ratio:

A relation between two quantities of the same kind (measured in same unit) is called ratio.

If  $a$  and  $b$  are two quantities of the same kind and  $b$  is not zero. Then the ratio of  $a$  and  $b$  is written as  $a:b$  or in fraction  $\frac{a}{b}$

### Remember that:

- The order of the element in a ratio is important.
- In ratio  $a:b$  the first term  $a$  is called antecedent and the second term  $b$  is called consequent.
- A ratio has no units.

### Proportional:

A proportional is a statement, which is expressed as an equivalence of two ratios.

If two ratio  $a:b :: c:d$  are equal, then we can write as  $a:b = c:d$

Where quantities  $a, d$  are called extremes, while  $b, c$  are called means.

Symbolically the proportional of  $a, b, c$ , and  $d$  is written as

$$a:b :: c:d \text{ or}$$

$$a:b = c:d$$

$$\frac{a}{b} = \frac{c}{d}$$

$$\text{i.e. } ad = bc$$

### Exercise 3.1

### Homework:

#### Question No.1

Express the following as ratio  $a:b$  and as a fraction in its simplest form.

(iv)

27min. 30 sec, 1 hour

$$\left( \begin{array}{l} 1650 \text{ sec} : 3600 \text{ sec} \\ 165 \text{ sec} : 360 \text{ sec} \\ 11 \text{ sec} : 24 \text{ sec} \\ \frac{11 \text{ sec}}{24 \text{ sec}} \\ \frac{11}{24} \text{ Ans.} \end{array} \right)$$

$$\therefore \left( \begin{array}{l} 1 \text{ hour} = 60 \text{ min} \\ 1 \text{ min} = 60 \text{ sec} \\ 27 \text{ min} = 27 \times 60 = 1620 \text{ s} \\ 27 \text{ min} 30 \text{ sec} = 1620 + 30 = 1650 \end{array} \right)$$

(v)

$$\begin{array}{l} 75^0, 225^0 \\ 75^0 : 225^0 \end{array}$$

$$3^0 : 9^0 \text{ Dividing by } 25$$

$$1^0 : 3^0 \text{ Dividing by } 3$$

$$\frac{1}{3} = 1:3$$

### Class Work:

#### Question No.4

Find the value of  $p$ , if the ratio  $2p+5:3p+4$  and  $3:4$  are equal.

#### Solution:

$$\frac{2p+5}{3p+4} = \frac{3}{4}$$

$$4(2p+5) = 3(3p+4)$$

$$8p+20 = 9p+12$$

$$20-12 = 9p-8p$$

$$8 = p$$

### Home Work:

#### Question No.5

if the ratio  $3x+1:6+$

$4x$  and  $2:5$  are equal find the value of  $x$ .

#### Solution:

$$\frac{3x+1}{6+4x} = \frac{2}{5}$$

$$5(3x+1) = 2(6+4x)$$

$$15x+5 = 12+8x$$

$$15x-8x = 12-5$$

$$7x = 7$$

$$x = \frac{7}{7}$$

$$x = 1$$

### Home Work:

#### Question No.7

If 10 is added in each number of the ratio

$4:13$  we get  $1:2$ .

What are the numbers?

#### Solution:

Let a number  $x$

According to the given Condition

$$\frac{4x+10}{13x+10} = \frac{1}{2}$$

$$\Rightarrow 2(4x+10) = 1(13x+10)$$

$$\Rightarrow 8x+20 = 13x+10$$

$$\Rightarrow 20-10 = 13x-8x$$

$$10 = 5x$$

$$x = \frac{10}{5} = 2$$

$$x = 2$$

$$\text{Now 1st number} = 4x = 4(2) = 8$$

$$\text{2nd number} = 13x = 13(2) = 26$$

### Class Work:

#### Question No.9

If  $a:b = 7:6$ , find the value of  $3a+5b:7b-5a$

#### Solution:

As given that  $a:b = 7:6$  or

$$\frac{a}{b} = \frac{7}{6}$$

Now

$$3a + 5b : 7b : 5a = \frac{3a + 5b}{7b - 5a}$$

Dividing numerators and denominator by b

$$\frac{\frac{3a + 5b}{b}}{\frac{7b - 5a}{b}} = \frac{3\left(\frac{a}{b}\right) + 5\left(\frac{b}{b}\right)}{7\left(\frac{b}{b}\right) - 5\left(\frac{a}{b}\right)} = \frac{3\left(\frac{a}{b}\right) + 5}{7 - 5\left(\frac{a}{b}\right)}$$

$$\because \frac{a}{b} = \frac{7}{6}$$

$$\text{so } \frac{3\left(\frac{7}{6}\right) + 5}{7 - 5\left(\frac{7}{6}\right)} = \frac{\frac{21}{6} + 5}{7 - \frac{35}{6}} = \frac{\frac{21 + 30}{6}}{\frac{42 - 35}{6}}$$

$$\frac{\frac{51}{6}}{\frac{7}{6}} = \frac{51}{6} \times \frac{6}{7} = \frac{51}{7} = 51:7$$

**Class Work:****Question No.11 Find x in the following proportions.****(iv)Class Work:**

$$P^2 + pq + q^2 :: \frac{p^3 - q^3}{p + q} : (p - q)^2$$

**Solution:**

$$P^2 + pq + q^2 :: \frac{p^3 - q^3}{p + q} : (p - q)^2$$

product of extremes = product of means

$$(p^2 + pq + q^2)(p - q)^2 = x \times \frac{p^3 - q^3}{p + q}$$

$$(p^2 + pq + q^2)(p - q)(p - q) = x \times \frac{p^3 - q^3}{p + q}$$

$$(p^3 - q^3)(p - q) = x \times \frac{p^3 - q^3}{p + q}$$

$$(p^3 - q^3)(p - q) \times \frac{p + q}{p^3 - q^3} = x$$

$$x = (p - q)(p + q)$$

$$x = p^2 - q^2$$

**(v) Home Work**

$$8 - x : 11 - x :: 16 - x : 25 - x$$

**Solution:**

product of extremes = product of means

$$(8 - x)(25 - x) = (11 - x)(16 - x)$$

$$200 - 8x - 25x + x^2 = 176 - 11x - 16x + x^2$$

$$200 - 33x + x^2 = 176 - 27x + x^2$$

$$-33x + x^2 + 27x - x^2 = 176 - 200$$

$$-6x = -24$$

$$x = 4$$

**Variation:**

The word variation is frequently used in all sciences.

There are two types of variations:

**Direct variation:**if two quantities are related in such a way that increase (decrease) in one quantity causes increase (decrease) in other quantity is called **direct variation**.**Inverse variation:**if two quantities are related in such a way that when one quantity increase, the other decrease is called **inverse variation**.**Exercise 3.2****Class work:****Question No.1**

if y varies directly as x, and y = 8 when x = 2 Find

(iii) x when y = 28

$$y = 4x$$

$$28 = 4x$$

$$\frac{28}{4} = x$$

$$x = 7$$

**Home Work:****Question No.2.**if  $y \propto x$ , and y = 7 when x = 3 find

(ii)

**x when y = 35 and y when x = 18**

When y = 35

$$y = \frac{7}{3} \times x$$

$$35 = \frac{7}{3} \times x$$

$$\frac{35 \times 3}{7} = x$$

$$5 \times 3 = x$$

$$15 = x$$

Put x = 18 in (i) we get

$$y = kx$$

$$y = \frac{7}{3} \times 18$$

$$y = 42$$

**Home Work:****Question No.5**if  $V \propto R^3$ , and V = 5 when R = 3 find when V = 625

$$V \propto R^3$$

$$V = kR^3$$

$$5 = k(3)^3$$

$$5 = 27k$$

$$k = \frac{5}{27}$$

So,

$$\text{eq (i) becomes } V = \frac{5}{27} R^3$$

Put  $V = 625$ 

$$625 = \frac{5}{27} \times R^3$$

$$R^3 = 625 \times \frac{27}{5}$$

$$R^3 = 125 \times 27$$

Taking cubes on both sides

$$R^{3 \times \frac{1}{3}} = (5^3)^{\frac{1}{3}} \times (3^3)^{\frac{1}{3}}$$

$$R = 5 \times 3 = 15$$

$$R = 15$$

**Class Work****Question No.8**if  $y \propto \frac{1}{x}$  and  $y = 4$  when  $x = 3$ , find  $x$  when  $y =$ 

24

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x}$$

when  $y = 4$  and  $x = 3$ 

$$y = \frac{k}{x}$$

$$4 = \frac{k}{3}$$

$$12 = k$$

$$\text{or } k = 12$$

So,

$$y = \frac{12}{x}$$

When  $y = 24$ 

$$24 = \frac{12}{x}$$

$$x = \frac{12}{24}$$

$$x = \frac{1}{2}$$

**Home Work:****Question No.10**if  $A \propto \frac{1}{r^2}$  and  $A = 2$  when  $r = 3$  find  $r$  when  $A$  $= 72$ 

$$A \propto \frac{1}{r^2}$$

$$A = \frac{k}{r^2}$$

when  $A = 2$  and  $r = 3$ 

$$A = \frac{k}{r^2}$$

$$2 = \frac{k}{(3)^2}$$

$$2 = \frac{k}{9}$$

$$2 \times 9 = k$$

$$k = 18$$

So,

$$A = \frac{18}{r^2}$$

When  $A = 72$ 

$$A = \frac{k}{r^2}$$

$$72 = \frac{18}{r^2}$$

$$r^2 = \frac{18}{72}$$

$$r^2 = \frac{1}{4}$$

Taking squaring root on both sides

$$r = \pm \frac{1}{2}$$

**Class Work****Question No.11**if  $a \propto \frac{1}{b^2}$  and  $a = 3$  when  $b = 4$ , find  $a$  when  $b$  $= 8$ 

Solution:

$$a \propto \frac{1}{b^2}$$

$$a = \frac{k}{b^2} \rightarrow (i)$$

when  $a = 3$  and  $b = 4$ 

$$3 = \frac{k}{(4)^2} = \frac{k}{16}$$

$$3 \times 16 = k$$

$$48 = k$$

$$\text{or } k = 48$$

So, (i) becomes

$$a = \frac{48}{b^2}$$

When  $b = 8$ 

$$a = \frac{k}{b^2}$$

$$a = \frac{48}{8^2}$$

$$a = \frac{48}{64}$$

$$a = \frac{3}{4}$$

**Home Work:****Question No.**13 if  $m \propto \frac{1}{n^3}$  and  $m = 2$  when  $n =$ 4, find  $m$  when $n = 6$  and  $n$  when  $m = 432$ 

Solution:

$$m \propto \frac{1}{n^3}$$

$$m = \frac{k}{n^3}$$

When  $m = 2$  and  $n = 4$

$$m = \frac{k}{n^3}$$

When  $m = 2$  and  $n = 4$

$$m = \frac{k}{n^3}$$

$$2 = \frac{k}{(4)^3}$$

$$2 = \frac{k}{64}$$

$$2 \times 64 = k$$

$$128 = k$$

So,

$$m = \frac{128}{n^3}$$

When  $n = 6$

$$m = \frac{128}{6^3}$$

$$m = \frac{128}{216}$$

$$m = \frac{16}{27}$$

When  $m = 432$

$$m = \frac{k}{n^3}$$

$$432 = \frac{128}{n^3}$$

$$n^3 = \frac{128}{432}$$

$$n^3 = \frac{8}{27}$$

Taking cube on both sides by

$$n^{3 \times \frac{1}{3}} = \frac{(2)^{3 \times \frac{1}{3}}}{(3)^{3 \times \frac{1}{3}}}$$

$$n = \frac{2}{3}$$

### Third proportional:

If three quantities  $a, b, c$  are related as

$a : b :: b : c$  then  $c$  is called third proportional.

### Fourth proportional:

If four quantities  $a, b, c$ , and  $d$  are related as

$$a : b :: c : d$$

Then  $d$  is called the fourth proportional.

### Mean proportional:

If three quantities  $a, b$  and  $c$  are related as

$$a : b :: b : c$$

Then  $b$  is called the mean proportional.

### Continued Proportional:

If three quantities  $a, b$ , and  $c$  are related as

$$a : b :: b : c$$

Where  $a$  is first,  $b$  is the mean and  $c$  is the third proportional, then  $a, b$  and  $c$  are in continued proportional.

## Exercise 3.3

Question No.1 find third proportional to

Class Work:

i. 6,12

let  $x$  be the third proportional to

$$6 : 12 :: 12 : x$$

product of Extremes = product of Means

$$6x = 12 \times 12$$

$$x = \frac{12 \times 12}{6}$$

$$x = 24$$

Home Work:

(iv)  $(x - y)^2, x^3 - y^3$

Solution

$$(x - y)^2, x^3 - y^3$$

let  $x$  be the third proportional to

$$(x - y)^2 : x^3 - y^3 :: x^3 - y^3 : c$$

product of Extremes = product of Means

$$(x - y)^2 \times c = x^3 - y^3 \times x^3 - y^3$$

$$c = \frac{x^3 - y^3 \times x^3 - y^3}{(x - y)^2}$$

$$c = \frac{(x - y)(x^2 + xy + y^2)(x - y)(x^2 + xy + y^2)}{(x - y)(x - y)}$$

$$x = (x^2 + xy + y^2)(x^2 + xy + y^2)$$

Home Work:

(vi)

$$\frac{p^2 - q^2}{p^3 + q^3}, \frac{p - q}{p^2 - pq + q^2}$$

let  $x$  be the third proportional to

$$\frac{p^2 - q^2}{p^3 + q^3} : \frac{p - q}{p^2 - pq + q^2} :: \frac{p - q}{p^2 - pq + q^2} : c$$

product of Extremes = product of Means

$$\frac{p^2 - q^2}{p^3 + q^3} \times c = \frac{p - q}{p^2 - pq + q^2} \times \frac{p - q}{p^2 - pq + q^2}$$

$$c = \frac{p - q}{p^2 - pq + q^2} \times \frac{p - q}{p^2 - pq + q^2} \times \frac{p^3 + q^3}{p^2 - q^2}$$

$$c = \frac{p - q}{p^2 - pq + q^2} \times \frac{p - q}{p^2 - pq + q^2} \times \frac{(p + q)(p^2 - pq + q^2)}{(p - q)(p + q)}$$

$$c = \frac{p - q}{p^2 - pq + q^2}$$

Question No.2 find a fourth proportional to

Home Work (ii)

$$x^2, 2x^3, 18x^5$$

let  $x$  be the fourth proportional

$$4x^2 : 2x^3 :: 18x^5 : c$$

product of Extremes = Product of means

$$4x^4 \times c = 2x^3 \times 18x^5$$

$$c = \frac{2x^3 \times 18x^5}{4x^4}$$

$$c = \frac{36x^8}{4x^4}$$

$$c = 9x^4$$

**Class Work: (iv)**

$$x^2 - 11x + 24, x - 3, 5x^4 - 40x^3$$

**Solution:***let x be the fourth proportional*

$$x^2 - 11x + 24 : x - 3 :: 5x^4 - 40x^3 : c$$

*product of Extremes**= Product of means*

$$x^2 - 11x + 24 \times c = (x - 3) \times (5x^4 - 40x^3)$$

$$c = \frac{(x - 3) \times (5x^4 - 40x^3)}{x^2 - 11x + 24}$$

$$c = \frac{(x - 3)5x^3(x - 8)}{x^2 - 8x - 3x + 24}$$

$$c = \frac{5x^3(x - 3)(x - 8)}{x(x - 8) - 3(x - 8)}$$

$$c = \frac{5x^3(x - 3)(x - 8)}{(x - 3)(x - 8)}$$

$$c = 5x^3$$

**Home Work: (v)**

$$p^3 + q^3, p^2 - q^2, p^2 - pq + q^2$$

**Solution:***let x be the fourth proportional*

$$p^3 + q^3 : p^2 - q^2 :: p^2 - pq + q^2 : c$$

*product of Extremes**= Product of means*

$$p^3 + q^3 \times c = p^2 - q^2 \times p^2 - pq + q^2$$

$$c = \frac{p^2 - q^2 \times p^2 - pq + q^2}{p^3 + q^3}$$

$$c = \frac{(p - q)(p + q)(p^2 - pq + q^2)}{p^3 + q^3}$$

$$c = \frac{(p - q)(p^3 + q^3)}{p^3 + q^3}$$

$$c = (p - q)$$

**Class Work: (vi)**

$$(p^2 - q^2)(p^2 + pq + q^2), p^3 + q^3, p^3 - q^3$$

**Solution:***let x be the fourth proportional*

$$(p^2 - q^2)(p^2 + pq + q^2) : p^3 + q^3 :$$

$$: p^3 - q^3 : c$$

$$(p^2 - q^2)(p^2 + pq + q^2) \times c$$

$$= p^3 + q^3 \times p^3 - q^3$$

$$c = \frac{p^3 + q^3 \times p^3 - q^3}{(p^2 - q^2)(p^2 + pq + q^2)}$$

$$c = \frac{(p + q)(p^2 - pq + q^2)(p - q)(p^2 + pq + q^2)}{(p + q)(p - q)(p^2 + pq + q^2)}$$

$$c = p^2 - pq + q^2$$

**Question No.3** find a mean proportional between**Class work:**

i. 20,45

*let x be the mean proportional*

$$20 : x :: x : 45$$

*product of means**= product of extremes*

$$x^2 = 20 \times 45$$

$$x^2 = 900$$

*Taking squaring root on both sides*

$$\sqrt{x^2} = \sqrt{(30)^2}$$

$$x = \pm 30$$

**Home Work(iv)**

$$x^2 - y^2, \frac{x - y}{x + y}$$

**Solution:***let x be the mean proportional*

$$x^2 - y^2 : c :: c : \frac{x - y}{x + y}$$

*product of means**= product of extremes*

$$c^2 = x^2 - y^2 \times \frac{x - y}{x + y}$$

$$c^2 = (x - y)(x + y) \times \frac{x - y}{x + y}$$

$$c^2 = (x - y)(x - y)$$

$$c^2 = (x - y)^2$$

*Taking squaring root on both sides*

$$\sqrt{c^2} = \sqrt{(x - y)^2}$$

$$x = \pm(x - y)$$

**Question No.4** find the values of the letter involved in the following continued proportional**Home Work:(ii)**

$$8, x, 18$$

$$8 : x :: x : 18$$

*product of means = product of extremes*

$$x^2 = 8 \times 18$$

$$x^2 = 144$$

*Squaring root on both sides*

$$\sqrt{x^2} = \sqrt{(12)^2}$$

$$x = \pm 12$$

**Class Work: (iii)**

$$12, 3p - 6, 27$$

$$12 : 3p - 6 :: 3p - 6 : 27$$

*product of means = product of extremes*

$$(3p - 6)^2 = 12 \times 27$$

$$(3p - 6)^2 = 324$$

*Squaring root on both sides*

$$\sqrt{(3p - 6)^2} = \sqrt{(18)^2}$$

$$3p - 6 = \pm 18$$

$$\begin{aligned}
 3p - 6 &= +18 \\
 3p &= 18 + 6 \\
 3p &= 24 \\
 \frac{24}{3} &= 8 \\
 p &= 8
 \end{aligned}$$

$$\begin{aligned}
 3p - 6 &= -18 \\
 3p &= -18 + 6 \\
 3p &= -12 \\
 \frac{-12}{3} &= -4 \\
 p &= -4
 \end{aligned}$$

**Theorems on proportional:**

**1. Theorem of Invertendo**

if  $a:b = c:d$  then  $b:a = d:c$

**2. Theorem of Alternando**

if  $a:b = c:d$ , then  $a:c = b:d$

**3. Theorem of Componendo**

if  $a:b = c:d$  then

(i).  $a + b : b = c + d : d$

(ii).  $a : a + b = c : c + d$

**4. Theorem of Dividendo**

if  $a:b = c:d$  then

(iii).  $a - b : b = c - d : d$

(iv).  $a : a - b = c : c - d$

### Exercise 3.4

Question No.1 prove that  $a:b = c:d$  if

**Class work:**

i.  $\frac{4a+5b}{4a-5b} = \frac{4c+5d}{4c-5d}$

Solution:

By componendo – dividendo

$$\begin{aligned}
 \frac{(4a + 5b) + (4a - 5b)}{(4a + 5b) - (4a - 5b)} &= \frac{(4c + 5d) + (4c - 5d)}{(4c + 5d) - (4c - 5d)} \\
 &= \frac{4a + 5b + 4a - 5b}{4a + 5b - 4a + 5b} = \frac{4c + 5d + 4c - 5d}{4c + 5d - 4c + 5d} \\
 \frac{8a}{10b} &= \frac{8c}{10d}
 \end{aligned}$$

Multiplying by  $\frac{10}{8}$

$$\begin{aligned}
 \frac{a}{b} &= \frac{c}{d} \\
 a:b &= c:d
 \end{aligned}$$

**Home work: (v)**

ii.  $\frac{pa+qb}{pa-qb} = \frac{pc+qd}{pc-qd}$

By componendo – dividendo

$$\begin{aligned}
 \frac{(pa + qb) + (pa - qb)}{(pa + qb) - (pa - qb)} &= \frac{(pc + qd) + (pc - qd)}{(pc + qd) - (pc - qd)} \\
 \frac{pa + qb + pa - qb}{pa + qb - pa + qb} &= \frac{pc + qd + pc - qd}{pc + qd - pc + qd} \\
 \frac{2pa}{2qb} &= \frac{2pc}{2qd}
 \end{aligned}$$

$$\frac{pa}{qb} = \frac{pc}{qd}$$

Multiplying by  $\frac{q}{p}$

$$\begin{aligned}
 \frac{a}{b} &= \frac{c}{d} \\
 a:b &= c:d
 \end{aligned}$$

**Home work: (viii)**

$$\frac{a^2 + b^2}{a^2 - b^2} = \frac{ac + bd}{ac - bd}$$

by componendo – dividendo

$$\begin{aligned}
 \frac{(a^2 + b^2) + (a^2 - b^2)}{(a^2 + b^2) - (a^2 - b^2)} &= \frac{(ac + bd) + (ac - bd)}{(ac + bd) - (ac - bd)} \\
 \frac{a^2 + b^2 + a^2 - b^2}{a^2 + b^2 - a^2 + b^2} &= \frac{ac + bd + ac - bd}{ac + bd - ac + bd} \\
 \frac{2a^2}{2b^2} &= \frac{2ac}{2bd} \\
 \frac{a^2}{b^2} &= \frac{ac}{bd}
 \end{aligned}$$

Multiplying by  $\frac{b}{a}$

$$\begin{aligned}
 \frac{a}{b} &= \frac{c}{d} \\
 a:b &= c:d
 \end{aligned}$$

**Question No.2**

using theorem of componendo – dividendo

**Home work: (ii)**

find the value of  $\frac{m+5n}{m-5n} + \frac{m+5p}{m-5p}$  if  $m = \frac{10np}{n+p}$

**Solution:**

$$m = \frac{10np}{n+p} \rightarrow (i)$$

From eq(i)

$$\begin{aligned}
 m &= \frac{5n \times 2p}{n+p} \\
 \frac{m}{5n} &= \frac{2p}{n+p}
 \end{aligned}$$

By applying componendo – dividendo theorem

$$\begin{aligned}
 \frac{m + 5n}{m - 5n} &= \frac{2p + n + p}{2p - n - p} \\
 \frac{m + 5n}{m - 5n} &= \frac{3p + n}{p - n} \rightarrow (ii)
 \end{aligned}$$

From eq.(i)

$$\begin{aligned}
 m &= \frac{2n \times 5p}{n+p} \\
 \frac{m}{5p} &= \frac{2n}{n+p}
 \end{aligned}$$

By applying componendo – dividendo theorem

$$\begin{aligned}
 \frac{m + 5p}{m - 5p} &= \frac{2n + n + p}{2n - n - p} \\
 \frac{m + 5p}{m - 5p} &= \frac{3n + p}{n - p} \rightarrow (iii)
 \end{aligned}$$

Adding (ii) and (iii)



$$\begin{aligned}
 \frac{m+5n}{m-5n} + \frac{m+5p}{m-5p} &= \frac{3p+n}{p-n} + \frac{3n+p}{n-p} \\
 &= -\frac{3p+n}{n-p} + \frac{3n+p}{n-p} \\
 &= \frac{3n+p}{n-p} - \frac{3p+n}{n-p} \\
 &= \frac{3n+p-3p-n}{n-p} \\
 &= \frac{n-p}{2n-2p} \\
 &= \frac{n-p}{n-p} \\
 \frac{2(n-p)}{n-p} &= 2
 \end{aligned}$$

**Class work: (iv)**

Find the value of  $\frac{x-3y}{x+3y} - \frac{x+3z}{x-3z}$ , if  $x = \frac{3yz}{y-z}$

$$x = \frac{3yz}{y-z} \rightarrow (i)$$

From equation (i)

$$x = \frac{3y \times z}{y-z}$$

$\frac{x}{3y} = \frac{z}{y-z}$  By applying componendo –  
dividendo theorem

$$\begin{aligned}
 \frac{x+3y}{x-3y} &= \frac{z+y-z}{z-y+z} \\
 \frac{x+3y}{x-3y} &= \frac{y}{2z-y} \\
 \frac{x-3y}{x+3y} &= \frac{2z-y}{y} \rightarrow (ii)
 \end{aligned}$$

From equation (i)

$$\begin{aligned}
 x &= \frac{3z \times y}{y-z} \\
 \frac{x}{3z} &= \frac{y}{y-z}
 \end{aligned}$$

By applying componendo – dividendo theorem

$$\begin{aligned}
 \frac{x+3z}{x-3z} &= \frac{y+y-z}{y-y+z} \\
 \frac{x+3z}{x-3z} &= \frac{2y-z}{2y-z} \\
 \frac{x-3z}{x+3z} &= \frac{z}{2y-z} \rightarrow (iii)
 \end{aligned}$$

Subtracting equation (iii) from eq. (ii)

$$\begin{aligned}
 \frac{x-3y}{x+3y} - \frac{x+3z}{x-3z} &= \frac{2z-y}{y} - \frac{2y-z}{z} \\
 &= \frac{z(2z-y) - y(2y-z)}{yz} \\
 &= \frac{2z^2 - zy - 2y^2 + yz}{yz} \\
 &= \frac{2(z^2 - y^2)}{yz}
 \end{aligned}$$

**Home work: (v)**

Find the value of  $\frac{s-3p}{s+3p} + \frac{s+3q}{s-3q}$ , if  $s = \frac{6pq}{p-q}$

$$s = \frac{6pq}{p-q} \rightarrow (i)$$

From eq. (i)

$$\begin{aligned}
 s &= \frac{3p \times 2p}{p-q} \\
 \frac{s}{3p} &= \frac{2p}{p-q}
 \end{aligned}$$

By applying componendo – dividendo theorem

$$\begin{aligned}
 \frac{s+3p}{s-3p} &= \frac{2p+p-q}{2p-p+q} \\
 \frac{s+3p}{s-3p} &= \frac{q+p}{3q-p} \\
 \frac{s-3p}{s+3p} &= \frac{3q-p}{p+q} \rightarrow (ii)
 \end{aligned}$$

From eq. (i)

$$\begin{aligned}
 s &= \frac{2p \times 3q}{p-q} \\
 \frac{s}{3q} &= \frac{2p}{p-q} \\
 \frac{s+3q}{s-3q} &= \frac{2p+p-q}{2p-p+q} \\
 \frac{s+3q}{s-3q} &= \frac{3p-q}{p+q} \rightarrow (iii)
 \end{aligned}$$

Adding equation (ii) and (i)

$$\begin{aligned}
 \frac{s-3p}{s+3p} + \frac{s+3q}{s-3q} &= \frac{3q-p}{p+q} + \frac{3p-q}{p+q} \\
 &= \frac{3q-p+3p-q}{p+q} \\
 &= \frac{2q+2p}{p+q} \\
 &= 2 \frac{p+q}{p+q} \\
 &= 2
 \end{aligned}$$

**Class work: (vii)**

Solve  $\frac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}} = 2$

**Solution:**

$$\frac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}} = 2$$

By applying componendo – dividendo theorem

$$\begin{aligned}
 \frac{\sqrt{x^2+2}+\sqrt{x^2-2}+\sqrt{x^2+2}-\sqrt{x^2-2}}{\sqrt{x^2+2}+\sqrt{x^2-2}-\sqrt{x^2+2}+\sqrt{x^2-2}} \\
 = \frac{2+1}{2-1} \\
 \frac{2\sqrt{x^2+2}}{2\sqrt{x^2-2}} &= \frac{3}{1} \\
 \frac{\sqrt{x^2+2}}{\sqrt{x^2-2}} &= 3
 \end{aligned}$$

Taking square on both sides



$$\begin{aligned}\frac{x^2 + 3}{x^2 - 2} &= 9 \\ x^2 + 2 &= 9(x^2 - 2) \\ x^2 + 2 &= 9x^2 - 18 \\ 2 + 18 &= 9x^2 - x^2 \\ 20 &= 8x^2 \\ x^2 &= \frac{20}{8} \\ x^2 &= \frac{5}{2} \\ &= \pm \sqrt{\frac{5}{2}}\end{aligned}$$

If we check the given equation for this value doesn't satisfy the equation so the given solution is extraneous.

So,

$$S.S = \{ \quad \}$$

### Joint variation:

A combination of direct and inverse variation of one or more than variables form joint variation.

If a variable  $y$  varies as  $x$  varies inversely as  $z$ .

$$\text{Then } y \propto x \text{ and } y \propto \frac{1}{z}$$

Then joint variation, we write it as

$$y \propto \frac{x}{z}$$

$$\text{i.e. } y = k \frac{x}{z}$$

where  $k \neq 0$  is the constant of variation.

### Exercise 3.5

#### Class Work:

**Question No.1** if  $S$  varies directly as  $u^2$  and inversely as  $v$  and  $s = 7$  when  $u = 3, v = 2$ . Find the value of  $S$  when  $u=6$

**Solution:**

$$\begin{aligned}s &\propto u^2 \\ s &\propto \frac{1}{v} \\ s &= k \frac{u^2}{v} \rightarrow (i) \\ \text{put } s &= 7, u = 3, v = 2 \\ 7 &= k \frac{3^2}{2} \\ 7 &= k \frac{9}{2} \\ \frac{7 \times 2}{9} &= k \\ \frac{14}{9} &= k \\ \text{or } k &= \frac{14}{9}\end{aligned}$$

So, equation (i) becomes

$$S = \frac{14u^2}{9v} \rightarrow (ii)$$

Put  $u = 6$  and  $v = 10$  in equation (ii)

$$\begin{aligned}S &= \frac{14(6)^2}{9(10)} \\ S &= \frac{14 \times 36}{9 \times 10} \\ S &= \frac{28}{5}\end{aligned}$$

#### Home Work:

**Question No.3** if  $Y$  varies directly as

$x^3$  and inversely as  $z^3$  and  $t$ , and  $y = 16$

When  $x = 4, z = 2, t =$

3. find the value of  $y$  when  $x = 2, z = 3$  and  $t = 4$

**Solution:**

$$\begin{aligned}y &\propto x^3 \\ y &\propto \frac{1}{z^2} \\ y &\propto \frac{1}{t} \\ y &= k \frac{x^3}{z^2 t} \rightarrow (i)\end{aligned}$$

Put  $y = 16, x = 4, z = 2, t = 3$

$$\begin{aligned}16 &= k \frac{4^3}{2^2(3)} \\ 16 &= k \frac{64}{12} \\ \frac{16 \times 12}{64} &= k \\ \frac{16}{4} &= k \\ 3 &= k \\ \text{or } k &= 3\end{aligned}$$

So, equation (i) becomes

$$y = \frac{3x^3}{z^2 t} \rightarrow (ii)$$

Put  $x = 2, z = 3$  and  $t = 4$  in equation(ii)

$$\begin{aligned}y &= \frac{3(2)^3}{(3)^2(4)} \\ y &= \frac{3(8)}{9(4)}\end{aligned}$$

Divide by 3

$$y = \frac{8}{12}$$

Divide by 4

$$y = \frac{2}{3}$$

#### Home Work:

**Question No.5** if  $v$  varies directly as the product

$xy^3$  and inversely as  $z^2$  and  $v = 27$  when

$X = 7, y = 6, z = 7$ . find the value of  $v$  when  $x = 6$

,  $y = 2, z = 3$

**Solution:**

$$v \propto xy^3$$

$$v \propto \frac{1}{z^2}$$

$$u = k \frac{xy^3}{z^2} \rightarrow (i)$$

put  $v = 27, x = 7, y = 6, z = 7$

$$27 = k \frac{(7)(6)^3}{(7)^2}$$

$$27 = k \frac{216}{7} \quad (\div \text{ by } 7)$$

$$\frac{27 \times 7}{216} = k$$

$$\frac{7}{8} = k \quad \text{by } (27 \div 216 = 8)$$

$$\text{or } k = \frac{7}{8}$$

so, Equation (i) becomes

$$u = \frac{7xy^3}{8z^2} \rightarrow (ii)$$

put  $x = 6, y = 2, z = 3$  in equation (ii)

$$u = \frac{7(6)(2)^3}{8(9)}$$

$$u = \frac{7 \times 6 \times 8}{8 \times 9}$$

$$u = \frac{7 \times 6}{9}$$

$$u = \frac{7 \times 2}{3}$$

$$u = \frac{14}{3}$$

### K-Method:

Uses k method to prove conditional equalities involving proportional.

if  $a : b :: c : d$  is a proportion, then putting each ratio equal to  $k$

$$\text{i.e. } \frac{a}{b} = \frac{c}{d} = k$$

$$\frac{a}{b} = k; \frac{c}{d} = k$$

$$a = bk \text{ and } c = dk$$

### Exercise 3.6

Question No1 if  $a : b = c : d$ , then show that

Home Work: (ii)

$$\frac{6a - 5b}{6a + 5b} = \frac{6c - 5d}{6c + 5d}$$

Solution:

As  $a : b = c : d$

$$\text{let } \frac{a}{b} = \frac{c}{d} = k$$

then  $a = bk$  and  $c = dk$

$$\frac{6a - 5b}{6a + 5b} = \frac{6c - 5d}{6c + 5d}$$

Putting these values

$$\frac{6(bk) - 5b}{6(bk) + 5b} = \frac{6(dk) - 5d}{6(dk) + 5d}$$

$$\begin{aligned} \frac{6bk - 5b}{6bk + 5b} &= \frac{6dk - 5d}{6dk + 5d} \\ \frac{b(6k - 5)}{b(6k + 5)} &= \frac{d(6k - 5)}{d(6k + 5)} \\ \frac{6k - 5}{6k + 5} &= \frac{6k - 5}{6k + 5} \\ L.H.S &= R.H.S \end{aligned}$$

Class Work: (iii)

$$\frac{a}{b} = \sqrt{\frac{a^2 + c^2}{b^2 + d^2}}$$

Solution:

As  $a : b = c : d$

$$\text{let } \frac{a}{b} = \frac{c}{d} = k$$

then  $a = bk$  and  $c = dk$

$$\frac{a}{b} = \sqrt{\frac{a^2 + c^2}{b^2 + d^2}}$$

Putting these values

$$\frac{bk}{b} = \sqrt{\frac{(bk)^2 + (dk)^2}{b^2 + d^2}}$$

$$\frac{bk}{b} = \sqrt{\frac{b^2k^2 + k^2d^2}{b^2 + d^2}}$$

$$k = \sqrt{\frac{k^2(b^2 + d^2)}{b^2 + d^2}}$$

$$k = \sqrt{k^2}$$

$$k = k$$

Home Work: (vi)

$$a^2 + b^2 : \frac{a^3}{a+b} = c^2 + d^2 : \frac{c^3}{c+d}$$

Solution:

$$\frac{a^2 + b^2}{\frac{a^3}{a+b}} = \frac{c^2 + d^2}{\frac{c^3}{c+d}}$$

As  $a : b = c : d$

$$\text{let } \frac{a}{b} = \frac{c}{d} = k$$

then  $a = bk$  and  $c = dk$

putting these values

$$\frac{(bk)^2 + b^2}{\frac{(bk)^3}{bk+b}} = \frac{(dk)^2 + d^2}{\frac{(dk)^3}{kd+d}}$$

$$\frac{b^2k^2 + b^2}{\frac{b^3k^3}{bk+b}} = \frac{d^2k^2 + d^2}{\frac{d^3k^3}{kd+d}}$$

$$\frac{b^2(k^2 + 1)}{\frac{b^3k^3}{bk+b}} = \frac{d^2(k^2 + 1)}{\frac{d^3k^3}{kd+d}}$$

$$\frac{b^2(k^2 + 1)}{\frac{b^2k^3}{(k+1)}} = \frac{d^2(k^2 + 1)}{\frac{d^2k^3}{k+1}}$$

$$\frac{b^2k^3}{(k+1)} = \frac{d^2k^3}{k+1}$$

$$\frac{(k^2 + 1)(k + 1)}{k^3} = \frac{(k^2 + 1)(k + 1)}{k^3}$$

$$L.H.S = R.H.S$$

**Class Work: (ii)**

$$\frac{ac + ce + ea}{bd + df + fb} = \left[ \frac{ace}{bdf} \right]^{\frac{2}{3}}$$

**Solution:**

$$\frac{ac + ce + ea}{bd + df + fb} = \left[ \frac{ace}{bdf} \right]^{\frac{2}{3}}$$

As  $a : b = c : d = e : f$ let  $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$ then  $a = bk$ ,  $c = dk$  and  $e = fk$ 

putting these values

$$\frac{bkdk + dkfk + fkbk}{bd + df + fb} = \left[ \frac{bkdkfk}{bdf} \right]^{\frac{2}{3}}$$

$$\frac{k^2bd + k^2df + k^2fb}{bd + df + fb} = \left[ \frac{k^3bdf}{bdf} \right]^{\frac{2}{3}}$$

$$\frac{k^2(bd + df + fb)}{bd + df + fb} = [k^3]^{\left(\frac{2}{3}\right)}$$

$$k^2 = k^2$$

$$L.H.S = R.H.S$$

**Exercise 3.7****Class Work:****Question No.2**

The surface area

 $S$  of a surface varies directly as the square of radius  $r$ , and  $S = 16\pi$  when  $r=2$ .Find  $r$  when  $S = 36\pi$ **Solution:**

$$S \propto r^2$$

$$S = kr^2 \rightarrow (i)$$

$$S = 16\pi \text{ when } r = 2$$

$$16\pi = k(2)^2$$

$$16\pi = 4k$$

$$\frac{16\pi}{4} = k$$

$$4\pi = k$$

$$\text{or } k = 4\pi$$

So, equation (i) becomes

$$S = 4\pi r^2$$

When  $S = 36\pi$  units

$$36\pi = 4\pi r^2$$

$$r^2 = 9$$

$$r = 3$$

**Home Work:**Question No.3 in Hook's law the force  $F$  applied to stretch a spring varies directly as the amount of elongation  $S$  and $F = 32 \text{ lb}$  when  $S = 1.6 \text{ inch}$ . find (i) when  $F = 50 \text{ lb}$ **Solution:**

$$F \propto S$$

$$F = kS \rightarrow (i)$$

$$F = 32 \text{ lb when } S = 1.6$$

$$(32) = k(1.6)$$

$$k = \frac{32}{1.6}$$

$$k = \frac{32}{16} \times 10$$

$$k = 20$$

So equation (i) becomes

$$F = 20S$$

when  $F = 50 \text{ lb}$ 

$$50 \text{ lb} = 20S$$

$$S = 2.5 \text{ in}$$

when  $S = 0.8 \text{ in}$ 

$$F = 20(0.8)$$

$$F = 16 \text{ lb}$$

**Home Work:****Question No.9**The kinetic energy ( $K.E$ ) of a body varies jointly as the mass " $m$ " of the body and the square of its velocity " $V$ " if the energy is  $4320 \text{ ft/lb}$  when the mass is  $45 \text{ lb}$  and the velocity is  $24 \text{ ft/sec}$ . Determine the kinetic energy of a  $3000 \text{ lb}$  automobile travelling  $44 \text{ ft/sec}$ .**Solution:**

$$K.E \propto mv^2$$

$$K.E = kmv^2 \rightarrow (i)$$

$$K.E = 4320 \text{ ft/lb, } m=45 \text{ lb, } v=24 \text{ ft/sec.}$$

$$4320 = k(45)(24)^2$$

$$k = \frac{4320}{45 \times 476}$$

$$k = \frac{4320}{25920}$$

$$k = \frac{1}{6}$$

So equation (i) becomes

$$K.E = \frac{mv^2}{6}$$

When  $m = 3000 \text{ lb, } v = 44 \text{ ft/sec}$ .

$$K.E = \frac{(3000)(44)^2}{6}$$

$$K.E = 500 \times 1936$$

$$K.E = 96800 \text{ ft/lb}$$

## Miscellaneous Exercise -3

### Q. 1 Multiple Choice Questions

Four possible answers are given for the following questions. Tick (✓) the correct answer.

- In a ratio a: b, a is called:**
  - relation
  - antecedent
  - consequent
  - None of these
- In a ratio x: y, y is called:**
  - relation
  - antecedent
  - consequent
  - None of these
- In a proportion a: b:: c: d, a and d are called:**
  - means
  - extremes
  - fourth proportional
  - None of these
- In a proportion a:b::c: d, b and c are called:**
  - means
  - extremes
  - fourth proportional
  - None of these
- In continued proportion a:b = b:c, ac= b<sup>2</sup>, b is said to be \_\_\_\_\_proportional.**
  - third
  - fourth
  - means
  - none of these
- In continued proportion a:b = b: c, c is said to be \_\_\_\_\_proportional to a and b.**
  - third
  - fourth
  - means
  - None of these
- Find x in proportion 4:x::5:15**
  - $\frac{75}{4}$
  - $\frac{4}{3}$
  - $\frac{3}{4}$
  - 12
- If  $u \propto v^2$ , then: (Board 2014)**
  - $u = v^2$
  - $u = kv^2$
  - $uv^2 = k$
  - $uv^2 = 1$
- If  $y^2 \propto \frac{1}{x^3}$ , then:**
  - $y^2 = \frac{k}{x^3}$
  - $y^2 = \frac{1}{x^3}$
  - $y^2 = x^2$
  - $y^2 = kx^3$
- If  $\frac{u}{v} = \frac{v}{w} = k$ , then:**
  - $u = wk^2$
  - $u = vk^2$
  - $u = w^2k$
  - $u = v^2k$
- The third proportional of x<sup>2</sup> and y<sup>2</sup> is:**
  - $\frac{y^2}{x^2}$
  - $x^2y^2$

$$(c) \frac{y^4}{x^2} \quad (d) \frac{y^2}{x^4}$$

12. The fourth proportional w of x:y::v:w is:

$$(a) \frac{xy}{v} \quad (b) \frac{vy}{x}$$

$$(c) xyv \quad (d) \frac{x}{vy}$$

13. If a: b=x: y, then alternando property is:

$$(a) \frac{a}{x} = \frac{b}{y} \quad (b) \frac{a}{b} = \frac{x}{y}$$

$$(c) \frac{a+b}{b} = \frac{x+y}{y} \quad (d) \frac{a-b}{x} = \frac{x-y}{y}$$

14. If a : b = x : y, then invertendo property is:

$$(a) \frac{a}{x} = \frac{b}{y} \quad (b) \frac{a}{a-b} = \frac{x}{x-y}$$

$$(c) \frac{a+b}{b} = \frac{x+y}{y} \quad (d) \frac{b}{a} = \frac{y}{x}$$

15. If  $\frac{a}{b} = \frac{c}{d}$ , then componendo property is:

$$(a) \frac{a}{a+b} = \frac{c}{c+d} \quad (b) \frac{a}{a-b} = \frac{c}{c-d}$$

$$(c) \frac{ad}{bc} \quad (d) \frac{a-b}{b} = \frac{c-d}{d}$$

16. The simplest form of the ratio  $\frac{(x+y)(x^2+xy+y^2)}{x^3-y^3}$  is:

$$(a) \frac{x-y}{x+y} \quad (b) \frac{x+y}{x-y}$$

$$(c) 1 \quad (d) 2$$

17. Newton's law of Gravitation is an example of:

- variation
- directvariation
- inversevariation
- jointvariation

18. The relation between radius and circumference of a circle is an example of:

- variation
- direct variation
- inverse variation
- joint variation

19. If  $\frac{24}{7} = \frac{6}{x}$ , then 4x = .....

- (a) 7 (b)  $\frac{7}{4}$   
(c) 4 (d)  $\frac{42}{24}$
20. If  $\frac{5a}{3x} = \frac{15b}{y}$ , then  $ay = \dots\dots$   
(a)  $\frac{9bx}{y}$  (b)  $\frac{9y}{9b}$   
(c)  $5ay = 45bx$  (d)  $9bx$
21. In proportion 7:4::p:8, p =.....  
(a) 1 (b) 28  
(c) 14 (d) 56
22. If 6: m:: 9: 12, then m = .....  
(a) 6 (b) 9  
(c) 1 (d) 8
23. If x and y varies directly, then x = .....  
(a) y (b) ky  
(c)  $\frac{k}{y}$  (d) k
24. If v varies directly as  $u^3$ , then  $u^3 = \dots\dots$   
(a) vk (b)  $\frac{k}{v}$   
(c)  $\frac{v}{k}$  (d)  $vk^3$
25. If w varies inversely as  $p^2$ , then k = .....  
(a)  $\frac{w}{P^2}$  (b)  $wp^2$   
(c)  $\frac{P^2}{w}$  (d) wp
26. A third proportional of 12 and 4, is:  
(a)  $\frac{3}{4}$  (b)  $\frac{4}{3}$   
(c) 12 (d) 16
27. The fourth proportional of 15, 6, 5 is:  
(a) 30 (b) 15  
(c) 2 (d) 1
28. The mean proportional of  $4m^2n^4$  and  $p^6$  is:  
(a)  $\pm 2mnp$  (b)  $\pm mnp$   
(c)  $\pm \frac{2m^2n}{P^3}$  (d)  $\pm 2mn^2p^3$
29. The continued proportion of 4, m, 9 is:  
(a) 4 : m :: m : 9  
(b) 4 : 9 :: 9 : m  
(c) 9 : 4 :: 4 : m  
(d) 9 : 4 :: m : m
30. Third proportional of 6, 12 is:  
(a) 24 (b) 2  
(c) 18 (d) 84

31. Third proportional of  $a^3, 3a^2$  is:  
(a)  $3a^5$  (b)  $9a$   
(c)  $9a^4$  (d)  $9a^7$
32. Fourth proportional of 5, 8, 15 is:  
(a) 120 (b) 40  
(c) 24 (d) 20
33. Fourth proportional of  $4x^4, 2x^3, 18x^5$  is:  
(a)  $36x^8$  (b)  $9x^2$   
(c)  $9x^{12}$  (d)  $9x^4$
34. Mean Proportional of 20 and 45 is:  
(a)  $\pm 30$  (b)  $\pm 25$   
(c)  $\pm 20$  (d)  $\pm 15$
35. Mean proportional of  $20x^3y^5, 5x^7y$  is:  
(a)  $\pm 10x^5y^6$  (b)  $\pm 10x^5y^3$   
(c)  $\pm 10x^{10}y^6$  (d)  $100x^{10}y^6$
36. What is the value of p in the continued proportion of 5, p, 45?  
(a) 225 (b)  $\pm 50$   
(c)  $\pm 15$  (d)  $\pm 9$
37. What is the value of x in the continued proportion of 8, x, 18?  
(a)  $\pm 144$  (b)  $\pm 8$   
(c)  $\pm 18$  (d)  $\pm 12$
38. If  $\frac{9pq}{2\ell m} = \frac{18p}{5m}$ , then  $5q = \dots\dots$   
(a) 4m (b) 4p  
(c)  $4\ell$  (d) 4q
39. The mean proportional of  $9p^6q^4$  and  $r^8$  is:  
(a)  $\pm 3p^3q^2r^4$  (b)  $\pm 9p^6q^2r^8$   
(c)  $\pm 9p^3q^2r^4$  (d)  $\pm 3p^6q^4q^8$
40. What is the value of P in continued proportion of 12, p, 3?  
(a)  $\pm 4$  (b)  $\pm 6$   
(c)  $\pm 30$  (d)  $\pm 2$

## (Answer key)

1.	b	2.	c	3.	b	4.	a	5.	c
6.	a	7.	d	8.	b	9.	a	10.	a
11.	c	12.	b	13.	a	14.	d	15.	a
16.	b	17.	d	18.	a	19.	a	20.	d
21.	c	22.	d	23.	b	24.	c	25.	b
26.	b	27.	c	28.	d	29.	a	30.	a
31.	b	32.	c	33.	d	34.	a	35.	b
36.	c	37.	d	38.	c	39.	a	40.	b

**Question No.2****Write short answer of the following questions:**

- i Define ratio and give one example.

**Ratio:**

A relation between two quantities of the same kind (measured in same unit) is called ratio.

If  $a$  and  $b$  are two quantities of the same kind and  $b$  is not zero. Then the ratio of  $a$  and  $b$  is written as  $a:b$  or in fraction  $\frac{a}{b}$

For example:

If a hockey team wins 4 games and losses 5, the ratio of the games won games lost is 4: 5 or in fraction

$$\frac{4}{5}$$

- ii Define proportion

Proportional:

A proportional is a statement, which is expressed as an equivalence of two ratios.

If two ratio  $a:b :: c:d$  are equal, then we can write as  $a:b = c:d$

Where quantities  $a, d$  are called extremes, while  $b, c$  are called means.

Symbolically the proportional of  $a, b, c$ , and  $d$  is written as

$$a:b :: c:d \text{ or}$$

$$a:b = c:d$$

$$\frac{a}{b} = \frac{c}{d}$$

$$\text{i.e } ad = bc$$

- iii Define direct variation

**Direct variation:**

if two quantities are relates in such a way that increase (decrease) in one quantity causes increase (decrease) in other quantity is called **direct variation**.

- iv Define inverse variation.

**Inverse variation:**

if two quantities are relates in such a way that when one quantity increase, the other decrease is called **inverse variation**.

- v State theorem of componendo-dividendo.

if  $a:b = c:d$  then

$$a+b:a-b = c+d:c-d$$

$$\text{or } \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

- vi Find  $x$ , if  $6:x :: 3:5$ .

$$6:x :: 3:5$$

$$6 \times 5 = 3x$$

$$3x = 30$$

$$x = 10$$

- vii If  $x$  and  $y^2$  varies directly, and  $x = 27$  when  $y = 4$ .

Find the value of  $y$  when  $x=3$ .

$$y^2 \propto x$$

$$y^2 = kx \rightarrow (i)$$

$x = 27$  and  $y = 4$  put in equation (i)

$$16 = k(27)$$

$$\frac{16}{27} = k$$

$$k = \frac{16}{27}$$

$$y^2 = \frac{16}{27}x \rightarrow (ii)$$

$x = 3$  put in equation (ii)

$$y^2 = \frac{16}{27} \times 3$$

$$y^2 = \frac{16}{9}$$

$$\sqrt{y^2} = \sqrt{\frac{16}{9}}$$

$$y = \pm \frac{4}{3}$$

- viii If  $u$  and  $v$  varies inversely, and  $u = 8$ , when  $v = 3$ , find  $v$  when  $u = 12$ .

According to given condition

$$v \propto \frac{1}{u}$$

$$v = \frac{k}{u} \rightarrow (i)$$

$$3 = \frac{k}{8}$$

$$k = 24$$

Put value of  $k$  in (i)

$$v = \frac{24}{u} \rightarrow (ii)$$

$u = 2$  put in (ii)

$$v = \frac{24}{12} = 2$$

- ix Find a fourth proportional to 8, 7, 6.

According to the given condition

$$6:7 :: 6:x$$

$$8x = 7 \times 6$$

$$8x = 42$$

$$x = \frac{42}{8}$$

$$x = \frac{21}{4}$$

- x Find a mean proportional to 16 and 49.

According to the Given

$$16:x :: x:49$$

product of Extremes = product of Means  
so

$$16 \times 49 = x^2$$

$$x^2 = 784$$

Taking both sides by square root.

$$\sqrt{x^2} = \sqrt{784}$$

$$x = \pm 28$$

Thus mean proportional mean is 28.

xi Find third proportional to 28 and 4.

According to the given condition

$$28:4 :: 4:x$$

product of Extremes = product of Means

$$\text{so } 28x = 16$$

$$x = \frac{16}{28}$$

$$x = \frac{4}{7}$$

Thus third proportional to  $\frac{4}{7}$

xii If  $y \propto \frac{x^2}{z}$  and  $y = 28$  when  $x = 7, z = 2$ ,

then find  $y$ .

according to the given condition

$$y \propto \frac{x^2}{z}$$

$$y = \frac{kx^2}{z} \rightarrow (i)$$

$x = 7, y = 28$  and  $z = 2$  put in (i)

$$28 = \frac{k(7)^2}{2}$$

$$k = \frac{28 \times 2}{49}$$

$$k = \frac{56}{49} = \frac{8}{7}$$

$$k = \frac{8}{7}$$

Put value of  $k$  in (i)

$$y = \frac{8x^2}{7z}$$

xiii If  $z \propto xy$  and  $z = 36$ , when  $x = 2$ ,

$y = 3$ , then find  $z$ .

According to the given condition

$$z \propto xy$$

$$z = kxy \rightarrow (i)$$

$x = 2, y = 3$  and  $z = 36$  put in (i)

$$\frac{36}{6} = k$$

$$k = 6$$

Put values of  $k$  in (i)

$$z = 6xy$$

xiv If  $w \propto \frac{1}{v^2}$  and  $w = 2$  when  $v = 3$  then find  $w$ .

According to the given condition

$$w \propto \frac{1}{v^2}$$

$$w = \frac{k}{v^2} \rightarrow (i)$$

$w = 2$  and  $v = 3$  put in (i)

$$2 = \frac{k}{(3)^2}$$

$$k = 2 \times 9$$

$$k = 18$$

Put values of  $k$  in (i)

$$w = \frac{18}{v^2}$$



# Unit-4

## PARTIAL FRACTION

### Fraction:

A fraction is an indicated quotient of two numbers or algebraic expressions.

Or

The quotient of two numbers or algebraic expression is called a fraction. The quotient is indicated by a bar (\_\_\_\_\_).

### Rational Fraction:

An expression of the form  $\frac{N(x)}{D(x)}$ , with  $D(x) \neq 0$  and  $N(x)$  and  $D(x)$  are polynomials in  $x$  expressed with real coefficients, is called a **rational fraction**. Every fraction expression can be expressed as a quotient of two polynomials.

For example:

$$\frac{2x}{(x-1)(x+2)}$$

### Proper Fraction:

A rational fraction  $\frac{N(x)}{D(x)}$ , with  $D(x) \neq 0$  is called a proper fraction if degree of the polynomial  $N(x)$  in the numerator is less than the degree of the polynomial  $D(x)$  in the denominator.

For example:

$$\frac{2}{x+1}$$

### Improper Fraction:

A rational fraction  $\frac{N(x)}{D(x)}$ , with  $D(x) \neq 0$  is called improper fraction if degree of the polynomial  $N(x)$  is greater or equal to the degree of the polynomial  $D(x)$ .

For example:

$$\frac{5x}{x+2}, \quad \frac{6x^4}{x^3+1}$$

### Identity:

An identity is an equation, which is satisfied by all the values of the variables involved. For example

$$2(x+1) = 2x+2$$

And  $\frac{2x^2}{2} = 2x$  are identities, as these equations are satisfied by all values of  $x$

## Exercise 4.1

Resolve into partial fractions.

### Home work:

#### Question No.2

$$\frac{x-11}{(x-4)(x+3)}$$

#### Solution:

$$\text{Let } \frac{x-11}{(x-4)(x+3)} = \frac{A}{(x-4)} + \frac{B}{(x+3)} \rightarrow (i)$$

Multiplying equation (i) by  $(x-4)(x+3)$  on

Both sides, we get

$$x-11 = A(x+3) + B(x-4) \rightarrow (ii)$$

$$\Rightarrow x+3=0 \Rightarrow x=-3 \text{ and } x-4=0 \Rightarrow x=4$$

putting  $x=-3$  and  $x=4$  put in eq(ii) we get

$$\text{For } x=-3$$

$$-3-11 = B(-3-4)$$

$$-14 = -7B$$

$$\Rightarrow \boxed{B=2}$$

$$\text{For } x=4$$

$$4-11 = A(4+3)$$

$$-7 = 7A$$

$$\Rightarrow \boxed{A=-1}$$

Putting the value of  $A$  and  $B$  in equation (i) We get the required partial fractions

$$\frac{-1}{(x-4)} + \frac{2}{(x+3)}$$

Hence the required partial fraction are

$$\frac{x-11}{(x-4)(x+3)} = \frac{-1}{(x-4)} + \frac{2}{(x+3)}$$

### Home work:

#### Question No.4

$$\frac{x-5}{x^2+2x-3}$$

#### Solution:

$$\begin{aligned} \frac{x-5}{x^2+2x-3} &= \frac{x-5}{x^2+3x-x-3} \\ &= \frac{x-5}{x(x+3)-1(x+3)} = \frac{x-5}{(x-1)(x+3)} \\ &= \frac{x-5}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3} \rightarrow (ii) \end{aligned}$$

Multiplying both sides by  $(x-1)(x+3)$ , we get

$$x-5 = A(x+3) + B(x-1) \rightarrow (ii)$$

$$\text{Let } x+3=0 \Rightarrow x=-3 \text{ and } x-1=0 \Rightarrow x=1$$

putting  $x=-3$  and  $x=1$  in equation(ii) we get

$$\text{For } x=-3$$

$$-3-5 = A(-3+3) + B(-3-1)$$

$$-8 = -4B$$

$$-8 = -4B$$

$$B = \frac{-8}{-4}$$

$$\Rightarrow \boxed{B=2}$$

$$\text{For } x=1$$

$$1-5 = A(1+3) + B(1-1)$$

$$-4 = 4A$$

$$-4 = 4A$$

$$A = \frac{-4}{4}$$

$$\Rightarrow \boxed{A=-1}$$

Hence the required partial fractions are

$$\frac{x-5}{(x-1)(x+3)} = \frac{-1}{x-1} + \frac{2}{x+3}$$

## Home work:

### Question No.7

$$\frac{x^2 + 2x + 1}{(x-2)(x+3)}$$

**Solution:**

$\frac{x^2+2x+1}{(x-2)(x+3)}$  is an important fraction.

First we resolve it into proper fraction.

By long division we get

$$\frac{x^2 + x - 6\sqrt{x^2 + 2x + 1}}{\pm x^2 \pm x \mp 1}$$

$$\frac{x+7}{x+3}$$

We have  $\frac{x^2+2x+1}{x^2+x-6} = 1 + \frac{x+7}{x^2+x-6}$

Let  $\frac{x+7}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3} \rightarrow (i)$

Multiplying both sides by  $(x-2)(x+3)$  we get

$$x+7 = A(x+3) + B(x-2) \rightarrow (ii)$$

Let  $x+3=0$  i.e  $x=-3$

and  $x-2=0$  i.e  $x=2$

For $x = -3$ $-3+7 = B(-3-2)$ $4 = -5B$ $\Rightarrow B = -\frac{4}{5}$	For $x = 2$ $2+7 = A(2+3)$ $9 = 5A$ $\Rightarrow A = \frac{9}{5}$
---	--

Hence the required partial fractions are

$$\frac{x^2 + 2x + 1}{x^2 + x - 6} = 1 + \frac{9}{5(x-2)} - \frac{4}{5(x+3)}$$

## Class work:

### Question No.8

$$\frac{6x^3 + 5x^2 - 7}{3x^2 - 2x - 1}$$

**Solution:**

$\frac{6x^3+5x^2-7}{3x^2-2x-1}$  is an improper fraction.

First we resolve in to proper fraction.

$$\frac{6x^3 + 5x^2 - 7}{3x^2 - 2x - 1} = \frac{2x+3}{1} + \frac{8x-4}{(3x+1)(x-1)}$$

Now, Let  $\frac{8x-4}{(3x+1)(x-1)} = \frac{A}{3x+1} + \frac{B}{x-1}$

Multiplying both sides by  $(3x+1)(x-1)$  we get

$$8x-4 = A(x-1) + B(3x+1) \rightarrow (ii)$$

Let  $x-1=0$  i.e  $x=1$

and  $3x+1=0$  i.e  $x=-\frac{1}{3}$

putting  $x=1$  and  $x=-\frac{1}{3}$  in equation (ii)

we get

For  $x=1$   
 $8(1)-4 = B[3(1)+1]$   
 $8-4 = 4B$   
 $4 = 4B$

$$4B = 4$$

$$B = \frac{4}{4}$$

$$\Rightarrow B = 1$$

For  $x = -\frac{1}{3}$   
 $8\left(-\frac{1}{3}\right) - 4 = A\left(-\frac{1}{3} - 1\right)$   
 $-\frac{8}{3} - 4 = A\left(\frac{-1-3}{3}\right)$   
 $\frac{-8-12}{3} = \frac{A(-4)}{3}$   
 $-\frac{20}{3} = \frac{A(-4)}{3}$   
 $\Rightarrow A = 5$

Hence the required of a fraction when  $D(x)$  consists of repeated linear factors are

$$\frac{6x^3 + 5x^2 - 7}{3x^2 - 2x - 1} = 2x + 3 + \frac{5}{3x+1} + \frac{1}{x-1}$$

Resolution of a fraction when  $D(x)$  consists of repeated linear factors.

Rule II:

If a linear factor  $(ax+b)$  occurs  $n$  times as a factor of  $D(x)$ , then there are  $n$  partial fractions of the form.

$$\frac{A_1}{(ax+b)} + \frac{A_2}{(ax+b)^2} + \dots + \frac{A_n}{(ax+b)^n}$$

Where  $A_1, A_2, \dots, A_n$  are constant and  $n \geq 2$  is a positive integer.

$$\therefore \frac{N(x)}{D(x)} = \frac{A_1}{(ax+b)} + \frac{A_2}{(ax+b)^2} + \dots + \frac{A_n}{(ax+b)^n}$$

## Exercise 4.2

Resolve into partial fractions:

### Question No.1 Home work:

$$\frac{x^2 - 3x + 1}{(x-1)^2(x-2)}$$

**Solution:**

Let  $\frac{x^2 - 3x + 1}{(x-1)^2(x-2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x-2} \rightarrow (i)$

Multiplying both sides by  $(x-1)^2(x-2)$  we get

$$x^2 - 3x + 1 = A(x-1)(x-2) + B(x-2) + C(x-1)^2 \rightarrow (ii)$$

$$x^2 - 3x + 1 = A(x^2 - 3x + 2) + B(x-2) + C(x^2 - 2x + 1)$$

Putting  $x-1=0$  i.e  $x=1$  in (ii) we get

$$(1)^2 - 3(1) + 1 = (1-2)$$

$$1 - 3 + 1 = B(-1)$$

$$-1 = -B$$

$$\Rightarrow B = 1$$

Putting  $x-2=0$  i.e  $x=2$  in (ii) we get

$$(2)^2 - 3 + 1 = C(2 - 1)^2$$

$$4 - 6 + 1 = C$$

$$-1 = C \Rightarrow -1$$

$$\Rightarrow \boxed{C = -1}$$

Equating the coefficient of  $x^2$  in (ii) we get

$$1 = A + C$$

$$1 = A - 1$$

$$\Rightarrow A = 1 + 1$$

$$\Rightarrow \boxed{A = 2}$$

Hence the required partial fractions are

$$\frac{x^2 - 3x + 1}{(x - 1)^2(x - 2)} = \frac{2}{x - 1} + \frac{1}{(x - 1)^2} + \frac{1}{x - 2}$$

### Class work:

#### Question No.2

$$\frac{x^2 + 7x + 11}{(x + 2)^2(x + 3)}$$

**Solution:**

$$\text{Let } \frac{x^2 + 7x + 11}{(x + 2)^2(x + 3)} = \frac{A}{x + 2} + \frac{B}{(x + 2)^2} + \frac{C}{x + 3} \rightarrow (i)$$

Multiplying both sides by  $(x + 2)^2(x + 3)$

$$\Rightarrow x^2 + 7x + 11 = A(x + 2)(x + 3) + B(x + 3) + C(x + 2)^2$$

$$\Rightarrow x^2 + 7x + 11 = A(x^2 + 5x + 6) + B(x + 3) + C(x^2 + 4x + 4) \rightarrow (ii)$$

Putting  $x + 2 = 0$  i.e  $x = -2$  in (ii) we get

$$(-2)^2 + 7(-2) + 11 = B(-2 + 3)$$

$$4 - 14 + 11 = B$$

$$\Rightarrow \boxed{B = 1}$$

Putting  $x + 3 = 0$  i.e  $x = -3$  in (ii) we get

$$(-3)^2 + 7(-3) + 11 = C(-3 + 2)^2$$

$$9 - 21 + 11 = C(-1)^2$$

$$20 - 21 = C(1)$$

$$-1 = C$$

$$\Rightarrow \boxed{C = -1}$$

Equating coefficient of  $x^2$  in (ii) we get

$$A + C = 1$$

$$A - 1 = 1$$

$$A = 1 + 1$$

$$\Rightarrow \boxed{A = 2}$$

Hence the required partial fractions are

$$\frac{x^2 + 7x + 11}{(x + 2)^2(x + 3)} = \frac{2}{x + 2} + \frac{1}{(x + 2)^2} - \frac{1}{x + 3}$$

### Class work:

#### Question No.6

$$\frac{1}{(x - 1)^2(x + 1)}$$

**Solution:**

$$\text{Let } \frac{1}{(x - 1)^2(x + 1)} = \frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{C}{x + 1} \rightarrow (ii)$$

Multiplying both sides by  $(x - 1)(x - 1)^2$  we get

$$1 = A(x - 1)(x + 1) + B(x + 1) + C(x - 1)^2 \rightarrow (ii)$$

Putting  $x - 1 = 0$  i.e  $x = 1$  in (ii) we get

$$1 = B(1 + 1)$$

$$1 = 2B$$

$$\Rightarrow \boxed{B = \frac{1}{2}}$$

Putting  $x + 1 = 0$  i.e  $x = -1$  in (ii) we get

$$1 = C(-1 - 1)^2$$

$$1 = C(-2)^2$$

$$1 = 4C$$

$$\Rightarrow \boxed{C = \frac{1}{4}}$$

Equating the coefficient of  $x^2$  in (ii) we get

$$A + C = 0$$

$$A = -C$$

$$A = -\left(\frac{1}{4}\right)$$

$$\Rightarrow \boxed{A = -\frac{1}{4}}$$

Putting the value of

$A, B, \text{ and } C$  in equation (i) we got required partial fractions

$$\frac{1}{(x - 1)^2(x + 1)} = \frac{-1}{4(x - 1)} + \frac{1}{2(x - 1)^2} + \frac{1}{4(x + 1)}$$

### Class work:

#### Question No.8

$$\frac{1}{(x^2 - 1)(x + 1)}$$

**Solution:**

$$\frac{1}{(x^2 - 1)(x + 1)} = \frac{1}{(x - 1)(x + 1)(x + 1)}$$

$$= \frac{1}{(x - 1)(x + 1)^2}$$

$$\text{Let } \frac{1}{(x - 1)(x + 1)^2} = \frac{A}{x - 1} + \frac{B}{x + 1} + \frac{C}{(x + 1)^2}$$

Multiplying both sides by  $(x - 1)(x + 1)^2$  we get

$$1 = A(x + 1)^2 + B(x + 1)(x - 1) + C(x - 1) \rightarrow (ii)$$

Putting  $x - 1 = 0$  i.e  $x = 1$  in (ii) we get

$$1 = A(1 + 1)^2$$

$$1 = A(2)^2$$

$$1 = 4A$$

$$\Rightarrow \boxed{A = \frac{1}{4}}$$

Putting  $x + 1 = 0$  i.e  $x = -1$  in (ii) we get

$$1 = C(-1 - 1)$$

$$1 = -2C$$

$$\Rightarrow \boxed{C = -\frac{1}{2}}$$

Equating the coefficient of  $x^2$  in equation (ii)

We get  $A + B = 0$

$$B = -A$$

$$B = -\left(\frac{1}{4}\right)$$

$$\Rightarrow \boxed{B = -\frac{1}{4}}$$

Putting the value of A and B in equation (ii)

We get required partial fractions.

$$\frac{1}{(x-1)(x+1)^2} = \frac{1}{4(x-1)} - \frac{1}{4(x+1)} - \frac{1}{2(x+1)^2}$$

**Resolution of fraction when  $D(x)$  consists of non-repeated irreducible quadratic factors:**

**Rule III**

If a quadratic factor  $(ax^2 + bx + c)$  with  $a \neq 0$  occur once as a factor of  $D(x)$ . The partial fraction is of the form  $\frac{Ax+B}{(ax^2+bx+c)}$  where A and B are constants to be found.

### Exercise 4.3

Resolve into partial fraction.

**Home Work:**

**Question No.1**

$$\frac{3x-11}{(x+3)(x^2+1)}$$

**Solution:**

$$\text{Let } \frac{3x-11}{(x+3)(x^2+1)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+1}$$

Multiplying both sides  $(x+3)(x^2+1)$ , we get

$$3x-11 = A(x^2+1) + (Bx+C)(x+3) \rightarrow (i)$$

$$3x-11 = A(x^2+1) + Bx(x+3) + C(x+3) \rightarrow (ii)$$

Putting  $x+3=0$  i.e  $x=-3$ , we get

$$3(-3)-11 = A[(-3)^2+1]$$

$$-9-11 = A(9+1)$$

$$-20 = 10A$$

$$A = \frac{-20}{10}$$

$$\Rightarrow \boxed{A = -2}$$

Now equating the coefficient of  $x^2$  and  $x$  we get from equation (iii)

$$A+B=0$$

$$-2+B=0$$

$$B=2$$

$$\Rightarrow \boxed{B=1}$$

$$3B+C=3$$

$$3(2)+C=3$$

$$6+C=3$$

$$C=3-6$$

$$\Rightarrow \boxed{C=-3}$$

Putting the value of A, B and C in equation (i) we get  
Required partial fractions.

$$\frac{3x-11}{(x+3)(x^2+1)} = \frac{-2}{x+3} + \frac{2x-3}{x^2+1}$$

**Home work:**

**Question No.6**

$$\frac{x^2}{(x+2)(x^2+4)}$$

**Solution:**

$$\text{Let } \frac{x^2}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+4} \rightarrow (i)$$

Multiplying both sides by  $(x+2)(x^2+4)$  we get

$$x^2 = A(x^2+4) + (Bx+C)(x+2)$$

$$x^2 = A(x^2+4) + Bx(x+2) + C(x+2) \rightarrow (ii)$$

Putting  $x+2=0$  i.e  $x=-2$  in (ii) we get

$$(-2)^2 = A[(-2)^2+4]$$

$$4 = A(4+4)$$

$$4 = 8 + A$$

$$\Rightarrow \boxed{A = \frac{1}{2}}$$

Equating the coefficients of  $x^3$  and  $x$  in equation (ii)

We get

$$A+B=1$$

$$\frac{1}{2}+B=1$$

$$B=1-\frac{1}{2}$$

$$B=\frac{1}{2}$$

$$\Rightarrow \boxed{B = \frac{1}{2}}$$

$$2B+C=0$$

$$2\left(\frac{1}{2}\right)+C=0$$

$$1+C=0$$

$$C=0-1=-1$$

$$\Rightarrow \boxed{C = -1}$$

Putting the value of A, B and C in equation (i) we get

Required partial fractions.

$$\frac{x^2}{(x+2)(x^2+4)} = \frac{1}{2(x+2)} + \frac{x-2}{2(x^2+4)}$$

**Class work:**

**Question No.8**

$$\frac{x^2+1}{x^3+1}$$

**Solution:**

$$\frac{x^2+1}{x^3+1} = \frac{x^2+1}{(x+1)(x^2-x+1)}$$

$$\text{Let } \frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} \rightarrow (i)$$

Multiplying both sides by  $(x+1)(x^2-x+1)$ , we get

$$x^2+1 = A(x^2-x+1) + (Bx+C)(x+1)$$

$$x^2+1 = A(x^2-x+1) + Bx(x+1) + C(x+1)$$

$\rightarrow (ii)$

Putting  $x+1=0$  i.e  $x=-1$  in (ii) we get

$$(-1)^2+1 = A[(-1)^2-(-1)+1]$$

$$1+1 = A(1+1+1)$$

$$2 = 3A$$

$$\Rightarrow \boxed{A = \frac{2}{3}}$$

Equating the coefficients of  $x^2$  and  $x$  in equation (ii) we get

$$\begin{array}{l|l} A + B = 1 & -A + B + C = 0 \\ \frac{2}{3} + B = 1 & \left(-\frac{2}{3}\right) + \frac{1}{3} + C = 0 \\ B = 1 - \frac{2}{3} & -\frac{1}{3} + C = 0 \\ \Rightarrow \boxed{B = \frac{1}{3}} & \Rightarrow \boxed{C = \frac{1}{3}} \end{array}$$

Putting the value of  $A, B$  and  $C$  in equation (i) we get required partial fractions.

$$\frac{x^2 + 1}{(x + 1)(x^2 - x + 1)} = \frac{2}{3(x + 1)} + \frac{x + 1}{3(x^2 - x + 1)}$$

**Resolution of a fraction when  $D(x)$  has repeated irreducible quadratic factor:**

Rule IV

If a quadratic factor  $(ax^2 + bx + c)$  with  $a \neq 0$  occurs twice in the denominator the corresponding partial fractions are,

$$\frac{Ax + B}{ax^2 + bx + c} + \frac{Cx + D}{(ax^2 + bx + c)^2}$$

The constants  $A, B$ , and  $C$  are found in the usual.

## Exercise 4.4

**Resolve into Partial Fractions.**

**Class work:**

**Question No.2**

$$\frac{x^4 + 3x^2 + x + 1}{(x + 1)(x^2 + 1)^2}$$

**Solution:**

$$\text{Let } \frac{x^4 + 3x^2 + x + 1}{(x + 1)(x^2 + 1)^2} = \frac{A}{x + 1} + \frac{Bx + C}{x^2 + 1} + \frac{Dx + E}{(x^2 + 1)^2} \rightarrow (i)$$

**Multiplying both sides by  $(x + 1)(x^2 + 1)^2$  we get**

$$x^4 + 3x^2 + x + 1 = A(x^2 + 1)^2 + (Bx + C)(x + 1)(x^2 + 1) + (Dx + E)(x + 1) \rightarrow (ii)$$

$$\begin{aligned} x^4 + 3x^2 + x + 1 &= A(x^4 + 2x^2 + 1) \\ &+ Bx(x^3 + x^2 + x + 1) + C(x^3 + x^2 + x + 1) \\ &+ Dx(x + 1) + E(x + 1) \\ x^4 + 3x^2 + x + 1 &= A(x^4 + 2x^2 + 1) \\ &+ B(x^4 + x^3 + x^2 + x) + C(x^3 + x^2 + x + 1) \\ &+ D(x^2 + x) + E(x + 1) \rightarrow (iii) \end{aligned}$$

Putting  $x + 1 = 0$  i.e  $x = -1$  in eq(ii) we get

$$\begin{aligned} (-1)^4 + 3(-1)^2 + (-1) + 1 &= A[(-1)^2 + 1]^2 \\ 1 + 3(1) - 1 + 1 &= A(1 + 1)^2 \end{aligned}$$

$$4 = 4A$$

$$\Rightarrow A = 1$$

Now equating the coefficients of  $x^4, x^3, x^2, x$  and constants, we get from equation (iii) coefficients of  $x^4$ :  $A + B = 1$

$$1 + B = 1$$

$$B = 1 - 1$$

$$\Rightarrow \boxed{B = 0}$$

Coefficients of  $x^3$ :  $B + C = 0$

$$0 + C = 0$$

$$\Rightarrow \boxed{C = 0}$$

Coefficients of  $x^2$ :  $2A + B + C + D = 3$

$$2(1) + 0 + 0 + D = 3$$

$$D = 3 - 2$$

$$D = 1$$

Coefficients of  $x$ :  $B + C + D + E = 1$

$$0 + 0 + 1 + E = 1$$

$$E = 1 - 1$$

$$\Rightarrow \boxed{E = 0}$$

Putting the value of  $A, B, C$  and  $D$  in equation (i)

We get required partial fractions.

$$\text{Let } \frac{x^4 + 3x^2 + x + 1}{(x + 1)(x^2 + 1)^2} = \frac{1}{x + 1} + \frac{x}{(x^2 + 1)^2}$$

**Home Work:**

**Question No.3**

$$\frac{x^2}{(x + 1)(x^2 + 1)^2}$$

**Solution:**

$$\text{Let } \frac{x^2}{(x + 1)(x^2 + 1)^2} = \frac{A}{x + 1} + \frac{Bx + C}{x^2 + 1} + \frac{Dx + E}{(x^2 + 1)^2} \rightarrow (i)$$

**Multiplying both sides by  $(x + 1)(x^2 + 1)^2$  we get**

$$x^2 = A(x^2 + 1)^2 + (Bx + C)(x + 1)(x^2 + 1) + (Dx + E)(x + 1) \rightarrow (ii)$$

$$x^2 = A(x^4 + 2x^2 + 1) + Bx(x^3 + x^2 + x + 1) +$$

$$C(x^3 + x^2 + x + 1) + Dx(x + 1) + E(x + 1)$$

$$x^2 = A(x^4 + 2x^2 + 1) + B(x^4 + x^3 + x^2 + x) +$$

$$+ C(x^3 + x^2 + x + 1) + D(x^2 + x) + E(x + 1) \rightarrow (iii)$$

Putting  $x + 1 = 0$  i.e  $x = -1$  in eq(ii) we get

$$(-1)^2 = A[(-1)^2 + 1]^2$$

$$1 = A(1 + 1)^2$$

$$1 = 4A$$

$$\Rightarrow \boxed{A = \frac{1}{4}}$$

Now equating the coefficients of  $x^4, x^3, x^2, x$  and constants, we get from equation (iii) coefficients of  $x^4$ :  $A + B = 0$

$$\frac{1}{4} + B = 1$$

$$\Rightarrow \boxed{B = -\frac{1}{4}}$$

Coefficients of  $x^3$ :  $B + C = 0$ 

$$-\frac{1}{4} + C = 0$$

$$\Rightarrow \boxed{C = \frac{1}{4}}$$

Coefficients of  $x^2$ :  $2A + B + C + D = 1$ 

$$2\left(\frac{1}{4}\right) - \frac{1}{4} + \frac{1}{4} + D = 1$$

$$\frac{1}{2} + D = 1$$

$$D = 1 - \frac{1}{2}$$

$$\Rightarrow D = \frac{2-1}{2}$$

$$\Rightarrow \boxed{D = \frac{1}{2}}$$

Coefficients of  $x$ :  $B + C + D + E = 0$ 

$$-\frac{1}{4} + \frac{1}{4} + \frac{1}{2} + E = 0$$

$$\frac{1}{2} + E = 0$$

$$\Rightarrow \boxed{E = -\frac{1}{2}}$$

Putting the value of  $A, B, C$  and  $D$  in equation (i)

We get required partial fractions.

$$\frac{x^2}{(x+1)(x^2+1)^2} = \frac{A}{4(x+1)} + \frac{x-1}{4(x^2+1)} + \frac{x-1}{2(x^2+1)^2}$$

**Home Work:****Question No.6**

$$\frac{x^5}{(x^2+1)^2}$$

**Solution:**

$$\frac{x^5}{(x^2+1)^2} = \frac{x^5}{x^4+2x^2+1} \text{ is an improper fraction.}$$

First we resolve it into proper fraction.

$$\frac{x^5}{(x^2+1)^2} = x + \frac{x^4+2x^2+1\sqrt{x^5}}{(x^2+1)^2}$$

$$\frac{x^5}{(x^2+1)^2} = x + \frac{\pm x^5 \pm 2x^3 \pm x}{-2x^3 - x}$$

$$\frac{x^5}{(x^2+1)^2} = x + \frac{-2x^3 - x}{(x^2+1)^2}$$

$$\text{Let } \frac{-2x^3 - x}{(x^2+1)^2} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2} \rightarrow (i)$$

Multiplying both sides by  $(x^2+1)^2$  we get

$$-2x^3 - x = (Ax+B)(x^2+1) + (Cx+D)$$

$$-2x^3 - x = A(x^3+x) + B(x^2+1) + Cx + D$$

Equating the coefficients of  $x^3, x^2, x$  and constants

We get

Coefficients of  $x^3$ :  $A = -2$ Coefficients of  $x^2$ :  $B = 0$ Coefficients of  $x$ :  $A + C = -1$ 

$$-2 + C = -1$$

$$C = -1 + 2$$

$$\Rightarrow \boxed{C = 1}$$

Constants:  $B + D = 0$ 

$$0 + D = 0$$

$$\Rightarrow \boxed{D = 0}$$

Hence the required partial fractions are

$$\frac{x^5}{(x^2+1)^2} = x + \frac{-2x}{x^2+1} + \frac{x}{(x^2+1)^2}$$

$$\frac{x^5}{(x^2+1)^2} = x - \frac{2x}{x^2+1} + \frac{x}{(x^2+1)^2}$$

**Miscellaneous Exercise -4****Q. 1 Multiple Choice Questions:**

Four possible answers are given for the following questions. Tick (✓) the correct answer.

1. The identity  $(5x+4)^2 = 25x^2 + 40x + 16$  is true for:

- (a) one value of  $x$       (b) two values of  $x$   
 (c) all values of  $x$       (d) none of these

2. A function of the form  $f(x) = \frac{N(x)}{D(x)}$ , with  $D(x) \neq 0$ , where  $N(x)$  and  $D(x)$  are polynomials in  $x$  is called:

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- (a) an identity      (b) an equation  
 (c) a fraction      (d) none of these

3. A fraction in which the degree of the numerator is greater or equal to the degree of denominator is called:

- (a) a proper fraction  
 (b) an improper fraction  
 (c) an equation  
 (d) algebraic relation

4. A fraction in which the degree of numerator is less than the degree of the denominator is called:

- (a) an equation  
 (b) an improper fraction  
 (c) an identity  
 (d) a proper fraction

5.  $\frac{2x+1}{(x+1)(x-1)}$  is:

- (a) an improper fraction  
 (b) an equation  
 (c) a proper fraction  
 (d) none of these

6.  $(x+3)^2 = x^2 + 6x + 9$  is:

- (a) a linear equation  
 (b) an equation  
 (c) an identity  
 (d) none of these

7.  $\frac{x^3+1}{(x-1)(x+2)}$  is:

- (a) a proper fraction  
 (b) an improper fraction  
 (c) an identity  
 (d) a constant term

8. Partial fractions of  $\frac{x-2}{(x-1)(x+2)}$  are of the

form:

- (a)  $\frac{A}{x-1} + \frac{B}{x+2}$  (b)  $\frac{Ax}{x-1} + \frac{B}{x+2}$   
 (c)  $\frac{A}{x-1} + \frac{Bx+C}{x+2}$  (d)  $\frac{Ax+B}{x-1} + \frac{C}{x+2}$

9. Partial fractions of  $\frac{x+2}{(x+1)(x^2+2)}$  are of

the form:

- (a)  $\frac{A}{x+1} + \frac{B}{x^2+2}$   
 (b)  $\frac{A}{x+1} + \frac{Bx+C}{x^2+2}$   
 (c)  $\frac{Ax+B}{x+1} + \frac{C}{x^2+2}$   
 (d)  $\frac{A}{x+1} + \frac{Bx}{x^2+2}$

10. Partial fractions of  $\frac{x^2+1}{(x+1)(x-1)}$  are of the form:

- (a)  $\frac{A}{x+1} + \frac{B}{x-1}$   
 (b)  $1 + \frac{A}{x+1} + \frac{Bx+C}{x-1}$   
 (c)  $1 + \frac{A}{x+1} + \frac{B}{x-1}$   
 (d)  $\frac{Ax+B}{(x+1)} + \frac{C}{x-1}$

(Answer key)

1.	c	2.	c	3.	b	4.	d	5.	c
6.	c	7.	b	8.	a	9.	b	10.	c

**Question No.2 Write a short answer of the following.**

**1. Define a rational fraction**

**Rational Fraction:**

An expression of the form  $\frac{N(x)}{D(x)}$ , with  $D(x) \neq 0$  and  $N(x)$  and  $D(x)$  are polynomials in  $x$  expressed with real coefficients, is called a **rational fraction**. Every fraction expression can be expressed as a quotient of two polynomials.

For example:

$$\frac{2x}{(x-1)(x+2)}$$

**2. What is a proper fraction?**

**Proper Fraction:**

A rational fraction  $\frac{N(x)}{D(x)}$ , with  $D(x) \neq 0$  is called a proper fraction if degree of the polynomial  $N(x)$  in the numerator is less than the degree of the polynomial  $D(x)$  in the denominator.

For example:

$$\frac{2}{x+1}$$

**3. What is an improper fraction.**

**Improper Fraction:**

A rational fraction  $\frac{N(x)}{D(x)}$ , with  $D(x) \neq 0$  is called improper fraction if degree of the polynomial  $N(x)$  is greater or equal to the degree of the polynomial  $D(x)$ .

For example:

$$\frac{5x}{x+2}, \quad \frac{6x^4}{x^3+1}$$

**4. What are partial fraction?**

Decomposition of resultant fraction  $\frac{N(x)}{D(x)}$  when

- (a) Denominator  $D(x)$  consists to non-repeated linear factors.  
 (b) Denominator  $D(x)$  consists of repeated linear factors.  
 (c) Denominator  $D(x)$  contains non repeated irreducible quadratic factor.  
 (d) Denominator  $D(x)$  has repeated quadratic factor.



# Unit-5

## SETS AND FUNCTIONS

### Sets.

A set is well defined collection objects and it is denoted by capital letters A,B,C etc.

**Recognize operations on sets ( $\cup, \cap, \setminus$ )**

#### a) Union of sets

The union of two sets  $A$  and  $B$  written as  $A \cup B$  (read as A union B) is the consisting of all the elements which are either in A or in B or in both.

Thus

$$A \cup B = \{x | x \in A \text{ or } x \in B \text{ or } x \in A \text{ and } B \text{ both}\}$$

**For example:**

If  $A = \{1, 2, 3, 4\}$  and  $B = \{4, 5, 6, 7\}$  then

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$$

#### b) Intersection of sets

The intersection of two sets A and B, written as  $A \cap B$  (read as A intersection B) is the set consisting of all the common elements of A and B. Thus

$$A \cap B = \{x | x \in A \text{ and } x \in B\}$$

Clearly  $x \in A \cap B \Rightarrow x \in A \text{ and } x \in B$

**For example:**

If  $A = \{1, 2, 3, 4\}$  and  $B = \{3, 4, 5, 6\}$  then

$$A \cap B = \{3, 4\}$$

#### c) Difference of Sets

If A and B are two sets, then their difference  $A - B$

Or  $A \setminus B$  is defined as

$$A - B = \{x | x \in A \text{ and } x \notin B\}$$

Similarly,

$$B - A = \{x | x \in B \text{ and } x \notin A\}$$

**For example:**

If  $A = \{1, 2, 3, 4\}$  and  $B = \{4, 5, 6, 7\}$  then

$$A - B = \{1, 2, 3\}$$

Similarly,

$$B - A = \{5, 6, 7\}$$

#### d) Complement of a set

If U is a universal set and A is a subset of U, then the complement of A is the set of those element of U, which are not contained in A and is denoted by  $A'$  or  $A^c$

$$A' = U - A = \{x | x \in U \text{ and } x \notin A\}$$

**For example:**

If  $U = \{1, 2, 3, \dots, 10\}$  and  $A = \{2, 4, 6, 8\}$  then

$$\begin{aligned} A' &= U - A \\ &= \{1, 2, 3, \dots, 10\} - \{2, 4, 6, 8\} \\ &= \{1, 3, 5, 7, 9, 10\} \end{aligned}$$

## EXERCISE 5.1

**Question No.1** If  $X = \{1, 4, 7, 9\}$  and

$$Y = \{2, 4, 5, 9\} \text{ then find:}$$

**Solution:**

**Class Work:**

(i)  $X \cup Y$

$$\begin{aligned} X \cup Y &= \{1, 4, 7, 9\} \cup \{2, 4, 5, 9\} \\ &= \{1, 2, 4, 5, 7, 9\} \end{aligned}$$

**Home Work:**

(ii)  $X \cap Y$

$$\begin{aligned} X \cap Y &= \{1, 4, 7, 9\} \cap \{2, 4, 5, 9\} \\ &= \{4, 9\} \end{aligned}$$

**Home Work:**

(iii)  $Y \cup X$

$$\begin{aligned} Y \cup X &= \{2, 4, 5, 9\} \cup \{1, 4, 7, 9\} \\ &= \{1, 2, 4, 5, 7, 9\} \end{aligned}$$

**Home Work:**

(iv)  $Y \cap X$

$$\begin{aligned} Y \cap X &= \{2, 4, 5, 9\} \cap \{1, 4, 7, 9\} \\ &= \{4, 9\} \end{aligned}$$

**Question No.3**

If  $X = \phi$   $Y = Z^+$   $T = O^+$  then find

**Class Work:**

(i)  $X \cup Y$

$$X = \phi \quad Y = \{0, 1, 2, 3, \dots\}$$

$$X \cup Y = \{ \} \cup \{0, 1, 2, 3, \dots\}$$

$$X \cup Y = \{0, 1, 2, 3, \dots\}$$

**Home Work:**

(ii)  $X \cup T$

$$X = \phi \quad T = \{1, 3, 5, \dots\}$$

$$X \cup T = \phi \cup \{1, 3, 5, \dots\}$$

$$X \cup T = \{1, 3, 5, \dots\}$$

**Home Work:**

(iii)  $Y \cup T$

$$Y = \{0, 1, 2, 3, \dots\} \quad T = \{1, 3, 5, 7, \dots\}$$

$$Y \cup T = \{0, 1, 2, 3, \dots\} \cup \{1, 3, 5, 7, \dots\}$$

$$Y \cup T = \{0, 1, 2, 3, 4, 5, \dots\}$$

**Home Work:**

(iv)  $X \cap Y$

$$X = \phi \quad Y = \{0, 1, 2, 3, \dots\}$$

$$X \cap Y = \{ \} \cap \{0, 1, 2, 3, \dots\}$$

$$X \cap Y = \{ \}$$

**Home Work+ Class work**(v)  $X \cap T$ 

$$X = \phi \quad T = \{1, 3, 5, 7, \dots\}$$

$$X \cap T = \{ \} \cap \{1, 3, 5, 7, \dots\}$$

$$X \cap T = \{ \} \text{ or } \phi$$

**Question No.4** If  $U = \{x | x \in N \wedge 3 < x \leq 25\}$ 

$$X = \{x | x \text{ is Prime } \wedge 8 < x < 25\}$$

$$Y = \{x | x \in W \wedge 4 \leq x \leq 17\}$$

then find the value of:

**Solution:**  $U = \{4, 5, 6, 7, \dots, 25\}$

$$X = \{11, 13, 17, 19, 23\}$$

$$Y = \{4, 5, 6, 7, \dots, 17\}$$

**Class work:**(i)  $(X \cup Y)'$ 

$$X \cup Y = \{11, 13, 17, 19, 23\} \cup \{4, 5, 6, 7, \dots, 17\}$$

$$= \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 23\}$$

$$(X \cup Y)' = U - (X \cup Y)$$

$$= \{4, 5, 6, 7, \dots, 25\} - \{4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 23\}$$

$$= \{18, 20, 21, 22, 24, 25\}$$

**Home work:**(iv)  $X' \cup Y'$ 

$$X' = U - X = \{4, 5, 6, 7, \dots, 25\} - \{11, 13, 17, 19, 23\}$$

$$= \{4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25\}$$

$$Y' = U - Y = \{4, 5, 6, 7, \dots, 25\} - \{4, 5, 6, 7, \dots, 17\}$$

$$= \{18, 19, 20, 21, 22, 23, 24, 25\}$$

$$X' \cup Y' = \{4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25\}$$

$$\cup \{18, 19, 20, 21, 22, 23, 24, 25\}$$

$$= \{4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25\}$$

**Question No.6** If  $A=N$  and  $B=W$  then find the value of**Solution:****Class Work:**(i)  $A - B$ 

$$A - B = N - W = \{1, 2, 3, \dots\} - \{0, 1, 2, 3, \dots\}$$

$$= \{ \}$$

**Home Work:**(ii)  $B - A$ 

$$B - A = W - N = \{0, 1, 2, 3, \dots\} - \{1, 2, 3, \dots\}$$

$$= \{0\}$$

**De-Morgan's Law**

For any two sets A and B

$$(i). (A \cup B)' = A' \cap B'$$

$$(ii). (A \cap B)' = A' \cup B'$$

**EXERCISE 5.2****Question No.1**

$$\text{if } X = \{1, 3, 5, 7, \dots, 19\} \quad Y = \{0, 2, 4, 6, 8, \dots, 20\}$$

$$Z = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

then find the following

**(iv) Home work**

$$(X \cap Y) \cap Y$$

$$= \{(1, 3, 5, 7, \dots, 19) \cap (0, 2, 4, 6, 8, \dots, 20)\} \cap (2, 3, 5, 7, 11, 13, 17, 19, 23)$$

$$= \{ \} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

$$= \{ \}$$

**(v) Class Work+ Home work**

$$X \cup (Y \cap Z)$$

$$= \{1, 3, 5, 7, \dots, 19\} \cup \left\{ (0, 2, 4, 6, 8, \dots, 20) \cap (2, 3, 5, 7, 11, 13, 17, 19, 23) \right\}$$

$$= \{1, 3, 5, 7, \dots, 19\} \cup \{2\}$$

$$= \{1, 2, 3, 5, 7, \dots, 19\}$$

**(vi) Home work**

$$(X \cup Y) \cap (X \cup Z)$$

$$X \cup Y = \{1, 3, 5, 7, \dots, 19\} \cup \{0, 2, 4, 6, 8, \dots, 20\}$$

$$= \{0, 1, 2, 3, 4, 5, 6, 7, \dots, 20\}$$

$$X \cup Z = \{1, 3, 5, 7, \dots, 19\} \cup \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

$$= \{1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19, 23\}$$

$$= (X \cup Y) \cap (X \cup Z)$$

$$= \{0, 1, 2, 3, 4, 5, 6, 7, \dots, 20\} \cap \{1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19, 23\}$$

$$= \{1, 2, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$$

**(vii) Home work**

$$X \cap (Y \cap Z)$$

$$= \{1, 3, 5, 7, \dots, 19\} \cap \{(0, 2, 4, 6, 8, \dots, 20) \cap (2, 3, 5, 7, 11, 13, 17, 19, 23)\}$$

$$= \{1, 3, 5, 7, \dots, 19\} \cap \{0, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 23\}$$

$$= \{3, 5, 7, 11, 13, 17, 19\}$$

**(viii) Home work**

$$(X \cap Y) \cup (X \cap Z)$$

$$X \cap Y = \{1, 3, 5, 7, \dots, 19\} \cap \{0, 2, 4, 6, 8, \dots, 20\}$$

$$X \cap Y = \{ \}$$

$$X \cap Z = \{1, 3, 5, 7, \dots, 19\} \cap \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$$

$$X \cap Z = \{3, 5, 7, 11, 13, 17, 19\}$$

$$(X \cap Y) \cup (X \cap Z) = \{ \} \cup \{3, 5, 7, 11, 13, 17, 19\}$$

$$(X \cap Y) \cup (X \cap Z) = \{3, 5, 7, 11, 13, 17, 19\}$$

$$= \{1, 2, 3, \dots, 10\} - \{1, 3, 5, 7, 9\}$$

$$= \{2, 4, 6, 8, 10\}$$

**Question No.2**

If  $A = \{1, 2, 3, 4, 5, 6\}$ ,  $B = \{2, 4, 6, 8\}$ ,  $C = \{1, 4, 8\}$

*prove that the following identities.*

**(iv) Class work:**

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

**Solution:**

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$\text{L.H.S} = A \cup (B \cap C)$$

$$= A \cup (\{2, 4, 6, 8\} \cap \{1, 4, 8\})$$

$$= \{1, 2, 3, 4, 5, 6\} \cap \{4, 8\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\}$$

$$\text{R.H.S} = (A \cup B) \cap (A \cup C)$$

$$A \cup B = \{1, 2, 3, 4, 5, 6\} \cup \{2, 4, 6, 8\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\}$$

$$A \cup C = \{1, 2, 3, 4, 5, 6\} \cup \{1, 4, 8\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\}$$

$$(A \cup B) \cap (A \cup C) = \{1, 2, 3, 4, 5, 6, 8\} \cap \{1, 2, 3, 4, 5, 6, 8\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$\text{So, } A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

**Home work:**

**Question No.3**

$$\text{If } U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{1, 3, 5, 7, 9\}, B = \{2, 3, 5, 7\}$$

then verify the De Morgan's laws i.e.,

$$(A \cup B)' = A' \cap B' \text{ and } (A \cap B)' = A' \cup B'$$

**Solution :**

$$(A \cup B)' = A' \cap B'$$

$$\text{L.H.S} = (A \cup B)'$$

$$A \cup B = \{1, 3, 5, 7, 9\} \cup \{2, 3, 5, 7\}$$

$$= \{1, 2, 3, 5, 7, 9\}$$

$$(A \cup B)' = U - (A \cup B)$$

$$= \{1, 2, 3, \dots, 10\} - \{1, 2, 3, 5, 7, 9\}$$

$$= \{4, 6, 8, 10\}$$

$$\text{R.H.S} = A' \cap B'$$

$$A' = U - A$$

$$B' = U - B$$

$$= \{1, 2, 3, \dots, 10\} - \{2, 3, 5, 7\}$$

$$= \{1, 4, 6, 8, 9, 10\}$$

$$A' \cap B' = \{2, 4, 6, 8, 10\} \cap \{1, 4, 6, 8, 9, 10\}$$

$$= \{4, 6, 8, 10\}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$(A \cup B)' = A' \cap B'$$

(ii)

$$(A \cap B)' = A' \cup B'$$

$$\text{L.H.S} = (A \cap B)'$$

$$A \cap B = \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 7\}$$

$$= \{3, 5, 7\}$$

$$(A \cap B)' = U - (A \cap B)$$

$$= \{1, 2, 3, \dots, 10\} - \{3, 5, 7\}$$

$$= \{1, 2, 4, 6, 8, 9, 10\}$$

$$\text{R.H.S} = A' \cup B'$$

$$A' = U - A$$

$$= \{1, 2, 3, \dots, 10\} - \{1, 3, 5, 7, 9\}$$

$$= \{2, 4, 6, 8, 10\}$$

$$B' = U - B$$

$$= \{1, 2, 3, \dots, 10\} - \{2, 3, 5, 7\}$$

$$= \{1, 4, 6, 8, 9, 10\}$$

$$A' \cup B' = \{2, 4, 6, 8, 10\} \cup \{1, 4, 6, 8, 9, 10\}$$

$$= \{1, 2, 4, 6, 8, 9, 10\}$$

$$\text{L.H.S} = \text{R.H.S}$$

$$(A \cap B)' = A' \cup B'$$

**Home work:**

**Question No.4**

$$\text{If } U = \{1, 2, 3, \dots, 20\}, X = \{1, 3, 7, 9, 15, 18, 20\}$$

$$Y = \{1, 3, 5, \dots, 17\} \text{ then show that}$$

$$(ii) Y - X = Y \cap X'$$

**Solution:**

$$\text{L.H.S} = Y - X$$

$$= \{1, 3, 5, \dots, 17\} - \{1, 3, 7, 9, 15, 18, 20\}$$

$$= \{5, 11, 13, 17\}$$

$$\text{R.H.S} = Y \cap X'$$

$$X' = U - X$$

$$\begin{aligned}
 &= \{1, 2, 3, \dots, 20\} - \{1, 3, 7, 9, 15, 18, 20\} \\
 &= \left\{ \begin{array}{l} 2, 4, 5, 6, 8, 10, 11, 12, 13, \\ 14, 16, 17, 19 \end{array} \right\} \\
 Y \cap X' &= \{1, 3, 5, \dots, 17\} \cap \left\{ \begin{array}{l} 2, 4, 5, 6, 8, \\ 10, 11, 12, 13, 14, 16, 17, 19 \end{array} \right\} \\
 &= \{5, 11, 13, 17\} \\
 \text{L.H.S.} &= \text{R.H.S} \\
 Y - X &= Y \cap X'
 \end{aligned}$$

**verify the fundamental properties for given sets:**

- A and B are any two subsets of U, then  
 $A \cup B = B \cup A$  (commutative Law)
- Commutative property of intersection
- If A, B and C are the subsets of U then  
 $(A \cup B) \cup C = A \cup (B \cup C)$
- If A, B and C are the subsets of U, then  
 $(A \cap B) \cap C = A \cap (B \cap C)$

### Distributive laws

- Union is distributive over intersection of sets
- Intersection is distributive over union of sets
- De Morgan's laws  $A \cap B)' = A' \cup B'$  and  $(A \cup B)' = A' \cap B'$

### Venn Diagram:

British mathematician John Venn (1834-1923) introduced rectangular for a universal set U and its subsets A and B are closed figures inside this rectangular.

## EXERCISE 5.3

### Question No.1

if  $U = \{1, 2, 3, 4, \dots, 10\}$ ,  $A = \{1, 3, 5, 7, 9\}$  and  $B = \{1, 4, 7, 10\}$

Then verify the following questions.

### Class work:

- $A - B = A \cap B'$   
 $\text{L.H.S} = A - B$   
 $= \{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\}$   
 $= \{3, 5, 9\}$   
 $\text{R.H.S} = A \cap B'$   
 $B' = U - B$   
 $= \{1, 2, 3, 4, \dots, 10\} - \{1, 4, 7, 10\}$   
 $B' = \{2, 3, 5, 6, 8, 9\}$   
 $A \cap B' = \{1, 3, 5, 7, 9\} \cap \{2, 3, 5, 6, 8, 9\}$   
 $= \{3, 5, 9\}$   
 So, L.H.S=R.H.S

$$(iii) (A \cup B)' = A' \cap B'$$

### Home work:

$$\begin{aligned}
 &(A \cup B)' = A' \cap B' \\
 \text{L.H.S} &= (A \cup B)' = U - (A \cup B) \\
 U - (A \cup B) &= \{1, 2, 3, 4, \dots, 10\} \\
 &\quad - (\{1, 3, 5, 7, 9\} \cup \{1, 4, 7, 10\}) \\
 U - (A \cup B) &= \{1, 2, 3, 4, \dots, 10\} \\
 &\quad - \{1, 3, 4, 5, 7, 9, 10\} \\
 \text{L.H.S} &= \{2, 6, 8\} \\
 \text{R.H.S} &= A' \cap B' \\
 A' &= U - A = \{1, 2, 3, 4, \dots, 10\} - \{1, 3, 5, 7, 9\} \\
 &= \{2, 4, 6, 8, 10\} \\
 B' &= U - B = \{1, 2, 3, 4, \dots, 10\} - \{1, 4, 7, 10\} \\
 &= \{2, 3, 5, 6, 8, 9\} \\
 A' \cap B' &= \{2, 4, 6, 8, 10\} \cap \{2, 3, 5, 6, 8, 9\} \\
 \text{R.H.S} &= \{2, 6, 8\}
 \end{aligned}$$

$$(v) (A - B)' = A' \cup B$$

### Home Work:

$$\begin{aligned}
 &(A - B)' = A' \cup B \\
 \text{L.H.S} &= (A - B)' \\
 &= U - (A - B) \\
 &= \{1, 2, 3, 4, \dots, 10\} - (\{1, 3, 5, 7, 9\} - \{1, 4, 7, 10\}) \\
 U - (A - B) &= \{1, 2, 3, 4, \dots, 10\} - \{3, 5, 9\} \\
 \text{L.H.S} &= \{1, 2, 4, 6, 7, 8, 10\} \\
 \text{R.H.S} &= A' \cup B \\
 A' &= U - A = \{1, 2, 3, 4, \dots, 10\} - \{1, 3, 5, 7, 9\} \\
 &= \{2, 4, 6, 8, 10\} \\
 A' \cup B &= \{2, 4, 6, 8, 10\} \cup \{1, 4, 7, 10\} \\
 \text{R.H.S} &= \{1, 2, 4, 6, 7, 8, 10\}
 \end{aligned}$$

So, L.H.S = R.H.S

### Question No.2

if  $U = \{1, 2, 3, 4, \dots, 10\}$ ,  $A = \{1, 3, 5, 7, 9\}$

$B = \{1, 4, 7, 10\}$ ,  $C = \{1, 5, 8, 10\}$

Then verify the following questions.

### Home work:

$$\begin{aligned}
 (ii) (A \cap B) \cap C &= A \cap (B \cap C) \\
 \text{L.H.S} &= (A \cap B) \cap C \\
 &= (\{1, 3, 5, 7, 9\} \cap \{1, 4, 7, 10\}) \cap \{1, 5, 8, 10\} \\
 &= \{1, 3, 4, 5, 7, 9\} \cap \{1, 10\} \\
 \text{L.H.S} &= \{1\} \\
 \text{R.H.S} &= A \cap (B \cap C) \\
 &= \{1, 3, 5, 7, 9\} \cap (\{1, 4, 7, 10\} \cap \{1, 5, 8, 10\}) \\
 &= \{1, 3, 5, 7, 9\} \cap \{1, 10\} \\
 \text{R.H.S} &= \{1\}
 \end{aligned}$$

So, L.H.S = R.H.S

### Class work:

$$\begin{aligned}
 (iii) A \cup (B \cap C) &= (A \cup B) \cap (A \cup C) \\
 \text{L.H.S} &= A \cup (B \cap C)
 \end{aligned}$$

$$\begin{aligned}
 &= (\{1,3,5,7,9\} \cup \{1,4,7,10\}) \cap \{1,5,8,10\} \\
 &= \{1,3,5,7,9\} \cup \{1,10\} \\
 L.H.S &= \{1,3,5,7,9,10\} \\
 R.H.S &= (A \cup B) \cap (A \cup C) \\
 &= (\{1,3,5,7,9\} \cup \{1,4,7,10\}) \cap (\{1,3,5,7,9\} \cup \{1,5,8,10\}) \\
 &= \{1,3,4,5,7,9,10\} \cap \{1,3,5,7,8,9,10\} \\
 R.H.S &= \{1,3,5,7,9,10\} \\
 \text{So } L.H.S &= R.H.S
 \end{aligned}$$

**Question No.4**

If  $U = \{1, 2, 3, 4, \dots, 10\}$ ,  $A = \{1, 3, 5, 7, 9\}$

$B = \{2, 3, 4, 5, 8\}$  then prove the following questions by Venn diagram:

**Class Work:**

(iii)

$$(A \cup B)' = A' \cap B'$$

$$L.H.S = (A \cup B)'$$

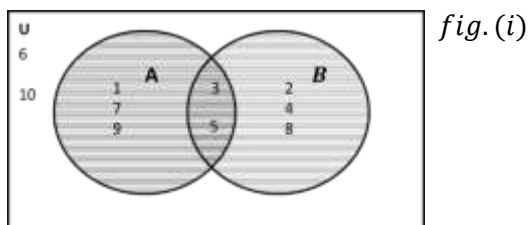
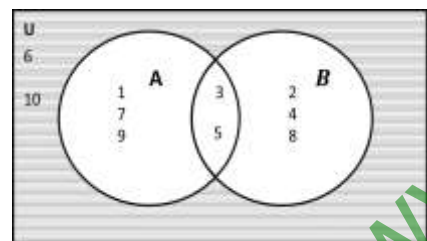


fig.(i)

$$A \cup B = \{1, 2, 3, 4, 5, 7, 8, 9\}$$

Fig (i) show that  $A \cup B$

$$(A \cup B)' = U - (A \cup B) = \{6, 10\}$$



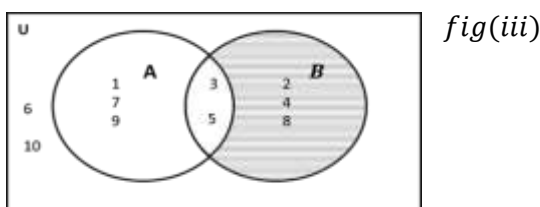
fig(ii)

$$(A \cup B)' = U - (A \cup B) = \{6, 10\}$$

fig(ii) show that  $(A \cup B)'$

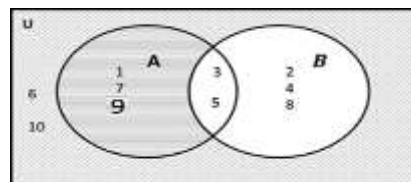
$$R.H.S = A' \cap B'$$

$$A' = U - A = \{2, 4, 6, 8, 10\}$$



fig(iii)

fig(iii) show that  $A'$

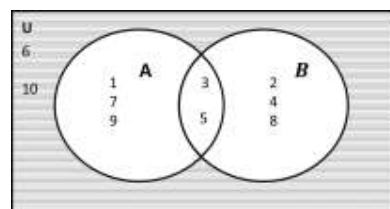


fig(iv)

$$B' = U - B = \{1, 6, 7, 9, 10\}$$

fig(iv) show that  $B'$

$$R.H.S = A' \cap B' = \{6, 10\}$$



fig(v)

fig(v) show that  $A' \cap B'$

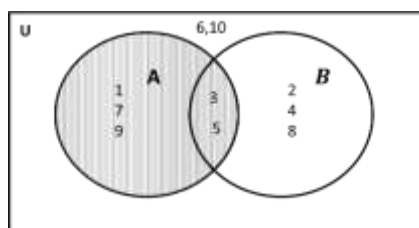
from fig(ii) and (v)

$$(A \cup B)' = A' \cap B'$$

**Home work:**

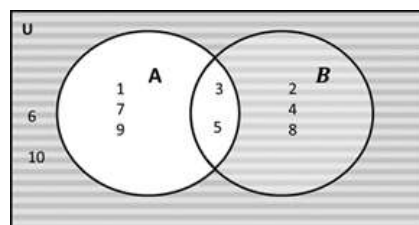
$$(v) (A - B)' = A' \cup B$$

$$\begin{aligned}
 L.H.S &= (A - B)' = U - (A - B) \\
 &= \{2, 3, 4, 5, 6, 8, 10\}
 \end{aligned}$$



fig(i)

$(A - B)$  shadow part.



fig(ii)

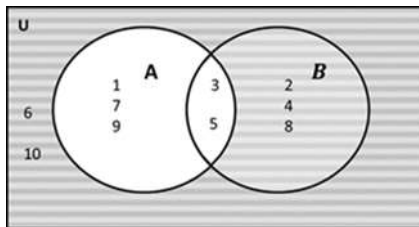
$(A - B)'$  shadow part

$$R.H.S = A' \cup B$$

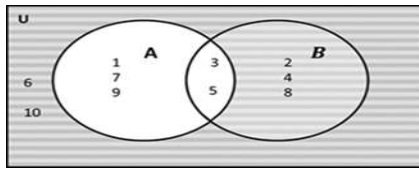
$$A' = \{2, 4, 6, 8, 10\}$$

$$A' \cup B = \{2, 4, 6, 8, 10\} \cup \{2, 3, 4, 5, 8\}$$

$$= \{2, 3, 4, 5, 6, 8, 10\}$$



$A'$  shadow part



From (ii) and (iv)  $A' \cup B$  shadow part

$$(A - B)' = A' \cup B$$

$$L.H.S = R.H.S$$

**Ordered pairs:**

Any two numbers  $x$  and  $y$  written in the form  $(x, y)$  is called an ordered pair. In an ordered pair  $(x, y)$ , the order of numbers is important, that is  $x$  is the first co-ordinate and  $y$  is the second co-ordinate.

For example,  $(3, 2)$  is different from  $(2, 3)$

Note that  $(x, y) = (s, t)$ , iff  $x = s$  and  $y = t$

**Cartesian Product:**

Cartesian product of two non-empty sets  $A$  and  $B$  denoted by  $A \times B$  consist of all ordered pair  $(x, y)$  such that  $x \in A$  and  $y \in B$

## EXERCISE 5.4

**Question No.3** Find  $a$  and  $b$  if

**Solution:**

**Class work:**

(iii)

$$(3 - 2a, b - 1) = (a - 7, 2b + 5)$$

$$3 - 2a = a - 7, \quad b - 1 = 2b + 5$$

$$3 + 7 = a + 2a, \quad -1 - 5 = 2b - b$$

$$10 = 3a, \quad -6 = b$$

$$\frac{10}{3} = a, \quad \boxed{b = -6}$$

$$\boxed{a = \frac{10}{3}}$$

**Home work:**

**Question.5** If  $X = \{a, b, c\}$  and  $Y = \{d, e\}$ , then find the number of elements in

**Solution:**

(ii) No. of elements in  $Y \times X = n(Y \times X) = 2 \times 3 = 6$

**Binary relation:**

If  $A$  and  $B$  are any two non-empty sets, then a subset  $R \subseteq A \times B$  is called binary relation from set  $A$  into set  $B$ . because there exist some relationship between first and second element of each ordered pair in  $R$ .

**Domain of relation** denoted by  $\text{Dom } R$  is the set consisting of all the first elements of each ordered pair in the relation.

**Range of relation** denoted by  $\text{Rang } R$  is the set consisting of all the second elements of each ordered pair in the relation.

**Function or Mapping**

Suppose  $A$  and  $B$  are two non-empty sets, then relation  $f: A \rightarrow B$  is called a function

if

(i).  $\text{Dom } f = A$

(ii).  $\forall x \in A$  we can associate some unique image element  $y = f(x) \in B$

**Domain, co-domain and Range of function**

If  $f: A \rightarrow B$  is a function, then  $A$  is called the domain of  $f$  and  $B$  is called the co-domain of  $f$

Domain  $f$  is the set consisting of all first elements of each ordered pair in  $f$  and range  $f$  is the set consisting of all second of each ordered pair in  $f$ .

**(a) Into function:**

A function  $f: A \rightarrow B$  is called an into function, if at least one element in  $B$  is not an image of some element of set  $A$

i.e

$$\text{Range of } f \subset \text{set } B$$

For example, we define a function

$f: A \rightarrow B$  such that

$$f = \{(0, 1), (1, 1), (2, 3), (3, 2)\}$$

Where  $A = \{0, 1, 2, 3\}$  and  $B = \{1, 2, 3, 4\}$

$f$  is an into function.

**(b) One-One Function**

A function  $f: A \rightarrow B$  is called one-one function, if all distinct elements of  $A$  have distinct images in  $B$

$$\text{i.e } f(x_1) = f(x_2)$$

$$\Rightarrow x_1 = x_2 \in A$$

$$\text{or } \forall x_1 \neq x_2 \in A$$

$$\Rightarrow f(x_1) \neq f(x_2)$$

For example, if  $A = \{0, 1, 2, 3\}$  and  $B = \{1, 2, 3, 4, 5\}$

Then we define a function  $f: A \rightarrow B$  such that

$$f = \{(x, y) | y = x + 1, \forall x \in A, y \in B\} \\ = \{(0, 1), (1, 2), (2, 3), (3, 4)\}$$

$f$  is one one function.

**(c) An onto or surjective function**

A function  $f: A \rightarrow B$  is called an onto function, if every element of set  $B$  as an image of at least one element of set  $A$

i.e Range of  $f = B$

for example

if  $A = \{0, 1, 2, 3\}$  and  $B = \{1, 2, 3\}$  then

$$f = \{(0, 1), (1, 2), (2, 3), (3, 2)\}$$



Range  $f = \{1, 2, 3\} = B$

Thus  $f$  so defined is an onto function.

**(d) Bijective Function or one to one corresponding**

a function  $f: A \rightarrow B$  is called bijective function iff function  $f$  is one-one and onto.

For example

$$\text{if } A = \{1, 2, 3, 4\} \text{ and } B = \{2, 3, 4, 5\}$$

We defined such that

$$f = \{(x, y) | y = x + 1, \forall x \in A, y \in B\}$$

Then  $f = \{(1, 2), (2, 3), (3, 4), (4, 5)\}$

**Evidently this function is one-one because distinct elements of A have distinct images in B. this is an onto function also because every element of B is the image of at least one element of A.**

Note:

- 1) Every function is a relation but convers may not be true
- 2) Every function may not be one-one.
- 3) Every function may not be onto.

## EXERCISE 5.5

**Question No.3** if  $L = \{a, b, c\}$  and  $L = \{d, e, f, g\}$  then find two binary relations in each:

**Class work:**

i.  $L \times L$

$$L \times L = \{a, b, c\} \times \{a, b, c\}$$

$$= \{(a, a), (a, b), (a, c), (b, a), (b, b), (b, c), (c, a), (c, b), (c, c)\}$$

$$R_1 = \{(a, a), (a, b)\}$$

$$R_2 = \{(b, c), (c, c)\}$$

**Home work:**

ii.  $L \times M$

$$L \times M = \{a, b, c\} \times \{d, e, f, g\}$$

$$= \{(a, d), (a, e), (a, f), (a, g), (b, d), (b, e), (b, f), (b, g), (c, d), (c, e), (c, f), (c, g)\}$$

$$R_1 = \{(a, d), (b, g)\}$$

$$R_2 = \{(a, f), (b, f), (c, f)\}$$

**Home work:**

iii.  $M \times M$

$$= \{d, e, f, g\} \times \{d, e, f, g\}$$

$$= \{(d, d), (d, e), (d, f), (d, g), (e, d), (e, e), (e, f), (e, g), (f, d), (f, e), (f, f), (f, g), (g, d), (g, e), (g, f), (g, g)\}$$

$$R_1 = \{(d, e), (d, f)\}$$

$$R_2 = \{(e, e), (f, f), (g, g)\}$$

**Question No.5** if  $L = \{x | x \in N \wedge x \leq 5\}$ ,

$M = \{x | x \in P \wedge x \leq 10\}$ , then make the following relations from L to M. write the domain and range of each relations.

**Solution:**

$$L = \{1, 2, 3, 4, 5\}$$

$$M = \{2, 3, 5, 7\}$$

$$L \times M = \{1, 2, 3, 4, 5\} \times \{2, 3, 5, 7\}$$

$$= \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7), (4, 2), (4, 3), (4, 5), (4, 7), (5, 2), (5, 3), (5, 5), (5, 7)\}$$

**Class work:**

$$(ii) R_2 = \{(x, y) | y = x\}$$

$$= \{(2, 2), (3, 3), (5, 5)\}$$

$$\text{Dom } R_2 = \{2, 3, 5\}$$

$$\text{Range } R_2 = \{2, 3, 5\}$$

**Home work:**

$$(iii) R_3 = \{(x, y) | x + y = 6\}$$

$$= \{(1, 5), (3, 3), (4, 2)\}$$

$$\text{Dom } R_3 = \{1, 5, 4\}$$

$$\text{Range } R_3 = \{5, 3, 2\}$$

## Miscellaneous Exercise

**Q.1 Multiple choice questions.** Four possible answers are given for the following questions. Tick mark (✓) the correct answer.

1. A collection of well-defined distinct objects is called:

- (a) subset
- (b) power set
- (c) set
- (d) none of these

2. A set  $Q = \left\{ \frac{a}{b} \mid a, b \in Z \wedge b \neq 0 \right\}$  is called a

set of :

- (a) Whole numbers
- (b) Natural numbers
- (c) Irrational numbers
- (d) Rational numbers

3. The different number of ways to describe a set are:

- (a) 1
- (b) 2
- (c) 3
- (d) 4

4. A set with no element is called:

- (a) subset
- (b) empty set
- (c) singleton set
- (d) super set

5. The set  $\{x / x \in W \wedge x \leq 101\}$  is:

- (a) infinite set
- (b) subset
- (c) Null set
- (d) finite set

6. The set having only one element is called:

- (a) Null set
- (b) power set
- (c) singleton set
- (d) subset

7. Power set of an empty set is:

- (a)  $\phi$
- (b)  $\{a\}$
- (c)  $\{\phi, \{a\}\}$
- (d)  $\{\phi\}$

8. The number of elements in power set  $\{1, 2, 3\}$  is:

- (a) 4
- (b) 6
- (c) 8
- (d) 9

9. If  $A \subseteq B$  then  $A \cup B$  is equal to:



- (a) A (b) B  
(c)  $\phi$  (d) None of these
10. If  $A \subseteq B$  then  $A \cap B$  is equal to:  
(a) A (b) B  
(c)  $\phi$  (d) None of these
11. If  $A \subseteq B$  then  $A - B$  is equal to:  
(a) A (b) B  
(c)  $\phi$  (d) None of these
12.  $(A \cup B) \cup C$  is equal to:  
(a)  $A \cap (B \cup C)$  (b)  $(A \cup B) \cap C$   
(c)  $A \cup (B \cup C)$  (d)  $A \cap (B \cap C)$
13.  $A \cup (B \cap C)$  is equal to:  
(a)  $(A \cup B) \cap (A \cup C)$   
(b)  $A \cap (B \cap C)$   
(c)  $(A \cap B) \cap (A \cap C)$   
(d)  $A \cup (B \cup C)$
14. If A and B are disjoint sets, then  $A \cup B$  is equal to: (Board 2014) 10305127  
(a) A (b) B  
(c)  $\phi$  (d)  $B \cup A$
15. If number of elements in set A is 3 and in set B is 4, then number of elements in  $A \times B$  is:  
(a) 3 (b) 4  
(c) 12 (d) 7
16. If number of elements in set A is 3 and in set B is 2, then number of binary relations in  $A \times B$  is:  
(a)  $2^3$  (b)  $2^6$   
(c)  $2^8$  (d)  $2^2$
17. The domain of R  
 $= \{(0,2), (2,3), (3,3), (3,4)\}$  is:  
(a)  $\{0,3,4\}$  (b)  $\{0,2,3\}$   
(c)  $\{0,2,4\}$  (d)  $\{2,3,4\}$
18. The Range of R  
 $= \{(1,3), (2,2), (3,1), (4,4)\}$  is:  
(a)  $\{1,2,4\}$  (b)  $\{3,2,4\}$   
(c)  $\{1,2,3,4\}$  (d)  $\{1,3,4\}$
19. Point  $(-1,4)$  lies in the quadrant:  
(a) I (b) II (Board 2014)  
(c) III (d) IV
20. The relation  $\{(1,2), (2,3), (3,3), (3,4)\}$  is:  
(a) onto function  
(b) into function  
(c) not a function  
(d) one-one function
21. If  $A \cap B = \phi$ , then set A and B are ....sets.  
(a) sub (b) over lapping  
(c) disjoint (d) power
22. If  $A \subseteq B$  and  $B \subseteq A$ , then:  
(a)  $A = B$  (b)  $A \neq B$   
(c)  $A \cap B = \phi$  (d)  $A \cup B = \phi$
23. The complement of U is:  
(a) U (b)  $\phi$   
(c) impossible (d) union
24. The complement of  $\phi$  is:  
(a) U (b)  $\phi$   
(c) impossible (d) union
25.  $A \cap A^c = \dots\dots$   
(a) U (b) A  
(c)  $A^c$  (d)  $\phi$
26.  $A \cup A^c = \dots\dots$   
(a) U (b) A  
(c)  $A^c$  (d)  $\phi$
27. The set  $\{x \mid x \in A \text{ and } x \notin B\}$  is:  
(a)  $A \cup B$  (b)  $A \cap B$   
(c)  $A - B$  (d)  $B - A$
28. The point  $(-5, -7)$  lies in ... quadrant.  
(a) I (b) II  
(c) III (d) IV
29. The point  $(4, -6)$  lies in .... Quadrant.  
(a) I (b) II  
(c) III (d) IV
30. y co-ordinate of every point on x-axis is:  
(a) +ve (b) -ve  
(c) zero (d) 1
31. x co-ordinate of every point on y-axis is:  
(a) +ve (b) -ve  
(c) zero (d) 1
32. The domain of  $\{(a,b), (b,c), (c,d)\}$  is:  
(a)  $\{a,b,c\}$  (b)  $\{b,c,d\}$   
(c)  $\{a,b\}$  (d)  $\{a,b,c,d\}$
33. The range of  $\{(a,a), (b,b), (c,c)\}$  is:  
(a)  $\{a,b\}$  (b)  $\{a,b,c\}$   
(c)  $\{a\}$  (d)  $\phi$
34. Venn diagram was first used by:  
(a) John Venn (b) Netwon  
(c) Arthur Cayley (d) John Napier
35. A subset of  $A \times A$  is called.....in A.  
(a) set (b) relation  
(c) function (d) into function
36. If  $f: A \rightarrow B$  and range of  $f = B$ , then f is an:  
(a) into function (b) onto function  
(c) bijective function (d) function
37. If  $f: A \rightarrow B$  and range of  $f \neq B$ , then f is an:  
(a) into function  
(b) onto function  
(c) bijective function  
(d) function

38. The relation  $\{(a,b), (b,c), (a,d)\}$  is:  
 (a) a function (b) not a function  
 (c) range (d) domain
39. By definition, which of the following is a set?  
 (a)  $\{a, b, c, a\}$  (b)  $\{1, 2, 3, 2\}$   
 (c)  $\{\ell, m, n, o\}$  (d)  $\{0, 1, 2, 3, 1\}$
40. Which of the following is true?  
 (a)  $W \subseteq N$  (b)  $Z \subseteq W$   
 (c)  $N \subseteq P$  (d)  $P \subseteq W$

(Answer key)

1.	c	2.	d	3.	c	4.	b	5.	d
6.	c	7.	d	8.	c	9.	b	10.	a
11.	c	12.	c	13.	a	14.	d	15.	c
16.	b	17.	b	18.	c	19.	b	20.	c
21.	c	22.	a	23.	b	24.	a	25.	d
26.	a	27.	c	28.	c	29.	d	30.	c
31.	c	32.	a	33.	b	34.	a	35.	b
36.	c	37.	a	38.	b	39.	c	40.	d

**Question No.2 write a short answer of the following question.**

**1) Define a subset and give example.**

Subset: If A and B are two sets and every element of A is an element of B then set A is called subset of a set B. it is denoted by  $A \subseteq B$

**Example:**

$A = \{1, 2\}, B = \{1, 2, 3, 4\}$  as all elements of set A are also present in Set B. therefore  $A \subseteq B$ .

**2) Write all subset of the set  $\{a, b\}$**

**Solution:**

All subset ( $2^n = 2^2 = 4$ )

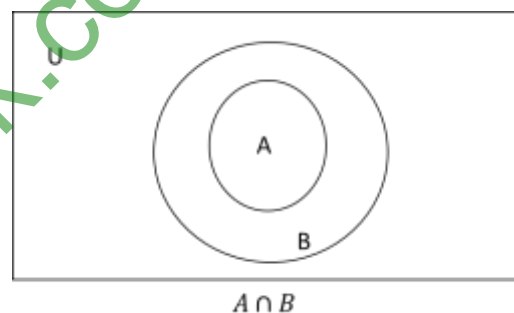
$\{\}, \{a\}, \{b\}, \{a, b\}$

**3) Show  $A \cap B$  by Venn diagram. When  $A \subseteq B$**

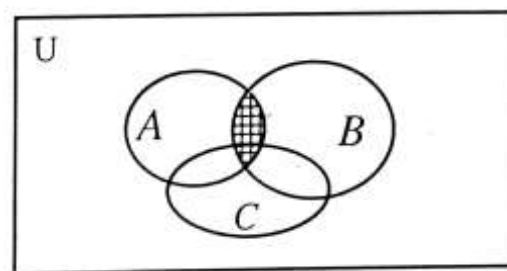
$A \subseteq B$

**Solution:**

if  $A \subseteq B$  then  $A \cap B = A$



**4) Show by Venn diagram  $A \cap (B \cup C)$**



- Horizontal the line segments and squares show  $B \cup C$
- $A \cap (B \cup C)$  is shown by squares.

**5) Define intersection of two sets.**

**Intersection of sets**

The intersection of two sets A and B, written as  $A \cap B$  (read as A intersection B) is the set consisting of all the common elements of A and B. Thus

$$A \cap B = \{x | x \in A \text{ and } x \in B\}$$

$$\text{Clearly } x \in A \cap B \Rightarrow x \in A \text{ and } x \in B$$

For example:

If  $A = \{1,2,3,4\}$  and  $B = \{3,4,5,6\}$  then

$$A \cap B = \{3,4\}$$

### 6) Define a function.

#### Function or Mapping

Suppose A and B are two non-empty sets, then

relation  $f: A \rightarrow B$  is called a function

if

(i).  $Dom f = A$

(ii).  $\forall x \in A$  we can associate some unique image element  $y = f(x) \in B$

### 7) Define one –one function.

#### One-One Function

A function  $f: A \rightarrow B$  is called one-one function, if all distinct elements of A have distinct images in B

$$i.e f(x_1) = f(x_2)$$

$$\Rightarrow x_1 = x_2 \in A$$

$$or \forall x_1 \neq x_2 \in A$$

$$\Rightarrow f(x_1) \neq f(x_2)$$

For example, if  $A = \{0,1,2,3\}$  and  $B = \{1,2,3,4,5\}$

Then we define a function  $f: A \rightarrow B$  such that

$$\begin{aligned} f &= \{(x,y) | y = x + 1, \forall x \in A, y \in B\} \\ &= \{(0,1), (1,2), (2,3), (3,4)\} \end{aligned}$$

### 8) Define an onto function.

#### An onto or surjective function

A function  $f: A \rightarrow B$  is called an onto function, if every element of set B as an image of at least one element of set A

$$i.e \text{ Range of } f = B$$

for example

if  $A = \{0,1,2,3\}$  and  $B = \{1,2,3\}$  then

$$f = \{(0,1), (1,2), (2,3), (3,2)\}$$

$$\text{Range } f = \{1,2,3\} = B$$

Thus  $f$  so defined is an onto function.

### 9) Define a bijective function.

#### Bijjective Function or one to one corresponding

a function  $f: A \rightarrow B$  is called bijective function iff function  $f$  is one-one and onto.

For example

$$if A = \{1,2,3,4\} \text{ and } B = \{2,3,4,5\}$$

We defined such that

$$f = \{(x,y) | y = x + 1, \forall x \in A, y \in B\}$$

$$\text{Then } f = \{(1,2), (2,3), (3,4), (4,5)\}$$

### 10) Write De Morgan's law.

De-Morgan's Law

For any two sets A and B

$$(i). (A \cup B)' = A' \cap B'$$

$$(ii). (A \cap B)' = A' \cup B'$$

# Unit-6

## BASIC STATISTICS

### Frequency Distribution:

A frequency distribution is a tabular arrangement for classifying data into different groups and the number of observation falling in each group corresponds to the respective group. In fact a frequency distribution is a method to summarize data.

#### Grouped Data:

The data presented in the form of frequency distribution is called **grouped data**.

#### Types of Frequency Distribution

On the basis of types of variable or data, there are two types of frequency distribution.

These are

- (a) Discrete Frequency Distribution
- (b) Continuous Frequency Distribution

#### (a) Discrete Frequency Table

Following steps are involved in making of a discrete frequency distribution:

- i Find the minimum and maximum observation in the data and write the values of the variable in the variable column from minimum to the maximum.
- ii Record the observations by using tally marks. (Vertical bar □)
- iii Count the tally marks write down the frequency in the frequency column.

#### (b) Continuous frequency distribution

The making of continuous frequency distribution involves the following steps:

- i Find the Range where  $\text{Range} = X_{\max} - X_{\min}$  (the difference between maximum and minimum observations).
- ii Decide about the number of groups (denote it by k) into which the data is to be classified (usually an integer between 5 and 20). Usually it depends upon the range. The larger the range the more the number of groups.
- iii Determine the size of class (denote by h) by using the formula:

$$h = \frac{\text{Range}}{k}$$

Note: The rule of approximation is relaxed in determining h. For example, h = 7.1 or h = 7.9 may be taken as 8.

- iv Start writing the classes or groups of the frequency distribution usually starting from the minimum observation and keeping in view the size of a class of tally marks
- v Record the observation from the data by using tally marks.
- vi Count the number and record them in the frequency column for each class.

### Concepts involved in a Continuous frequency table.

The following terms are frequently used in a continuous frequency distribution.

**(a) Class Limits:** The minimum and the maximum values defined for a class or group are called class limits. The minimum value is called the lower class limit and the maximum value is called the upper class limit of that class. For example in the group (5-10), 5 is lower class limit and 10 is called upper class limit.

#### (b) Class Boundaries:

The real class limits of a class are called class boundaries. A class boundary is obtained by adding two successive class limits and dividing the sum by 2. The value so obtained is taken as upper class boundary for the previous class and lower class boundary for the next class.

#### (c) Midpoint or Class Mark:

For a given class the average of that class obtained by dividing the sum of upper and lower class limits by 2, is called the midpoint or class mark of that class.

For example the midpoint of a class (6-10) is 8. i.e.

$$\text{Class Marks } x = \frac{6+10}{2} = \frac{16}{2} = 8$$

#### (d) Cumulative frequency

The total of frequency up to an upper class limit or boundary is called cumulative frequency.

## Exercise 6.1

### Class work:

The following data shows the number of members in various families. Construct frequency distribution. Also find cumulative frequencies.

9, 11, 4, 5, 6, 8, 3, 4, 9, 12, 8, 9, 10, 6, 7, 7, 11, 4, 4, 8, 4, 3, 2, 7, 9, 10, 9, 7, 6, 9, 5, 7

### Solution:

Frequency distribution of numbers of family members

Numbers of members	Talley marks	Frequency	Commulative
2	I	1	1
3	III	3	1+3=4
4	I III	6	4+6=10
5	IIII	4	10+4=14
6	III	3	14+3=17
7	I III	6	17+6=23
8	III	5	23+5=28
9	III	6	28+6=34
10	II	2	34+2=36
11	II	2	36+2=38
12	I	1	38+1=39
Total		39	

### Home work:

Question No.3 from the following data representing the salaries of 30 teachers of a school. Make a frequency distribution taking class interval size of Rs. 100, 450, 500, 550, 580, 1020, 1130, 1220, 760, 690, 710, 750, 1120, 760, 1240.

(Hint: Make classes 450 – 549, 550 – 649, ...).

### Solution:

Class Limits	Talley marks	Frequency
450 – 549	II	2
550 – 649	II	2
650 – 749	IIII	4
750 – 849	III	5
850 – 949	III	3
950 – 1049	IIII	4
1050 – 1149	III	5
1150 – 1249	III	5
	Total =	30

**Measures of Central Tendency:** A specific values of the variable around which the majority of the observation tend to concentrate, this comprehensive shows the tendency or behavior of the distribution of the variable under study. This value is called average or of the central value. The measure or techniques that are used to determine this central value are called Measures of Central Tendency.

The following measures of central tendency will be discussed in this section:

1. Arithmetic Mean
2. Median
3. Mode
4. Geometric mean
5. Harmonic mean
6. Quartiles

**Arithmetic Mean:**

Arithmetic mean (or simply called mean) is a measure that determines a value (observation) of the variable under study by dividing the sum of all values (observations) of the variable by their number of observations. We denote Arithmetic mean by  $\bar{X}$  in symbols we define:

Arithmetic mean

$$\bar{X} = \frac{\sum X}{n} = \frac{\text{sum of all values of observation}}{\text{No. of observation}}$$

### Computation of Arithmetic Mean:

There are two types of data, ungrouped and grouped. We, therefore have different methods to determine Mean for the two types of data.

### Ungrouped Data:

For ungrouped data we use three approaches to find mean. These are as follows.

#### i. Direct Method (By definition)

The formula under this method is given by:

$$\bar{X} = \frac{\sum X}{n} = \frac{\text{sum of all values of observation}}{\text{No. of observation}}$$

#### ii. Indirect, Short Cut or Coding Methods:

There are two approaches under indirect method. There are used to find mean when data set consist of large values or large number of values. The purpose is to simplify the computation of Mean. These approaches exist in theory but are not used in practice as many statistical software are available now to handle **large data**. However a student should have knowledge of these two approaches. These are:

- I. Using an Assume or provisional Mean

- II. Using a provisional mean and changing scale of the variable.

Deviation is defined as difference of any value of the variable from any constant "A" for Example we say,  
Deviation from Mean of  $X = (x_1 - \bar{X})$  for  $i = 1, 2, \dots, n$   
Deviation from any constant A =  $(x_1 - A)$  for  $i = 1, 2, \dots, n$

The formulae used under indirect method are:

$$(i) \quad \bar{X} = A + \frac{\sum_{i=1}^n D_1}{n}$$

$$(ii) \quad \bar{X} = A + \frac{\sum_{i=1}^n U_1}{n} \times h$$

Where

$D_1 = (x_1 - A)$ , A is any assumed value of X called **Assumed or provisional mean**.

$U_1 = \frac{x_1 - A}{h}$ , h is a constant multiple of the values of X.

### **Grouped data:**

A data in the form of frequency distributive is called grouped data. For the grouped data we define formulae under direct and indirect methods are given below.

(a) Using Direct method

$$\bar{X} = \frac{\sum fx}{\sum f}$$

Using indirect method

$$(i) \quad \bar{X} = A + \frac{\sum fD}{\sum f} \quad (ii) \quad \bar{X} = A + \frac{\sum fu}{\sum f} \times h$$

Where "X" denotes the midpoint of a class or group if class intervals are given and "h" is the class interval size.

### **(b) Median:**

Median is the middle most observation in an arranged data set. It divides the data set into two equal parts.  $\bar{X}$  is used to represent median. We determine Median by using the following formulae.

### **Ungrouped data:**

#### **Case-1:**

When the number of observations is odd of a set of data arranged in order of magnitude the median (middle most observation) is located by the formula given below:

$$\text{Median} = \text{size of } \left(\frac{n+1}{2}\right)^{\text{th}} \text{ observation}$$

#### **Case-2:**

When the number of observation is even of a set of data arranged in order of magnitude the median is

the arithmetic mean of the two middle observation. That is, median is average of

$$\frac{n}{2} \text{ and } \left(\frac{n}{2} + 1\right)^{\text{th}} \text{ Values.}$$

$$\text{Median} = \frac{1}{2} \left[ \text{size of } \left(\frac{n}{2}^{\text{th}} + \frac{n+1}{2}^{\text{th}}\right) \text{ observation} \right]$$

### **Grouped Data (Discrete)**

The following steps are involved in determining median for grouped data (discrete)

- Make cumulative frequency column.
- Determine the median observation using cumulative frequency, i.e the class containing  $\left(\frac{n}{2}\right)^{\text{th}}$  observation.

### **Grouped Data (continuous):**

The following steps are involved in determining median for grouped data(continuous)

- Determine Class boundaries
- Make cumulative frequency column.

Determine the median class using cumulative frequency. i.e the class containing  $\left(\frac{n}{2}\right)^{\text{th}}$  observation.

Use the formula:

$$\text{Median} = l + \frac{h}{f} \left\{ \frac{n}{2} - c \right\}$$

Where  $l$  = lower class boundary of the median class

$h$  = class interval size of the median class

$f$  = frequency of the median class,

$c$  = cumulative frequency of the class preceding the median class.

### **Mode:**

Mode is defined as the most frequent occurring observation in the data. It is the observation that occur maximum number of times is given data. The following formula is used to determine mode.

- Ungrouped data and Discrete data  
Mode = the most frequent observation

### **ii. Grouped Data (continuous)**

The following steps are involved in determine mode for grouped data:

Find the group that has the maximum frequency.

Use the formula

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

Where  $l$  = lower class boundary of the model class or group,



$h$  = class interval size of the model class,

$f_m$  = frequency of the model class,

$f_1$  = frequency of the class preceding the model class.

$f_2$  = frequency of the class succeeding the model class

### Geometric Mean:

Geometric Mean of a variable  $X$  is the  $n^{th}$  positive root of the product of the  $x_1, x_2, x_3, \dots, x_n$  observations. In symbols we write,

$$G.M = (x_1, x_2, x_3, \dots, x_n)^{\frac{1}{n}}$$

The above formula can also be written by using logarithm.

For Ungrouped data

$$G.M = \text{Antilog} \left( \frac{\sum \log X}{n} \right)$$

For Grouped data

$$G.M = \text{antilog} \left( \frac{\sum \log X}{\sum f} \right)$$

### Harmonic Mean:

Harmonic Mean refers to the value obtained by reciprocating the mean of the reciprocal of  $x_1, x_2, x_3, \dots, x_n$  observation. In symbols, for ungrouped data,

$$H.M = \frac{n}{\sum \frac{1}{x}}$$

Properties of Arithmetic Mean:

- Mean of a variable with similar observation say constant  $k$  is the constant  $k$  itself.
- Mean is affected by change in origin.
- Mean is affected by change in scale.
- Sum of the deviations of the variables  $X$  from its mean is always zero.

Calculation of Weighted Mean and Moving Average:

The Weighted Arithmetic Mean:

The relative importance of a number is called its weight. When numbers

$x_1, x_2, \dots, x_n$  are not equally

Important, we associate them with certain weights  $w_1, w_2, w_3, \dots, w_n$

Depending on the importance or significance.

$$\bar{X} = \frac{w_1 x_1 + w_2 x_2 + \dots + w_n x_n}{w_1 + w_2 + \dots + w_n} = \frac{\sum wx}{\sum w}$$

Is called the weighted arithmetic mean.

## Exercise 6.2

### Class work:

**Question No.3** Find arithmetic mean by direct method for the following set of data:

- 12, 14, 17, 20, 24, 29, 35, 45
- 200, 225, 350, 375, 270, 320, 290

Solution:

$$\text{i. } A.M = \bar{X} = \frac{\sum X}{n} = \frac{12+14+17+20+24+29+35+45}{8} = \frac{196}{8} = 24.5$$

$$\text{ii. } A.M = \bar{X} = \frac{\sum X}{n} = \frac{200+225+350+375+270+320+290}{7} = \frac{2030}{7} = 290$$

### Class Work:

**Question No.7.** The following data shows the number of children in which in various families. Find mode and median.

9, 11, 4, 5, 6, 8, 4, 3, 7, 8, 5, 5, 8, 3, 4, 9, 12, 8, 9, 10, 6, 1, 7, 11, 3, 2, 7, 9, 10, 9, 7, 6, 9, 5

Solution:

Writing the observation in Ascending order

2, 3, 3, 3, 4, 4, 4, 4, 4, 5, 5, 5, 5, 6, 6, 6, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 9, 9, 9, 10, 10, 11, 11

Mode: the most frequent observation = 9, 4

Number of observation = 38

Therefore, median is the mean of 19<sup>th</sup> and 20<sup>th</sup> observation =  $\frac{7+7}{2} = 7$

### Home work:

**Question No.11.** On a vacation trip a family bought 21.3 liters of petrol at 39.90 rupees per liter, 18.7 liters at 42.90 rupees per liter, and 23.5 liters at 40.90 rupees per liter find the mean price paid per liter.

Solution:

$X$	$W$	$XW$
21.3	39.90	(21.3)(39.90) = 849.87
18.7	42.90	(18.7)(42.90) = 802.13
23.5	40.90	(23.5)(40.90) = 961.15
$\sum x = 63.5$		$\sum xW = 2613.15$

Mean price =  $\frac{\sum XW}{\sum X} = \frac{2613.15}{63.5} = 41.15$  rupees per liter



**Home Work:**

**Question 12.** Calculator simple moving average of 3 years from the following data;

Years	2001	2002	2003	2004	2005	2006	2007
Valves	102	108	130	140	1158	180	196
Years	2008	2009	2010				
Valves	210	220	230				

**Solution:**

Years	Values	3-years moving total	3- years moving average
2001	102	-	-
2002	108	340	340/3=113.33
2003	130	378	378/3=126.00
2004	140	428	428/3=142.67
2005	158	478	$\frac{478}{3} = 159.33$
2006	180	534	534/3=178.00
2007	196	586	586/3=195.33
2008	210	626	626/3=208.67
2009	220	660	660/3=220.00
2010	230	-	

**1. Range:**

Range measure the extent of variation between two extreme observations of a data set.

It is given by the formula:

$$X_{max} - X_{min} = X_m - X_o$$

Where

$X_{max} = X_m$  = the maximum, highest or largest observation.

$X_{min} = X_o$  = the minimum lowest or smallest observation.

The formula to find range for grouped continuous data us given below.

Range=

(Upper class boundary of last group)-(Lower class boundary of first group).

**2. Variance:**

Variance is defined as the mean of the squared deviation of  $x_i (i = 1, 2, 3, \dots, n)$  observation from their arithmetic mean. In symbols,

$$\text{Variance of } X = \text{Var}(X) = S^2 = \frac{\sum (X - \bar{X})^2}{n}$$

**3. Standard Deviation**

Standard deviation is defined as the positive square root of mean of the squared deviations of  $X_i (i = 1, 2, 3, \dots, n)$  observations from their arithmetic mean. In symbols we write

$$\text{standard Deviation of } X = S.D(X) = S$$

$$= \sqrt{\frac{\sum (X - \bar{X})^2}{n}}$$

Computations of Variance and Standard Devotions

We uses the following to compute Variance and standard Deviations for Ungrouped and Grouped Data.

**Ungrouped Data:**

The formula of Variance is given by

$$\text{Var}(X) = S^2 = \frac{\sum X^2}{n} - \left( \frac{\sum X}{n} \right)^2$$

And standard Deviation

$$S.D(X) = S = \sqrt{\left[ \frac{\sum X^2}{n} - \left( \frac{\sum X}{n} \right)^2 \right]}$$

**Exercise 6.3****Home Work:**

**Question Mo.4** The salaries of five teachers in Rupees are as follows.

**11500, 12400, 15000, 14500, 14800.**

**find Range and Standard deviations**

**Solution:**

$X = 11500, 12400, 15000, 14500, 14800.$

Here  $X_{min} = 11500$ ,  $X_{max} = 15000$

$$\text{Range} = X_{max} - X_{min}$$

$$= 15000 - 11500$$

$$= 3500$$

$$\bar{X} = \frac{\sum x}{n}$$

$$= \frac{11500 + 12400 + 15000 + 14500 + 14800}{5}$$

$$= \frac{68200}{5} = 13640$$

$X$	$X - \bar{X}$	$(X - \bar{X})^2$
11500	-2140	4579600
12400	-1240	1537600
15000	1360	1849600
14500	860	739600
14800	1160	1345600

$$\sum (X - \bar{X})^2 = 10052000, \quad n = 5$$

$$S.D(X) = S = \sqrt{\left[ \frac{\sum X^2}{n} - \left( \frac{\sum X}{n} \right)^2 \right]}$$

$$= \sqrt{\frac{10052000}{5}}$$

$$= \sqrt{2010400}$$

$$= 1417.88$$

**Class work:**

**Question No.5. (a)** Find the standard deviation "S" of each set of numbers:

(ii) **9, 3, 8, 8, 9, 8, 9, 18.**

$X$	$X - \bar{X}$	$(X - \bar{X})^2$
9	0	0
3	-6	36
8	-1	1
8	-1	1
9	0	0
8	-1	1
9	0	0
18	9	81

$$\sum X = 72 \quad \sum (X - \bar{X})^2 = 120, n = 8$$

$$\bar{X} = \frac{\sum X}{n} = \frac{72}{8} = 9$$

$$S.D(X) = S = \sqrt{\left[ \frac{\sum X^2}{n} - \left( \frac{\sum X}{n} \right)^2 \right]}$$

$$= \sqrt{\frac{120}{8}}$$

$$= \sqrt{15} = 3.87$$

**Home Work:**

**Question No.7.** For the following distribution of marks calculator Range

	Frequency/No.
33 – 40	28
41 – 50	31
51 – 60	12
61 – 70	9
71 – 75	5

**Solution:**

$C.I$	Class Boundaries	$f$
33 – 40	32.5 – 40.5	28
41 – 50	40.5 – 50.5	32
51 – 60	50.5 – 60.5	12
61 – 70	60.5 – 70.5	9
71 – 75	70.5 – 75.5	5

Here

$$X_{max} = 75.5$$

$$X_{min} = 32.5$$

$$Range = X_{max} - X_{min}$$

$$= 75.5 - 32.5 = 43$$

**Miscellaneous Exercise**

- A grouped frequency table is also called:
  - data
  - frequency distribution
  - frequency polygon
  - Histogram
- A histogram is a set of adjacent:
  - squares
  - rectangles
  - circles
  - Dots
- A frequency polygon is a many sided:
  - closed figure
  - rectangle
  - square
  - Circles
- A cumulative frequency table is also called:
  - frequency distribution
  - data
  - less than cumulative frequency distribution
  - Histogram
- In a cumulative frequency polygon frequencies are plotted against:
  - midpoints
  - upper class boundaries
  - class limits
  - frequencies
- Arithmetic mean is a measure that determines a value of the variable under study by dividing the sum of all values of the variable by their:
  - number
  - group
  - denominator
  - numerator
- A deviation is defined as a difference of any value of the variable from a:
  - constant
  - histogram
  - sum
  - frequency
- A data in the form of frequency distribution is called:
  - Grouped data
  - Ungrouped data
  - Histogram
  - Dispersion
- Mean of a variable with similar observations say constant k is:
  - negative
  - k itself
  - zero
  - one
- Mean is affected by change in:
  - value
  - ratio
  - origin
  - none of these
- Mean is affected by change in:
  - place
  - scale
  - rate
  - none of these

12. Sum of the deviations of the variable  $x$  from its mean is always:

- (a) zero (b) one  
(c) same (d) negative

13. The  $n^{\text{th}}$  positive root of the product of the  $x_1, x_2, x_3, \dots, x_n$  observations is called:

- (a) Mode (b) Mean  
(c) Geometric mean (d) median

14. The value obtained by reciprocating the mean of the reciprocal of  $x_1, x_2, x_3, \dots, x_n$  observations is called:

- (a) Geometric mean  
(b) Median  
(c) Harmonic mean  
(d) S.D

15. The most frequent occurring observation in a data set is called:

- (a) Mode (b) Median  
(c) Harmonic mean (d) Mean

16. The measure which determines the middlemost observation in a data set is called:

- (a) median (b) mode  
(c) mean (d) variance

17. The observation that divide a data set into four equal part, are called:

- (a) deciles (b) quartiles  
(c) percentiles (d) mode

18. The spread or scatterings of observations in a data set is called:

- (a) average  
(b) dispersion  
(c) central tendency  
(d) quartile

19. The measures that are used to determine the degree or extent of variation in a data set are called measures of:

- (a) dispersion (b) central tendency  
(c) average (d) quartile

20. The extent of variation between two extreme observations of a data set is measured by:

- (a) average (b) range  
(c) quartiles (d) mode

21. The mean of the squared deviations of  $x_i$  ( $i = 1, 2, \dots, n$ ) observations from their arithmetic mean is called:

- (a) variance  
(b) standard deviation  
(c) range (d) mode

22. The positive square root of mean of the squared deviations of  $x_i$  ( $i = 1, 2, \dots, n$ ) observations from their arithmetic mean is called:

- (a) harmonic mean (b) range  
(c) S.D (d) variance

23. The size of class interval (6–10) is:

- (a) 4 (b) 5  
(c) 8 (d) 10

24. The arrangement of data is necessary to find the value of:

- (a) Mean (b) Median  
(c) Mode (d) Range

25. The class having maximum frequency is called .....class.

- (a) Modal (b) Median  
(c) Lower (d) Upper

26. The class containing  $\frac{n}{2}$ th observation is called ..... class.

- (a) Modal (b) Median  
(c) Boundary of (d) Size of

27. During frequency distribution number of groups should be between:

- (a) 5 and 10 (b) 10 and 15  
(c) 10 and 20 (d) 5 and 15

28. Direct formula to find mean from ungrouped data.

- (a)  $\bar{X} = \frac{\sum x}{n}$  (b)  $\bar{X} = \frac{\sum fx}{\sum f}$   
(c)  $\bar{X} = A + \frac{\sum D}{n}$  (d)  $\bar{X} = A + \frac{\sum fD}{\sum f}$

29. Direct formula to find mean from grouped data is:

- (a)  $\bar{X} = \frac{\sum x}{n}$  (b)  $\bar{X} = \frac{\sum fx}{\sum f}$   
(c)  $\bar{X} = A + \frac{\sum D}{n}$  (d)  $\bar{X} = A + \frac{\sum fD}{\sum f}$

30. Short formula to find mean from ungrouped data is:

- (a)  $\bar{X} = \frac{\sum x}{n}$  (b)  $\bar{X} = \frac{\sum fx}{\sum f}$   
(c)  $\bar{X} = A + \frac{\sum D}{n}$  (d)  $\bar{X} = A + \frac{\sum fD}{\sum f}$

31. Short formula to find mean from grouped data is:

- (a)  $\bar{X} = \frac{\sum x}{n}$  (b)  $\bar{X} = \frac{\sum fx}{\sum f}$   
(c)  $\bar{X} = A + \frac{\sum D}{n}$  (d)  $\bar{X} = A + \frac{\sum fD}{\sum f}$

32. Coding formula to find mean from ungrouped data is:

$$(a) \bar{X} = \frac{n}{\sum \frac{1}{x}} \quad (b) \bar{X} = \frac{n}{\sum \frac{f}{x}}$$

$$(c) \bar{X} = A + \frac{\sum u}{n} \times h \quad (d) \bar{X} = A + \frac{\sum fu}{\sum f} \times h$$

33. Coding formula to find mean from grouped data is:

$$(a) \bar{X} = \frac{n}{\sum \frac{1}{x}} \quad (b) \bar{X} = \frac{n}{\sum \frac{f}{x}}$$

$$(c) \bar{X} = A + \frac{\sum u}{n} \times h \quad (d) \bar{X} = A + \frac{\sum fu}{\sum f} \times h$$

34. Formula to find Harmonic mean from ungrouped data is:

$$(a) \bar{X} = \frac{n}{\sum \frac{1}{x}} \quad (b) \bar{X} = \frac{n}{\sum \frac{f}{x}}$$

$$(c) \bar{X} = A + \frac{\sum fu}{n} \times h$$

$$(d) \bar{X} = A + \frac{\sum fu}{\sum f} \times h$$

35. Formula to find Harmonic mean from grouped data is:

$$(a) \bar{X} = \frac{n}{\sum \frac{1}{x}} \quad (b) \bar{X} = \frac{n}{\sum \frac{f}{x}}$$

$$(c) \bar{X} = A + \frac{\sum fu}{n} \times h \quad (d) \bar{X} = A + \frac{\sum fu}{\sum f} \times h$$

36. The concept of antilogarithm is used to find the value of:

- (a) A.M (b) G.M  
(c) H. M (d) Mode

37. Variance is denoted by:

- (a) V (b) S  
(c)  $S^2$  (d)  $\bar{X}$

38. Standard deviation is denoted by:

- (a) X (b) S  
(c)  $S^2$  (d)  $\bar{X}$

39. Median is denoted by:

- (a)  $\bar{X}$  (b) X  
(c) S (d)  $S^2$

40. On the basis of types of variable or data, the types of frequency distribution are:

- (a) 2 (b) 3  
(c) 4 (d) 5

41. In class (10 – 19), upper class limit is:

- (a) 10 (b) 19  
(c) 29 (d) 14.5

42. In class (30–39), lower class limit is:

- (a) 39 (b) 9 10306123  
(c) 30 (d) 34.5

43. In class (20–29), Midpoint or class mark is:

- (a) 20.5 (b) 24.5  
(c) 29 (d) 49

44. Types of measures of central tendency are:

- (a) 3 (b) 4  
(c) 5 (d) 6

45. Median from the data 82,93,86,92 and 79 is:

- (a) 82 (b) 86  
(c) 92 (d) 93

1.	b	2.	b	3.	a	4.	c	5.	b
6.	a	7.	a	8.	a	9.	b	10.	c
11.	b	12.	a	13.	c	14.	c	15.	a
16.	a	17.	b	18.	b	19.	a	20.	b
21.	a	22.	c	23.	b	24.	b	25.	a
26.	b	27.	d	28.	a	29.	b	30.	c
31.	d	32.	c	33.	d	34.	a	35.	b
36.	b	37.	c	38.	b	39.	b	40.	a
41.	b	42.	c	43.	b	44.	c	45.	b

**Question No.2 Write short answers of the following questions.**

### 1. Define Class limit.

The minimum and the maximum values defined for a class or group are called class limits. The minimum value is called the lower class limit and the maximum value is called the upper class limit of that class. For example in the group (5-10). 5 is lower class limit 10 is called upper class limit.

### 2. Define class mark Midpoint or Class Mark:

For a given class the average of that class obtained by dividing the sum of upper and lower class limit by 2. is called the midpoint or class mark of

### 3. What is cumulative frequency? Cumulative

Frequency: The total of frequency up to an upper class limit or boundary is called the Cumulative frequency

### 4. Define a frequency distribution.

Frequency Distribution

A frequency distribution is a tabular arrangement for classifying data into different groups and the number of observation falling in each group corresponds to the respective group. In fact a frequency distribution is method to summarize data

### 5. What is Histogram?

Histogram A Histogram is a graph of adjacent rectangles constructed on XY-plane. It is a graph of

frequency distribution. In practice both discrete and continuous frequency distribution are represented by means of histogram.

6. **Name two measures of central tendency.**

1 - Arithmetic Mean 2- Median 3- Mode

7. **Define Arithmetic mean?**

**Arithmetic Mean:** Arithmetic mean is a measure that determines a value of the variable under study by dividing the sum of all values observations of the variable by their number of observations. We denote Arithmetic mean by

$\bar{X}$ . In symbols we define

**Arithmetic mean**

$$\bar{X} = \frac{\sum X}{n} = \frac{\text{sum of all values of observation}}{\text{No. of observation}}$$

8. **Write three properties of Arithmetic mean.**

- Mean of a variable with similar observation say constant k is the constant k itself.
- Mean is affected by change in origin.
- Mean is affected by change in scale.

9. **Define Median.**

Median is the middle most observation in an arranged data set. It divides the data set into two equal parts.  $\bar{X}$  is used to represent median. We determine Median by using the following formulae.

**Ungrouped data:**

**Case-1:**

When the number of observations is odd

$$\text{Median} = \text{size of } \left(\frac{n+1}{2}\right)^{\text{th}} \text{ observation}$$

**Case-2:**

When the number of observation is

$$\text{Median} = \frac{1}{2} \left[ \text{size of } \left(\frac{n}{2}\right)^{\text{th}} + \left(\frac{n+1}{2}\right)^{\text{th}} \text{ observation} \right]$$

**10. Define Mode:**

**Mode:** Mode is defined as the most frequent occurring observation in the data. It is the observation that occur maximum number of times is given data. The following formula is used to determine mode.

- Ungrouped data and Discrete data  
Mode = the most frequent observation
- Grouped Data (continuous)  
The following steps are involved in determine mode for grouped data:  
Find the group that has the maximum frequency.  
Use the formula

$$\text{Mode} = l + \frac{f_m - f_1}{2f_m - f_1 - f_2} \times h$$

**11. What do you mean by Harmonic mean?**

**Harmonic Mean:** Harmonic Mean refers to the value obtained by reciprocating the mean of the reciprocal of  $x_1, x_2, x_3, \dots, x_n$  observation. In symbols, for ungrouped data,

$$H.M = \frac{n}{\sum \frac{1}{x}}$$

**12. Define Geometric mean.**

**Geometric Mean:**

Geometric Mean of a variable  $X$  is the  $n^{\text{th}}$  positive root of the product of the  $x_1, x_2, x_3, \dots, x_n$  observations. In symbols we write,

$$G.M = (x_1, x_2, x_3, \dots, x_n)^{\frac{1}{n}}$$

The above formula can also be written by using logarithm.

For Ungrouped data

$$G.M = \text{Antilog} \left( \frac{\sum \log X}{n} \right)$$

For Grouped data

$$G.M = \text{antilog} \left( \frac{\sum \log X}{\sum f} \right)$$

**13. What is range?**

**Range:**

Range measure the extent of variation between two extreme observations of a data set.

It is given by the formula:

$$X_{\max} - X_{\min} = X_m - X_o$$

Where

$$X_{\max} = X_m$$

= the maximum, highest or largest observation.

$$X_{\min} = X_o$$

= the minimum lowest or smallest observation.

The formula to find range for grouped continuous data is given below.

$$\text{Range} = (\text{Upper class boundary of last group}) - (\text{Lower class boundary of first group}).$$

**14. Define standard deviation.**

**Standard Deviation**

Standard deviation is defined as the positive square root of mean of the squared deviations of

$X_i (i = 1, 2, 3, \dots, n)$  observations from their arithmetic mean. In symbols we write

$$\text{standard Deviation of } X = S.D(X)$$

$$= S = \sqrt{\frac{\sum (X - \bar{X})^2}{n}}$$

Computations of Variance and Standard Deviations

We use the following to compute Variance and standard Deviations for Ungrouped and Grouped Data.

Ungrouped Data:

The formula of Variance is given by

$$\text{Var}(X) = S^2 = \frac{\sum X^2}{n} - \left( \frac{\sum X}{n} \right)^2$$

And standard Deviation

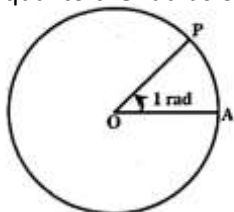
$$S.D(X) = S = \sqrt{\left[ \frac{\sum X^2}{n} - \left( \frac{\sum X}{n} \right)^2 \right]}$$

# Unit-7

## INTRODUCTION TO TRIGONOMETRY

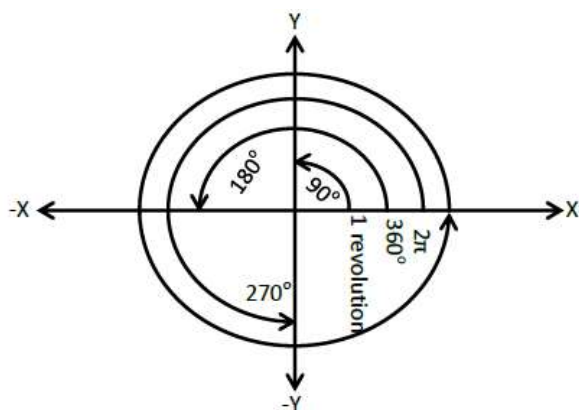
### Radian:

The angle subtended at the Centre of the circle by an arc, whose length is equal to the radius of the circle is called one radian.



### Relationship between radians and degree:

Things to know:



So,

$$1 \text{ revolution} = 2\pi \text{ radian} = 360^\circ$$

$$2\pi \text{ radian} = 360^\circ$$

to get 1 radian divide by  $2\pi$

$$1 \text{ radian} = \frac{360^\circ}{2\pi}$$

$$1 \text{ radian} = \frac{2\pi}{180^\circ}$$

$$1 \text{ radian} = \frac{\pi}{180^\circ}$$

To get 1 degree divide by 360

$$\frac{2\pi}{360} \text{ rad} = 1^\circ$$

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

### Remember that:

$$1 \text{ radian} = \frac{180}{3.1416} = 57.295795^\circ \approx 57^\circ 17' 45''$$

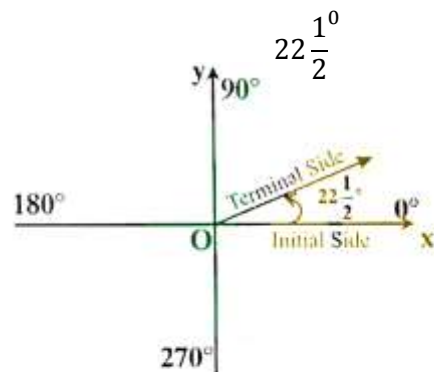
$$1^\circ = \frac{3.1416}{180} = 0.0175 \text{ radians}$$

## Exercise 7.1

Question No.1 Locate the following angles:

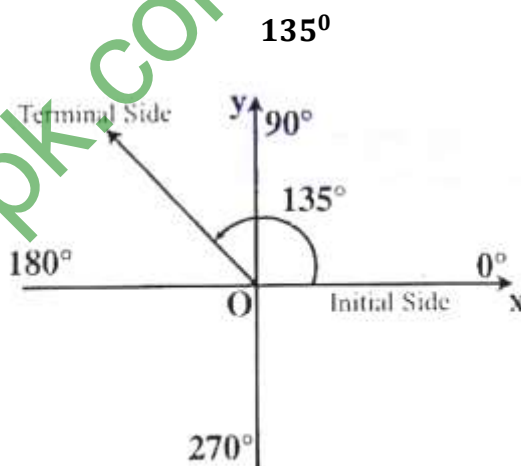
Home Work:

(ii)



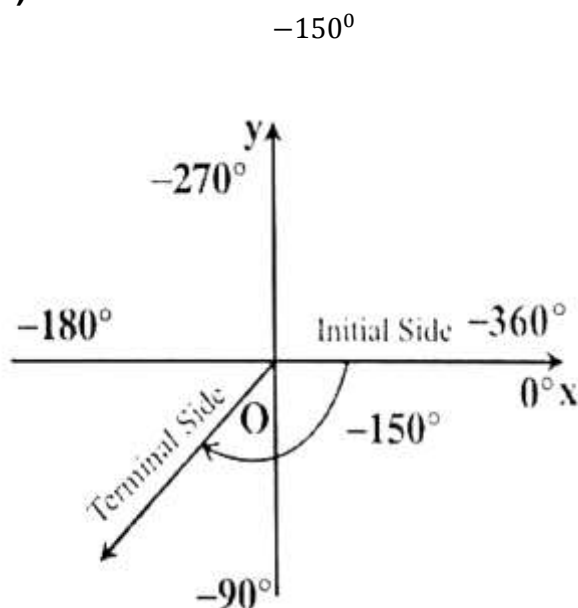
Home Work

(iii)



Class work:

(vii)





**Question No.2**

Express the following sexagesimal measures of angles in decimal form.

**Class work:**

$$60^{\circ}30'30''$$

Solution:

$$\begin{aligned} &= 60^{\circ} + \frac{30^{\circ}}{60^{\circ}} + \frac{30^{\circ}}{60^{\circ} \times 60^{\circ}} \\ &= 60^{\circ} + 0.5^{\circ} + 0.008^{\circ} \\ &= 60.508^{\circ} \end{aligned}$$

**Question No.3 Express the following in  $D^{\circ}M'S''$ :****Home Work (ii)**

$$125.45^{\circ}$$

Solution:

$$\begin{aligned} &= 125^{\circ} + 0.45^{\circ} \\ &= 125^{\circ} + (0.45 \times 60)' \\ &= 225^{\circ} + 27' \\ &225^{\circ}27'0'' \end{aligned}$$

**Home Work (iv)**

$$-22.5^{\circ}$$

Solution:

$$\begin{aligned} &= -[22^{\circ} + 0.5^{\circ}] \\ &= -[22^{\circ} + (0.5 \times 60)'] \\ &= -[22^{\circ} + 30'] \\ &= -22^{\circ}30' \end{aligned}$$

**Class Work (v)**

$$-67.58^{\circ}$$

Solution:

$$\begin{aligned} &= -(67^{\circ} + 0.58^{\circ}) \\ &= -[67^{\circ} + (0.58 \times 60)'] \\ &= -[67^{\circ} + 34' + 0.8'] \\ &= [67^{\circ} + 34' + (0.8 \times 60)''] \\ &= -[67^{\circ} + 34' + 48''] \\ &= -67^{\circ}34'48'' \end{aligned}$$

**Question N.4 Express the following angles into radians.****Home work: (ii)**

$$\begin{aligned} &60^{\circ} \\ &= 60 \times \frac{\pi}{180} \text{ radian} \\ &= 60 \frac{\pi}{60 \times 3} \text{ radian} \\ &= \frac{\pi}{3} \text{ radians} \end{aligned}$$

**Home work: (iii)**

$$135^{\circ}$$

Solution:

$$\begin{aligned} &225^{\circ} \\ &= 225 \frac{\pi}{180} \text{ radians} \\ &= 45 \times 3 \frac{\pi}{45 \times 4} \text{ radians} \end{aligned}$$

$$= \frac{3\pi}{4} \text{ radians}$$

**Class work: (viii)**

$$315^{\circ}$$

Solution:

$$\begin{aligned} &= 315 \frac{\pi}{180} \text{ radians} \\ &= 45 \times 7 \frac{\pi}{45 \times 4} \text{ radians} \\ &= \frac{7\pi}{4} \text{ radians} \end{aligned}$$

**Question No.5 Convert the following to degree.****Class work: (iii)**

$$\begin{aligned} &\frac{7\pi}{8} \text{ radians} \\ &= \frac{7\pi}{8} \frac{180}{\pi} \text{ degree} \\ &= \frac{7 \times 180}{8} \text{ degree} \\ &= \frac{1260}{8} \text{ degrees} \\ &= 157.5^{\circ} \end{aligned}$$

**Home Work: (vii)**

$$-\frac{7\pi}{8}$$

Solution:

$$\begin{aligned} &-\frac{7\pi}{8} \text{ radians} \\ &= -\frac{7\pi}{8} \frac{180}{\pi} \text{ degree} \\ &= \frac{-1260}{8} \text{ degrees} \\ &= 157.5^{\circ} \end{aligned}$$

**Home Work: (viii)**

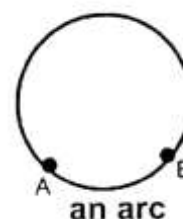
$$-\frac{13}{16}\pi$$

Solution:

$$\begin{aligned} &-\frac{13\pi}{16} \text{ radians} \\ &= -\frac{13\pi}{16} \frac{180}{\pi} \text{ degree} \\ &= \frac{-2340}{16} \text{ degrees} \\ &= 146.25^{\circ} \end{aligned}$$

**Sector of a Circle:****i. Arc of a circle:**

A part of the circumference of a circle is called an arc.





ii. **Segment of a circle:**

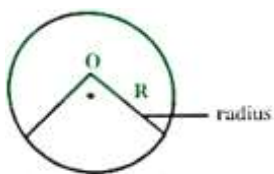
A part of the circular region bounded by an arc and a chord is called segment of a circle:



a segment

iii. **Sector of a circle:**

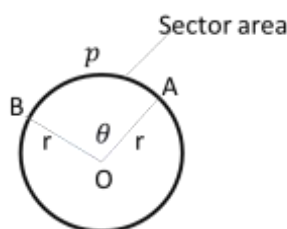
A part of a circular region bounded by the two radii and an arc is called sector of the circle.



a sector

**Area of the Circular sector:**

Consider a circle of radius  $r$  and an arc of length  $I$  units, subtended an angle  $\theta$  at  $O$ .

**Exercise 7.2**

**Question No.1** find  $\theta$  when:

**Class work:** (ii)

$$I = 4.5m, \quad r = 2.5m$$

**Solution:** using rule

$$I = r\theta$$

$$\frac{I}{r} = \theta$$

$$\frac{4.5}{2.5} = \theta$$

$$\theta = 1.8 \text{radian}$$

**Question No.3** find  $r$  when

**Home work:** (i)

$$\theta = 180^\circ, r = 4.9cm$$

**Solution:**

As  $\theta$  should be in radius so

$$\theta = 180^\circ$$

$$= 180 \frac{\pi}{180} \text{radian}$$

$$= \pi \text{radian}$$

Using rule  $I = r\theta$

$$= 4.9cm \times \pi$$

$$= 15.4cm$$

**Home work:**

**Question No.5**

**Question No.5** In a circle of radius 10m, find the distance travelled by a point moving on this circle if the point makes 3.5 revolution.

**Solution:**

$$\text{Radius} = r = 10m$$

$$\text{Number of revolutions} = 3.5$$

$$\text{Angle of one revolution} = 2\pi$$

$$\text{Angle of 3.5 revolution} = \theta$$

$$= 3.5 \times 2\pi \text{radian}$$

$$\theta = 7\pi \text{radian}$$

$$\text{Distance travelled} = I = ?$$

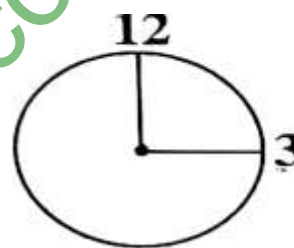
$$\text{Using rule } I = r\theta$$

$$I = 10m \times 7\pi$$

$$I = 220m$$

**Home Work:**

**Question No.6** What is the circular measure of the angle between the hands of the watch at 3 O' clock?



**Solution:**

At 3 O' clock the minute hand will be at 12 and hour hand will be at 3 i.e the angle between the hands of watch will be one quarter of the central angle of full circle.

$$\text{i.e.} = \frac{1}{4} \text{ of } 360^\circ$$

$$\frac{1}{4} \times 360^\circ$$

$$= 90^\circ$$

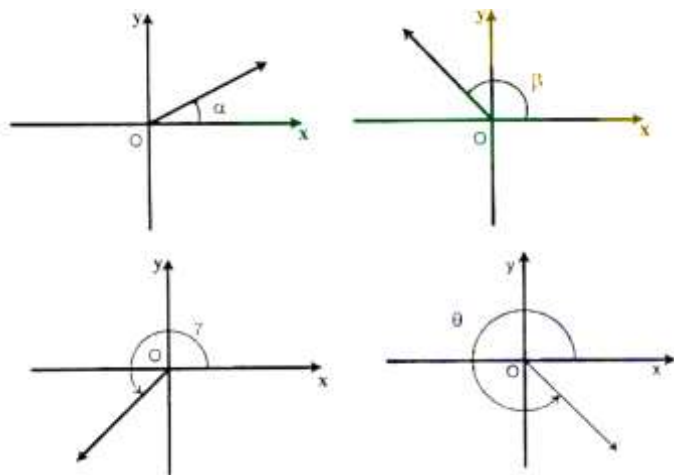
$$= 90 \frac{\pi}{180} \text{radian}$$

$$= \frac{\pi}{2} \text{radian}$$

**Angle in Standard position:**

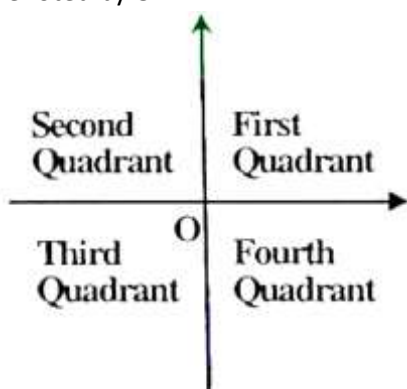
A general angle is said to be in standard position if its vertex is at the origin and its initial side is directed along the positive direction of the  $x$ -axis of a rectangular coordinates system.

The position of the terminal side of a angle in standard position remains the same if measure of an angle is increased or decreased by a multiple of  $2\pi$  Some Standard angles are shown in the following figures.



### The Quadrants and Quadrantal Angles:

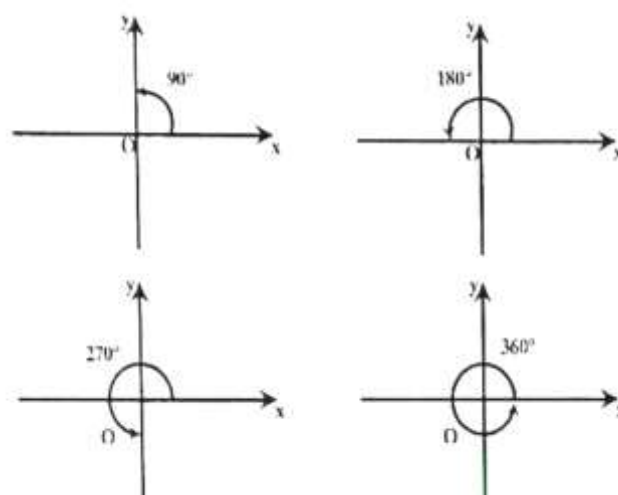
The  $x$  - axis and  $y$  - axis divides the plane in four regions. Called quadrants, When they intersect each other at right angle. The point of intersection is called origin and is denoted by O.



- Angle between  $0^\circ$  and  $90^\circ$  are in the first quadrant.
- Angle between  $90^\circ$  and  $180^\circ$  are in the second quadrant.
- Angle between  $180^\circ$  and  $270^\circ$  are in the third quadrant.
- Angle between  $270^\circ$  and  $360^\circ$  are in the fourth quadrant.
- An angle in standard position is said to lie in that quadrant if its terminal side lies in that quadrant. Angles  $\alpha, \beta, \gamma$  and  $\theta$  lie in I, II, III and IV quadrant respectively.

### Quadrantal Angles:

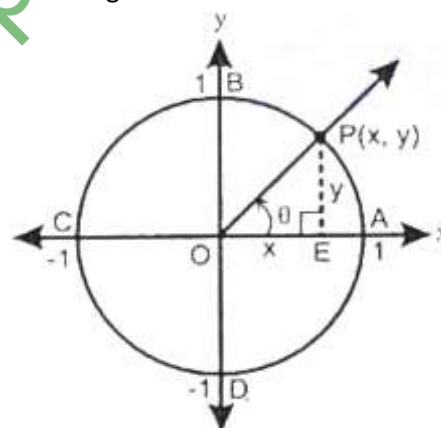
If the terminal side of an angle in standard position falls on  $x$  - axis or  $y$  - axis then it is called a quadrantal angle. The quadrantal angles are shown as below.'



### Trigonometric Ratios and their reciprocals with the help of a unit circle:

There are six fundamental trigonometric ratios called sine, cosine, tangent, cotangent, secant and cosecant. To define these functions we use circular approaches which involve the unit circle.

Let  $\theta$  be a real number, which represents the radian measure of an angle in standard position. Let  $P(x, y)$  be any point on the unit circle lying on the terminal side of  $\theta$  as shown in figure.



We define sine of  $\theta$ , written as  $\sin\theta$  and cosine of  $\theta$  written as

$$\sin\theta = \frac{EP}{OP} = \frac{y}{1} = \sin\theta = y$$

$$\cos\theta = \frac{OE}{OP} = \frac{x}{1} = \cos\theta = x$$

i.e.  $\cos\theta$  and  $\sin\theta$  are the  $x$  - coordinates and  $y$  - coordinates of the point P on the unit circle. The equations  $x = \cos\theta$  and  $y = \sin\theta$  are called circular or Trigonometric functions.

The remaining trigonometric functions tangent, cotangent, secant and cosecant will be denoted by  $\tan\theta$ ,  $\cot\theta$ ,  $\sec\theta$ , and  $\csc\theta$  for any real angle  $\theta$ .

$$\bullet \quad \tan\theta = \frac{EP}{OE} = \frac{y}{x} \quad \tan\theta = \frac{y}{x} (x \neq 0)$$

$$\text{As } y = \sin\theta \text{ and } x = \cos\theta \quad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\bullet \cot\theta = \frac{x}{y} (y \neq 0) \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\bullet \sec\theta = \frac{1}{\cos\theta} (x \neq 0) \text{ and } \operatorname{cosec}\theta = \frac{1}{\sin\theta} (y \neq 0)$$

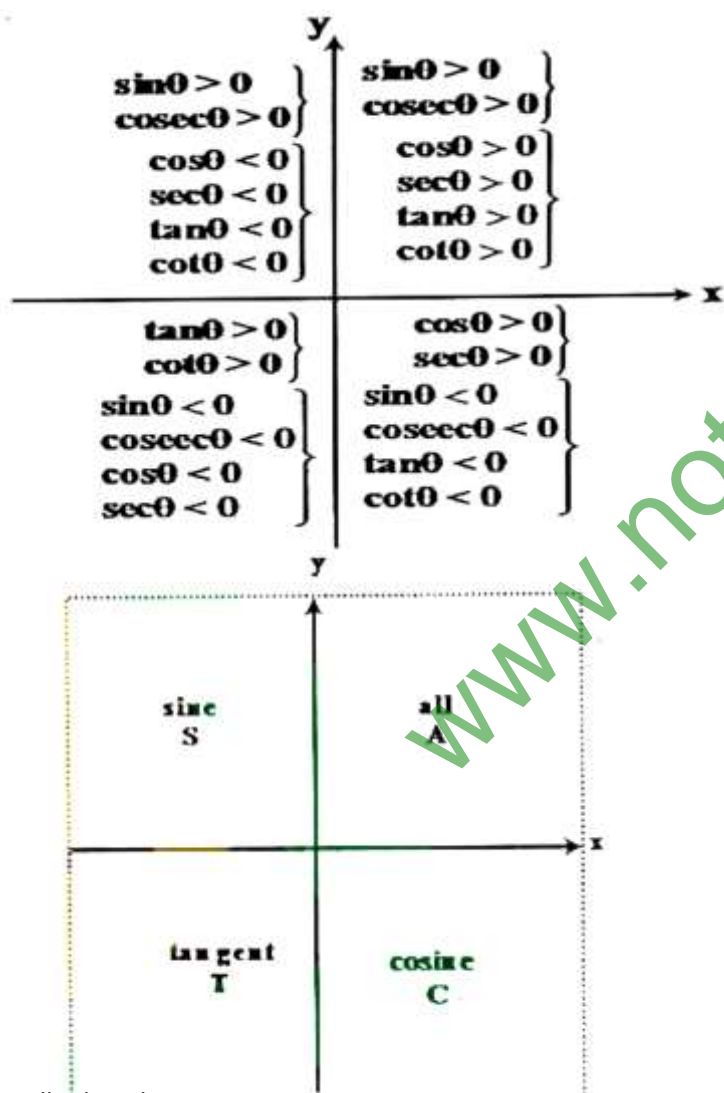
$$\bullet \sec\theta = \frac{1}{\cos\theta} \text{ and } \operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

**Reciprocal Identities:**

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta} \quad \text{or} \quad \operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \quad \text{or} \quad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \quad \text{or} \quad \cot\theta = \frac{1}{\tan\theta}$$

**Signs of trigonometric ratios in different Quadrants:**

Allied angles:

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\sin(90 - \theta) = \cos\theta$$

$$\cos(90 - \theta) = \sin\theta$$

$$\sin(90 + \theta) = \cos\theta$$

$$\sin(90 + \theta) = \sin\theta$$

$$\sin(180 - \theta) = \sin\theta$$

$$\cos(180 - \theta) = -\cos\theta$$

$$\sin(180 + \theta) = -\sin\theta$$

$$\cos(180 + \theta) = -\cos\theta$$

**Exercise 7.3**

**Question No.1** Locate each of the following angles in standard position using a protector or fair free hand guess, also find a positive and a negative angle conterminal with each given angle:

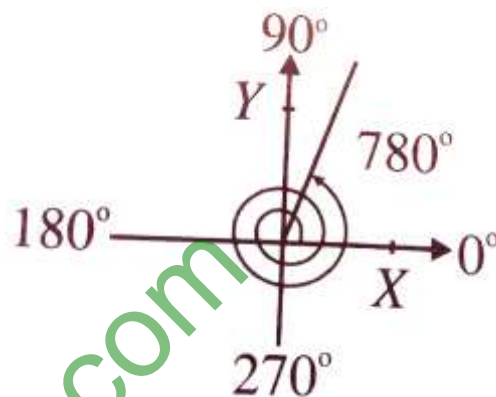
Solution:

Class work: (ii)

$$780^\circ$$

$$\text{Positive coterminal angle } 780^\circ + 2[360^\circ] = 60^\circ$$

$$\text{Negative coterminal angle} = -300^\circ$$

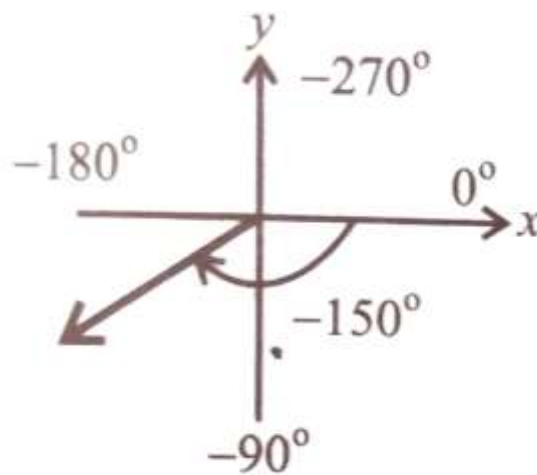


Home work: (iii)

$$-100^\circ$$

$$\text{Positive coterminal angle } 260^\circ$$

$$\text{Negative coterminal angle} = -360^\circ - 100^\circ = -460^\circ$$



**Question No.2** Identify closest quadrantile angles between which the following angles lie.

Class work (i)

$$156^\circ$$

Answer:  $90^\circ$  and  $180^\circ$ 

Home work: (ii)

$$318^\circ$$

Answer:  $270^\circ$  and  $360^\circ$

**Question No.3** Write the closest Quadrantal angles between which the angles lie. Write your answer in radian measure.

**Home work: (iii)**

$$-\frac{\pi}{2}$$

Answer:  $0$  and  $-\frac{\pi}{2}$

**Class work: (iv)**

$$-\frac{3\pi}{4}$$

Answer:  $-\frac{\pi}{2}$  and  $-\pi$

**Question No.4** in which quadrant  $\theta$  lies, when

**Home work: (ii)**

$$\cos\theta < 0, \sin\theta < 0$$

Answer: *III quadrant*

**Class work: (iv)**

$$\cos\theta < 0, \tan\theta < 0$$

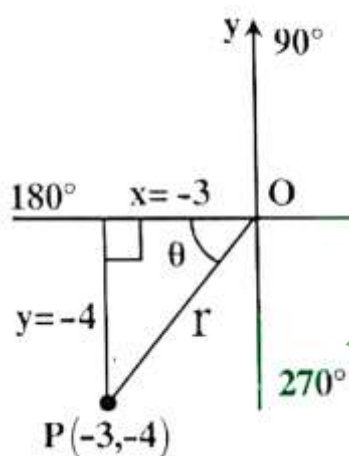
Answer: *II quadrant*

**Home work:**

**Question No.8** if  $\tan\theta = \frac{4}{3}$  and  $\sin\theta < 0$ , find the values of other trigonometric functions at  $\theta$

**Solution:**

As  $\tan\theta = \frac{4}{3}$  and  $\sin\theta$  is  $-ve$ , which is possible in quadrant *III* only. We complete the figure.



From the figure  $x = -3$  and  $y = -4$

By Pythagorean theorem

$$\begin{aligned} r^2 &= x^2 + y^2 \\ r &= \sqrt{x^2 + y^2} \\ r &= \sqrt{(-3)^2 + (-4)^2} \\ r &= \sqrt{9 + 16} \\ r &= \sqrt{25} \\ r &= 5 \end{aligned}$$

Now,

$$\sin\theta = \frac{y}{r} = -\frac{4}{5} \quad \left| \quad \csc\theta = \frac{r}{y} = -\frac{5}{4} \right.$$

$$\begin{aligned} \cos\theta &= \frac{x}{r} = -\frac{3}{5} \\ \tan\theta &= \frac{y}{x} = \frac{4}{3} \end{aligned}$$

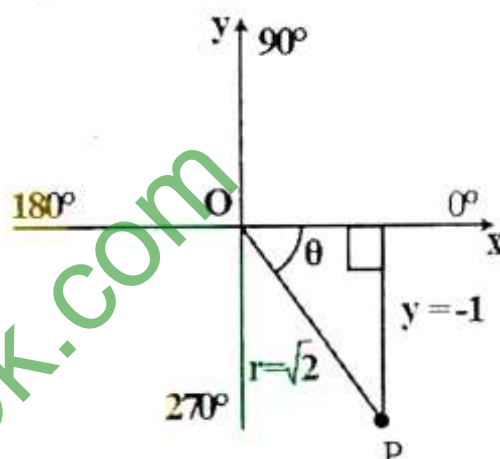
$$\begin{aligned} \sec\theta &= \frac{r}{x} = -\frac{5}{3} \\ \cot\theta &= \frac{3}{4} \end{aligned}$$

**Class work:**

**Question No. 9** if  $\sin\theta = -\frac{1}{\sqrt{2}}$  and terminal side of the angle is not in quadrant *III*, find the values of  $\tan\theta$ ,  $\sec\theta$  and  $\csc\theta$ .

**Solution:**

As  $\sin = -\frac{1}{\sqrt{2}}$  and terminal side of angle is not in *III* quadrant, so it lies in quadrant *IV*.



From the figure  $y = -1$  and  $r = \sqrt{2}$

By Pythagorean theorem

$$\begin{aligned} r^2 &= x^2 + y^2 \\ x^2 &= r^2 - y^2 \\ x &= \sqrt{r^2 - y^2} \\ r &= \sqrt{(\sqrt{2})^2 - (-1)^2} \\ r &= \sqrt{2 - 1} \\ r &= \sqrt{1} \\ r &= 1 \end{aligned}$$

Now,

$$\begin{aligned} \tan\theta &= \frac{y}{x} = -\frac{1}{1} = -1 \\ \sec\theta &= \frac{r}{x} = \frac{\sqrt{2}}{1} = \sqrt{2} \\ \csc\theta &= \frac{r}{y} = \frac{\sqrt{2}}{-1} = -\sqrt{2} \end{aligned}$$

**Question No.12** Find the value of the trigonometric functions. Do not use trigonometric table or calculator.

**Solution:**

**Home work: (i)**

we know that  $2k\pi + \theta = \theta$ , where  $k \in \mathbb{Z}$

$$30^\circ = 30 \frac{\pi}{180} \text{ radian} = \frac{\pi}{6} \text{ radian}$$

$$\tan 30^\circ = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

Home work: (v)

$$\cos \frac{2\pi}{3}$$

$$\cos(120^\circ) = -\frac{1}{2}$$

Class work: (viii)

$$\begin{aligned}\tan(-9\pi) &= \tan(-8\pi - \pi) \\ &= \tan[2(-4)\pi - \pi] \\ &= \tan(-8\pi + (-\pi)) \\ &= \tan(-\pi) \\ &= 0\end{aligned}$$

Home work: (xi)

$$\begin{aligned}\cot \frac{7\pi}{6} &= \cot \left[ 2\pi + \left( -\frac{5\pi}{6} \right) \right] \\ &= \cot \left( -\frac{5\pi}{6} \right) \\ &= \frac{1}{\tan \left( -\frac{5\pi}{6} \right)} = \frac{1}{\tan(-150^\circ)} = \frac{1}{-\frac{1}{\sqrt{3}}} = -\sqrt{3}\end{aligned}$$

## Exercise 7.4

Question No.7. Class work

verify the identities  $(1 - \sin\theta)(1 + \sin\theta) = \theta$ 

Solution:

$$\begin{aligned}L.H.S &= (1 - \sin\theta)(1 + \sin\theta) \\ &= 1 - \sin^2\theta \\ &= \cos^2\theta \\ &= R.H.S\end{aligned}$$

Home work

Question No.10

$$(\cot\theta + \operatorname{cosec}\theta)(\tan\theta - \sin\theta) = \sec\theta - \cos\theta$$

Solution:

$$\begin{aligned}L.H.S &= (\cot\theta + \operatorname{cosec}\theta)(\tan\theta - \sin\theta) \\ &= \left( \frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} \right) \left( \frac{\sin\theta}{\cos\theta} - \sin\theta \right) \\ &= \left( \frac{\cos\theta + 1}{\sin\theta} \right) \left( \frac{\sin\theta - \sin\theta \cos\theta}{\cos\theta} \right) \\ &= \left( \frac{1 + \cos\theta}{\sin\theta} \right) \left( \frac{\sin\theta(1 - \cos\theta)}{\cos\theta} \right) \\ &= (1 + \cos\theta) \frac{(1 - \cos\theta)}{\cos\theta} \\ &= \frac{1 - \cos^2\theta}{\cos\theta} \\ &= \frac{1}{\cos\theta} - \frac{\cos^2\theta}{\cos\theta} \\ &= \sec\theta - \cos\theta \\ &= R.H.S\end{aligned}$$

Home work:

Question No.11

$$\frac{\sin\theta + \cos\theta}{\tan^2\theta - 1} = \frac{\cos^2\theta}{\sin\theta - \cos\theta}$$

Solution:

$$\begin{aligned}L.H.S &= \frac{\sin\theta + \cos\theta}{\tan^2\theta - 1} \\ &= \frac{\sin\theta + \cos\theta}{\frac{\sin^2\theta}{\cos^2\theta} - 1} \\ &= \frac{\sin\theta + \cos\theta}{\frac{\sin^2\theta - \cos^2\theta}{\cos^2\theta}} \\ &= \frac{\sin\theta + \cos\theta}{\sin^2\theta - \cos^2\theta} \times \cos^2\theta \\ &= \frac{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)}{(\sin\theta + \cos\theta)(\sin\theta - \cos\theta)} \times \cos^2\theta \\ &= \frac{1}{\sin\theta - \cos\theta} \times \cos^2\theta \\ &= \frac{\cos^2\theta}{\sin\theta - \cos\theta} \\ &= R.H.S\end{aligned}$$

Home work:

Question No.16

$$(\tan\theta + \cot\theta)(\cos\theta + \sin\theta) = \sec\theta + \operatorname{cosec}\theta$$

Solution:

$$\begin{aligned}L.H.S &= (\tan\theta + \cot\theta)(\cos\theta + \sin\theta) \\ &= \left( \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \right) (\cos\theta + \sin\theta) \\ &= \left( \frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta} \right) (\cos\theta + \sin\theta) \\ &= \left( \frac{1}{\cos\theta \sin\theta} \right) (\cos\theta + \sin\theta) \\ &= \frac{\cos\theta}{\cos\theta \sin\theta} + \frac{\sin\theta}{\cos\theta \sin\theta} \\ &= \frac{1}{\sin\theta} + \frac{1}{\cos\theta} \\ &= \operatorname{cosec}\theta + \sec\theta \\ &= R.H.S\end{aligned}$$

Class work:

Question No.20

$$\frac{1 + \sin\theta}{1 - \sin\theta} - \frac{1 - \sin\theta}{1 + \sin\theta} = 4\tan\theta \sec\theta$$

Solution:

$$\begin{aligned}L.H.S &= \frac{1 + \sin\theta}{1 - \sin\theta} - \frac{1 - \sin\theta}{1 + \sin\theta} \\ &= \frac{(1 + \sin\theta)^2 - (1 - \sin\theta)^2}{(1 - \sin\theta)(1 + \sin\theta)} \\ &= \frac{1 + 2\sin\theta + \sin^2\theta - 1 + 2\sin\theta - \sin^2\theta}{1 - \sin^2\theta} \\ &= \frac{4\sin\theta}{\cos^2\theta} \\ &= \frac{4\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta}\end{aligned}$$



$$= 4 \tan \theta \sec \theta$$

Home work:

Question No.24

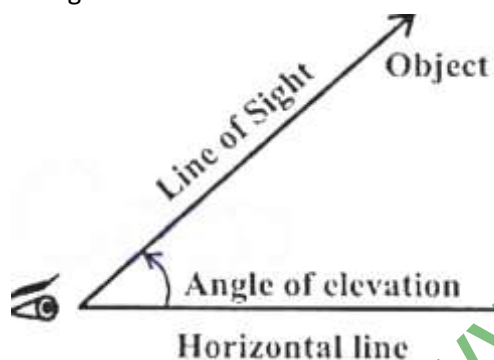
$$\sqrt{\frac{\sec \theta + 1}{\sec \theta - 1}} = \frac{\sec \theta + 1}{\tan \theta}$$

Solution:

$$\begin{aligned} &= \sqrt{\frac{\sec \theta + 1}{\sec \theta - 1}} \\ &= \sqrt{\frac{(\sec \theta + 1)(\sec \theta + 1)}{(\sec \theta - 1)(\sec \theta + 1)}} \\ &= \sqrt{\frac{(\sec \theta + 1)^2}{\sec^2 \theta - 1}} \\ &= \sqrt{\frac{(\sec \theta + 1)^2}{\tan^2 \theta}} \\ &\quad R.H.S \end{aligned}$$

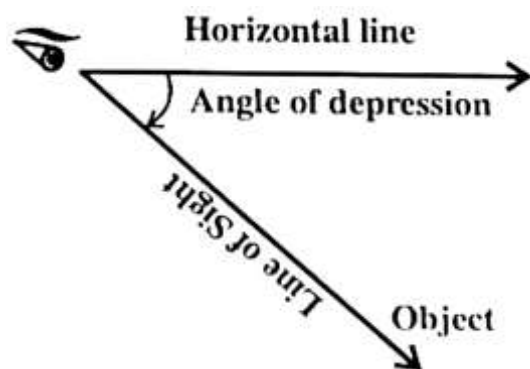
**Angle of Elevation:**

The angle between the horizontal line through eye and a line from eye to the object above the horizontal line is called angle of elevation.



**Angle of depression:**

The angle between the horizontal line through eye and a line from eye to the object below the horizontal line is called angle of depression.

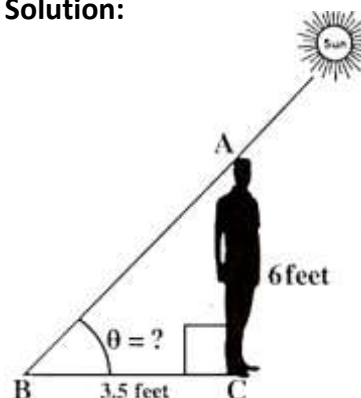


## Exercise 7.5

Class work:

Question No.1 Find the angle of elevation of the sun if a 6feet man casts a 3.5 feet shadow.

Solution:



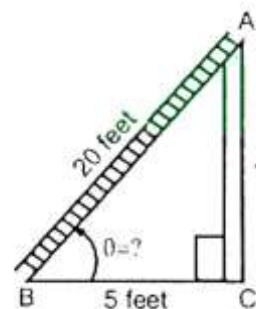
From figure we have

$$\begin{aligned} \tan \theta &= \frac{AB}{BC} \\ \tan \theta &= \frac{6}{3.5} \\ \tan \theta &= 1.714 \\ \theta &= \tan^{-1}(1.7143) \\ \theta &= 59.7437^\circ \\ \theta &= 59^\circ 44' 37'' \end{aligned}$$

Home work:

Question No.3. A 20 feet long ladder is leaning against a wall. The bottom of the wall. Find the acute angle (angle of elevation) the ladder makes with the ground.

Solution:



from the figure

Length of ladder =  $m \overline{AB} = 20 \text{ feet}$

Distance of ladder from the wall =  $m \overline{BC} = 5 \text{ feet}$

Angle of elevation =  $\theta = ?$

Using the fact that

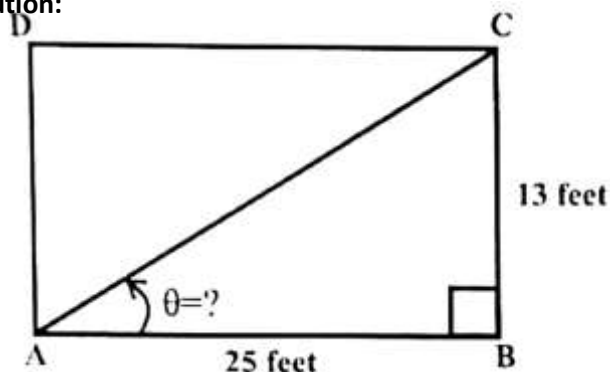
$$\begin{aligned} \cos \theta &= \frac{m \overline{BC}}{m \overline{AB}} \\ \cos \theta &= \frac{5 \text{ ft.}}{20 \text{ ft.}} \\ \cos \theta &= 0.25 \\ \theta &= \cos^{-1} 0.25 \\ \theta &= 75.5225 \\ \theta &= 75.5^\circ \\ \text{or } \theta &= 75^\circ 30' \end{aligned}$$

So, angle of elevation is  $75^{\circ}30'$

**Home work:**

**Question No.4** The base of rectangular is 25 feet and the height of rectangular is 13 feet. Find the angle that diagram of the rectangular makes with the base.

**Solution:**



From the figure

Base of rectangular =  $m\overline{BC} = 25\text{ feet}$

Height of rectangular =  $m\overline{BC} = 13\text{ feet}$

Diagonal  $\overline{AC}$  is taken

Angle between diagonal and base =  $\theta$

Using the fact that

$$\tan\theta = \frac{m\overline{BC}}{m\overline{AB}}$$

$$\tan\theta = \frac{13}{25}$$

$$\theta = \tan^{-1} \frac{13}{25}$$

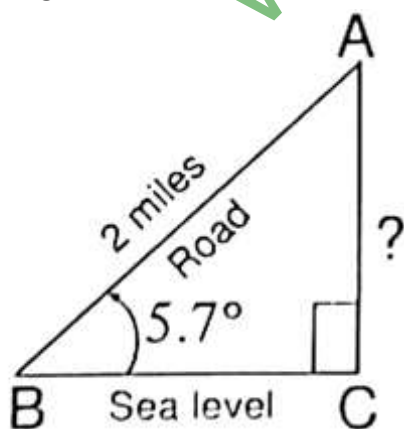
$$\theta = 27.4744^{\circ}$$

$$\theta = 27.47^{\circ}$$

**Home work:**

**Question No.8** A road is inclined at an angle  $5.7^{\circ}$  suppose that we drive 2 miles up this road starting from sea level. How high above sea level are we?

**Solution:**



**From the figure**

Distance covered on road =  $m\overline{AB} = 2\text{ miles}$

Angle of inclination =  $\theta = 5.7^{\circ}$

Height from sea level =  $m\overline{AC} = ?$

Using the fact that,

$$\sin\theta = \frac{m\overline{AC}}{m\overline{AB}}$$

$$\sin 5.7^{\circ} = \frac{m\overline{AC}}{2}$$

$$m\overline{AC} = 2 \times \sin 5.7^{\circ}$$

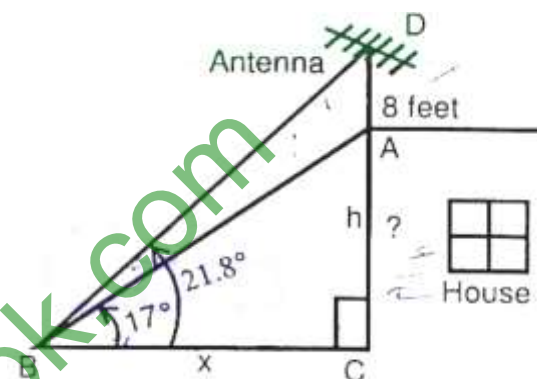
$$m\overline{AC} = 0.199\text{ mile}$$

Hence, we are at height of 0.199 mile from the sea level.

**Class Work:**

**Question No.9.** A television antenna of 8 feet height is point on the top of a house. From a point on the ground the angle of elevations to the top of the house is  $17^{\circ}$  And the angle of elevation to the top of antenna is  $21.8^{\circ}$ . find the height of the house.

**Solution:**



**From the figure**

Distance of point from house  $m\overline{BC} = x$

Height of house =  $m\overline{AC} = h = ?$

Height of antenna =  $m\overline{AD} = 8\text{ feet}$

Angle of elevation of top of house =  $17^{\circ}$

Angle of elevation of top of antenna =  $21.8^{\circ}$

In right angled  $\triangle ABC$

$$\tan 17^{\circ} = \frac{m\overline{AC}}{m\overline{BC}}$$

$$\tan 17^{\circ} = \frac{h}{x}$$

$$x = \frac{1}{\tan 17^{\circ}} \times h$$

$$x = 3.271 \times h \rightarrow (i)$$

Now in right angle  $\triangle DBC$

$$\tan 21.8^{\circ} = \frac{m\overline{CD}}{m\overline{BC}}$$

$$\tan 21.8^{\circ} = \frac{m\overline{AD} + m\overline{AC}}{m\overline{BC}}$$

$$\tan 21.8^{\circ} = \frac{8 + h}{x}$$

$$0.40 = \frac{8 + h}{3.271h} \text{ from (i)}$$

$$0.40 \times 3.271h = 8 + h$$

$$1.3084h - h = 8$$

$$(1.3084 - 1)h = 8$$

$$0.3084h = 8$$



$$h = \frac{8}{0.3084}$$

$$h = \frac{8}{0.3084} = 25.94 \text{ feet}$$

### MISCELLANEOUS EXERCISE -7

**Multiple choice questions: Four possible answers are given for the following questions. Tick (✓) the correct answer.**

- The union of two non-collinear rays, which have common end point is called:
  - an angle
  - a degree
  - a minute
  - a radian
- The system of measurement in which the angle is measured in radians is called:
  - CGS system
  - sexagesimal system
  - MKS system
  - circular system
- $20^\circ = \dots\dots\dots$ 
  - $360'$
  - $630'$
  - $1200'$
  - $3600'$
- $\frac{3\pi}{4}$  radians =
  - $115^\circ$
  - $135^\circ$
  - $150^\circ$
  - $30^\circ$
- If  $\tan \theta = \sqrt{3}$ , then  $\theta$  is equal to:
  - $90^\circ$
  - $45^\circ$
  - $60^\circ$
  - $30^\circ$
- $\sec^2 \theta =$ 
  - $1 - \sin^2 \theta$
  - $1 + \tan^2 \theta$
  - $1 + \cos^2 \theta$
  - $1 - \tan^2 \theta$
- $\frac{1}{1 + \sin \theta} + \frac{1}{1 - \sin \theta} =$ 
  - $2\sec^2 \theta$
  - $2\cos^2 \theta$
  - $\sec^2 \theta$
  - $\cos \theta$
- $\frac{1}{2} \operatorname{cosec} 45^\circ$ 
  - $\frac{1}{2\sqrt{2}}$
  - $\frac{1}{\sqrt{2}}$
  - $\sqrt{2}$
  - $\frac{\sqrt{3}}{2}$
- $\sec \theta \cot \theta =$ 
  - $\sin \theta$
  - $\frac{1}{\cos \theta}$
  - $\frac{1}{\sin \theta}$
  - $\frac{\sin \theta}{\cos \theta}$
- $\operatorname{cosec}^2 \theta - \cot^2 \theta =$ 
  - 1
  - 1
  - 0
  - $\tan \theta$
- In degree measurement,  $1^\circ$  is equal to:
  - $1'$
  - $60'$
  - $90'$
  - $360'$

- $1'$
  - $60'$
  - $90'$
  - $360'$
- In degree measurement,  $1'$  is equal to:
    - $1''$
    - $60''$
    - $90''$
    - $360''$
  - How many right angles are there in 360 degrees?
    - two
    - four
    - six
    - eight
  - If 'r' is the radius of a circle, then its circumference is:
    - $\frac{\pi}{2} r$
    - $\pi r$
    - $2\pi r$
    - $4\pi r$
  - The radian measure of an angle that form a complete circle is:
    - $\frac{\pi}{2}$
    - $\pi$
    - $2\pi$
    - $4\pi$
  - $2\pi$  radians =
    - $0^\circ$
    - $90^\circ$
    - $180^\circ$
    - $360^\circ$
  - $\pi$  radians =
    - $0^\circ$
    - $90^\circ$
    - $180^\circ$
    - $360^\circ$
  - $1^\circ =$ 
    - $180\pi$  radian
    - $\pi$  radian
    - $\frac{\pi}{180}$  radian
    - $\frac{180}{\pi}$  radian
  - 1 radian =
    - $(180\pi)^\circ$
    - $(180)^\circ$
    - $\left(\frac{\pi}{180}\right)^\circ$
    - $\left(\frac{180}{\pi}\right)^\circ$
  - $\frac{\pi}{2}$  radians =
    - $30^\circ$
    - $45^\circ$
    - $60^\circ$
    - $90^\circ$
  - $\frac{\pi}{3}$  radians =
    - $30^\circ$
    - $45^\circ$
    - $60^\circ$
    - $90^\circ$
  - $\frac{\pi}{4}$  radians =
    - $30^\circ$
    - $45^\circ$
    - $60^\circ$
    - $90^\circ$
  - $\frac{\pi}{6}$  radians =
    - $30^\circ$
    - $45^\circ$

- (c)  $60^\circ$  (d)  $90^\circ$
24.  $\frac{3\pi}{2}$  radians =  
 (a)  $90^\circ$  (b)  $180^\circ$   
 (c)  $270^\circ$  (d)  $360^\circ$
25.  $1^\circ =$   
 (a) 0.0175 radians  
 (b) 0.175 radians  
 (c) 1.75 radians  
 (d) 175 radians
26. A part of circumference of a circle is called:  
 10307210  
 (a) radius (b) chord  
 (c) sector (d) arc
27. Formula for arc length is:  
 (a)  $\ell = r\theta$  (b)  $r = \ell\theta$   
 (c)  $\theta = \ell r$  (d)  $\ell = \frac{r}{\theta}$
28. Area of a circular sector =  
 (a)  $r\theta$  (b)  $r^2\theta$   
 (c)  $\frac{1}{2}r\theta$  (d)  $\frac{1}{2}r^2\theta$
29.  $\frac{1}{\sin\theta} =$   
 (a)  $\cos\theta$  (b)  $\sec\theta$   
 (c)  $\operatorname{cosec}\theta$  (d)  $\cot\theta$
30.  $\frac{1}{\cos\theta} =$   
 (a)  $\sin\theta$  (b)  $\sec\theta$   
 (c)  $\operatorname{cosec}\theta$  (d)  $\cot\theta$
31.  $\frac{1}{\tan\theta} =$   
 (a)  $\tan\theta$  (b)  $\sec\theta$   
 (c)  $\operatorname{cosec}\theta$  (d)  $\cot\theta$
32.  $\sin 45^\circ =$   
 (a) 1 (b)  $\sqrt{2}$   
 (c)  $\frac{1}{\sqrt{2}}$  (d) 0
33.  $\cos 45^\circ =$   
 (a) 1 (b)  $\sqrt{2}$   
 (c)  $\frac{1}{\sqrt{2}}$  (d) 0
34.  $\tan 45^\circ =$   
 (a) 1 (b)  $\sqrt{2}$   
 (c)  $\frac{1}{\sqrt{2}}$  (d) 0
35.  $\operatorname{cosec} 45^\circ =$   
 (a) 1 (b)  $\sqrt{2}$

- (c)  $\frac{1}{\sqrt{2}}$  (d) 0
36.  $\sec 45^\circ =$   
 (a) 1 (b)  $\sqrt{2}$   
 (c)  $\frac{1}{\sqrt{2}}$  (d) 0
37.  $\cot 45^\circ =$   
 (a) 1 (b)  $\sqrt{2}$   
 (c)  $\frac{1}{\sqrt{2}}$  (d) 0
38.  $\sin 30^\circ =$   
 (a)  $\frac{1}{2}$  (b)  $\frac{\sqrt{3}}{2}$   
 (c) 2 (d)  $\frac{2}{\sqrt{3}}$
39.  $\cos 30^\circ =$   
 (a)  $\frac{1}{2}$  (b)  $\frac{\sqrt{3}}{2}$   
 (c) 2 (d)  $\frac{2}{\sqrt{3}}$
40.  $\tan 30^\circ =$   
 (a)  $\frac{1}{2}$  (b)  $\frac{\sqrt{3}}{2}$   
 (c)  $\sqrt{3}$  (d)  $\frac{1}{\sqrt{3}}$

**(Answer key)**

1.	a	2.	d	3.	c	4.	b	5.	c
6.	b	7.	a	8.	b	9.	c	10.	b
11.	b	12.	b	13.	b	14.	c	15.	c
16.	d	17.	c	18.	c	19.	d	20.	d
21.	c	22.	b	23.	a	24.	c	25.	a
26.	d	27.	a	28.	d	29.	c	30.	b
31.	d	32.	c	33.	c	34.	a	35.	b
36.	b	37.	a	38.	a	39.	b	40.	d

**Question No.2. write a short answer of the following questions.**

- Define an angle:  
 An angle is defined as the union of two non-collinear rays with some common end point. The rays are called arms of the angle the common end point is known as vertex of the angle.
- What is sexagesimal system of measurement of angles?

Ans. In sexagesimal system of measurement of angles. We find angles in degree minute and second.

$$1 \text{ degree} = 60 \text{ minutes}$$

$$1 \text{ minute} = 60 \text{ seconds}$$

3. How many minutes are in two right angles?

Ans.

Two right angles means  $90^\circ + 90^\circ = 180^\circ$

$$1^\circ = 60 \text{ minutes}$$

$$180^\circ = 60 \times 180 \text{ minutes}$$

$$= 10800 \text{ minutes.}$$

4. Define radian measure of an angle.

Ans.

**Radian:**

The angle subtended at the Centre of the circle by an arc, whose length is equal to the radius of the circle is called one radian.

$$\pi \text{ radian} = 180^\circ$$

5. Convert  $\frac{\pi}{4}$  radian to degree measure.

Ans.

$$\text{since } \pi \text{ radian} = 180^\circ$$

$$1 \text{ radian} = \frac{180}{\pi}$$

$$\frac{\pi}{4} \text{ radian} = \frac{180^\circ}{\pi} \times \frac{\pi}{4}$$

$$= \frac{\pi}{4} \times \frac{4 \times 45}{\pi} \text{ degree}$$

$$= 45^\circ$$

6. Convert  $15^\circ$  to radians.

Ans.

$$15^\circ = 15 \frac{\pi}{180} \text{ radian}$$

$$= 15 \frac{\pi}{15 \times 12} \text{ radian}$$

$$= \frac{\pi}{12} \text{ radian.}$$

**Convert  $15^\circ$  to radians** 10307261

7. What is the radian measure of the central angle of an arc 50m long on the circle of radius 25m.

Solution:

$$l = 50 \text{ m}$$

$$r = 25 \text{ m}$$

$$\theta = ?$$

$$\theta = \frac{l}{r}$$

$$\theta = \frac{50}{25} = 2$$

$$\theta = 2 \text{ radians.}$$

8. Find  $r$  when  $l = 56 \text{ cm}$  and  $\theta = 45^\circ$

Solution:

$$l = 56 \text{ m}$$

$$\theta = 45^\circ = 45 \times \frac{\pi}{180} \text{ radians}$$

$$= \frac{\pi}{4} \text{ radians}$$

$$r = ?$$

$$r = \frac{l}{\theta}$$

$$= \frac{56}{\frac{\pi}{4}}$$

$$= \frac{56}{\frac{\pi}{4}} = \frac{56 \times 4 \times 7}{22}$$

$$= \frac{787}{11}$$

$$= 71.27 \text{ cm}$$

9. Find  $\tan \theta$  when  $\cos \theta = \frac{9}{41}$  and terminal side of

the angle  $\theta$  is in fourth quadrant.

Solution:

$$\cos \theta = \frac{9}{41}$$

$$\Rightarrow x = 9 \text{ and } r = 41$$

by pythagorous theorem, we have

$$r^2 = x^2 + y^2$$

$$y^2 = r^2 - x^2$$

$$y = \sqrt{r^2 - x^2}$$

$$= \sqrt{(41)^2 - (9)^2}$$

$$= \sqrt{1681 - 81}$$

$$y = \sqrt{1600}$$

$$y = \pm 40$$

$$\text{Now } \tan \theta = \frac{\text{per.}}{\text{base}} = -\frac{40}{9}$$

-ve sign show  $\tan \theta$  is -ve in IV quadrant.

10. Prove that  $(1 - \sin^2 \theta)(1 + \tan^2 \theta) = 1$

Solution:

$$L.H.S = (1 - \sin^2 \theta)(1 + \tan^2 \theta) = 1$$

$$= \cos^2 \theta \cdot \frac{1}{\cos^2 \theta} = 1$$

$$L.H.S = R.H.S$$

# Unit-8

## PROJECTION OF SIDE OF A TRIANGLE

### THEOREM 2 Home Work +Class Work

In any triangle, the square on the side opposite to acute angle is equal to sum of the squares on the sides containing that acute angle diminished by twice the rectangle contained by one of those sides and the projection on it of the other.

**Given:**

$\triangle ABC$  with an acute angle  $CAB$  at  $A$ .

Take  $\overline{BC} = a$   $\overline{CA} = b$  and  $\overline{AB} = c$

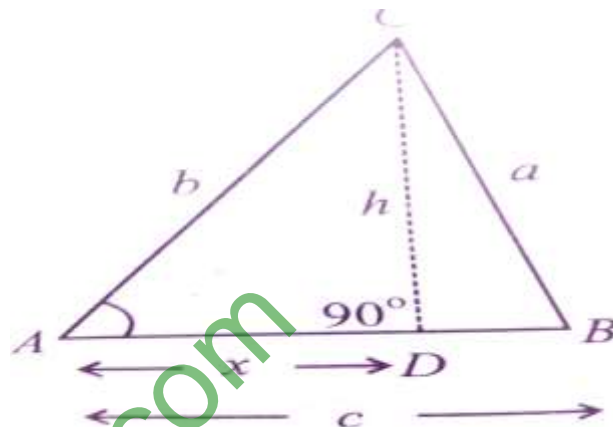
Draw  $\overline{CD} \perp \overline{AB}$  so that  $\overline{AD}$  is projection of  $\overline{AC}$  on  $\overline{AB}$

Also,  $\overline{AD} = x$  and  $\overline{CD} = h$

**To prove:**

$$(\overline{BC})^2 = (\overline{AC})^2 + (\overline{AB})^2 - 2(\overline{AB})(\overline{AD})$$

$$\text{i.e., } a^2 = b^2 + c^2 - 2cx$$



**Proof:**

Statement	Reasons
In $\triangle CDA$	
$\angle CDA = 90^\circ$	Given
$(AC)^2 = (AD)^2 + (CD)^2$	Pythagoras theorem
i.e., $b^2 = x^2 + h^2 \rightarrow (i)$	
In $\triangle CDB$ ,	
$\angle CDB = 90^\circ$	Given
$(BC)^2 = (BD)^2 + (CD)^2$	
$a^2 = (c - x)^2 + h^2$	From the figure
or $a^2 = c^2 - 2cx + x^2 + h^2 \rightarrow (ii)$	
$a^2 = b^2 + c^2 - 2cx$	Using (i) and (ii)
Hence $a^2 = b^2 + c^2 - 2cx$	

$$\text{i.e., } (\overline{BC})^2 = (\overline{AC})^2 + (\overline{AB})^2 - 2(\overline{AB})(\overline{AD})$$

**Question No.3** In a  $\triangle ABC$ , calculate  $\overline{BC}$  when  $\overline{AB} = 5\text{cm}$ ,  $\overline{AC} = 4\text{cm}$ ,  $\angle A = 60^\circ$

**Solution:**

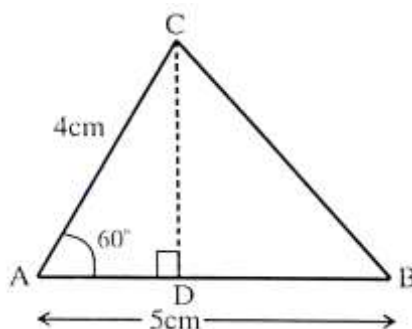
$$\frac{\overline{AD}}{\overline{AC}} = \cos 60^\circ$$

$$\overline{AD} = \overline{AC} \cos 60^\circ$$

$$\overline{AD} = \frac{1}{2} \times 4 = 2$$

$$\overline{AD} = 2$$

By the statement



$$(\overline{BC})^2 = (\overline{AC})^2 + (\overline{AB})^2 - 2(\overline{AB})(\overline{AD})$$

By putting values

$$\begin{aligned}
 (\overline{BC})^2 &= (4)^2 + (5)^2 - 2(5)(2) \\
 &= 14 + 25 - 20 \\
 (\overline{BC})^2 &= 21\text{cm} \\
 m\overline{BC} &= 4.58\text{cm}
 \end{aligned}$$

**Question No.5 Class Work +Home Work:**

In a triangle ABC,  $m\overline{BC} = 21\text{cm}$ ,  $m\overline{AC} = 17\text{cm}$ ,  $m\overline{AB} = 10\text{cm}$ . Measure the length of projection of  $\overline{AC}$  upon  $\overline{BC}$

**BC Solution:**

$$(\overline{AB})^2 = (\overline{AC})^2 + (\overline{BC})^2 - 2\overline{BC} \cdot \overline{DC}$$

Putting the values

$$(10)^2 = (17)^2 + (21)^2 - 2 \times 21 \times \overline{DC}$$

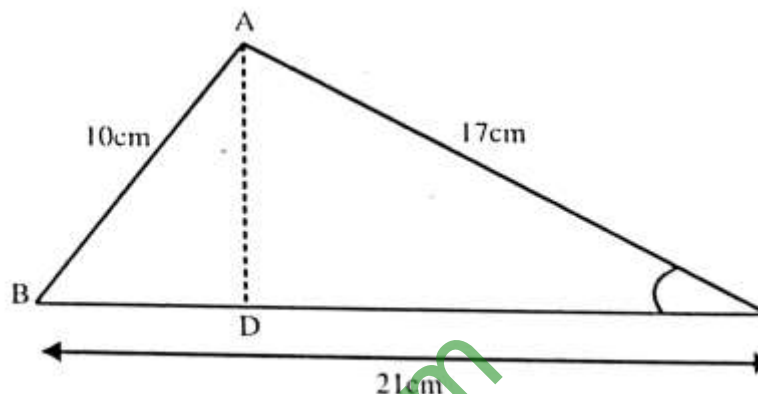
$$100 = 289 + 441 - 2 \times 21 \times \overline{DC}$$

$$100 = 730 - 42\overline{DC}$$

$$42\overline{DC} = 730 - 100$$

$$\overline{DC} = \frac{630}{42}$$

$$\overline{DC} = 15\text{cm}$$

**Question No.8**

In a  $\triangle ABC$ ,  $a = 17\text{cm}$ ,  $b = 15\text{cm}$  and  $c = 8\text{cm}$  find  $m\angle B$ .

**Solution:**

From the figure

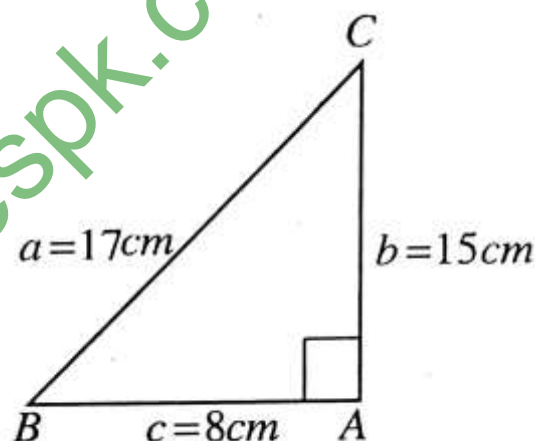
$$\frac{\text{Base}}{\text{hyper.}} = \cos B$$

$$\cos B = \frac{8}{17}$$

$$\cos B = 0.470$$

$$\angle \cos^{-1}(0.470)$$

$$\angle B = 61.9^\circ$$



# Unit-9

## CHORDS OF A CIRCLE

**Class Work+ Home Work:****THEOREM 2**

A straight line, drawn from the *centre* of a circle to bisect a chord (which is not a diameter) is perpendicular to the chord.

**Given:**

M is the midpoint of any chord  $\overline{AB}$  of a circle with *centre* at O.

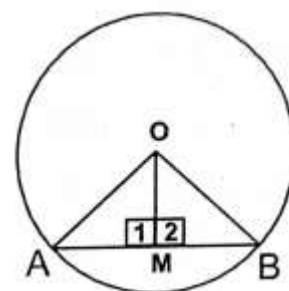
Where chord  $\overline{AB}$  is not the diameter of the circle.

**To prove:**

$\overline{OM} \perp$  the chord  $\overline{AB}$ .

**Construction:**

Join A and B with *centre* O. write  $\angle 1$  and  $\angle 2$  as shown in the figure.



Statements	Reasons
In $\triangle OAM \leftrightarrow \triangle OBM$	
$m\overline{OA} = m\overline{OB}$	Radii of the same circle.
$m\overline{AM} = m\overline{BM}$	Given
$m\overline{OM} = m\overline{OM}$	Common
$\triangle OAM \cong \triangle OBM$	$S.S.S \cong S.S.S$
$\Rightarrow m\angle 1 = m\angle 2 \text{ --- (i)}$	Corresponding sides of congruent triangles.
i.e., $m\angle 1 + m\angle 2 = m\angle AMB = 180^\circ \text{ --- (ii)}$	Adjacent supplementary angles.
$\therefore m\angle 1 = m\angle 2 = 90^\circ$	From (i) and (ii)
i.e., $\overline{OM} \perp \overline{AB}$	Hence Proved.

**THEOREM 4**

If two chords of a circle are congruent then they will be equidistant from the *centre*.

**Given:**

$\overline{AB}$  and  $\overline{CD}$  are two equal chords of a circle with centre at  $O$ .

So that  $\overline{OH} \perp \overline{AB}$  and  $\overline{OK} \perp \overline{CD}$ .

**To Prove:**

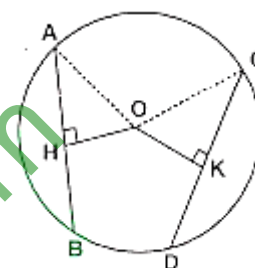
$m\overline{OH} = m\overline{OK}$

**Construction:**

Join  $O$  with  $A$  and  $O$  with  $C$ .

So that we have  $\triangle OAH$  and  $\triangle OCK$ .

**Proof:**



Statements	Reasons
$\overline{OH}$ bisects chord $\overline{AB}$	$\overline{OH} \perp \overline{AB}$ By theorem 3
$m\overline{AH} = \frac{1}{2}m\overline{AB} \text{ --- (i)}$	
Similarly $\overline{OK}$ bisects chord $\overline{CD}$	$\overline{OK} \perp \overline{CD}$ By theorem 3
$m\overline{CK} = \frac{1}{2}m\overline{CD} \text{ --- (ii)}$	
But $m\overline{AB} = m\overline{CD} \text{ --- (iii)}$	Given
Hence $m\overline{AH} = m\overline{CK} \text{ --- (iv)}$	Using (i), (ii) & (iii)
Now in $\triangle OAH \leftrightarrow \triangle OCK$	Given $\overline{OH} \perp \overline{AB}$ and $\overline{OK} \perp \overline{CD}$
hyp $\overline{OA} = \text{hyp } \overline{OC}$	Radii of the same circle
$m\overline{AH} = m\overline{CK}$	Already proved in (iv)
$\triangle OAH \cong \triangle OCK$	$H.S$ postulate.
$\Rightarrow m\overline{OH} = m\overline{OK}$	

**MISCELLANEOUS EXERCISE -9****From Exercise MCQ'S Start number 5****5. Radii of a circle are (Board 2014)**

- (a) all equal
- (b) double of the diameter
- (c) all unequal
- (d) half of any chord

**6. A chord Passing through the centre of a circle is called:**

- (a) radius
- (b) diameter
- (c) circumference
- (d) secant

**7. Right bisector of the chord of a circle always passes through the:**

- (a) radius
- (b) circumference
- (c) centre
- (d) diameter

**8. The circular region bounded by two radii and the corresponding arc is called:**

- (a) circumference of a circle
- (b) sector of a circle
- (c) diameter of a circle
- (d) segment of a circle

**9. The distance of any point of the circle to its centre is called:**

- (a) radius
- (b) diameter
- (c) a chord
- (d) an arc

**10. Line segment joining any point of the circle to the centre is called: (Board 2014)**

- (a) circumference
- (b) diameter
- (c) radial segment
- (d) perimeter

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**11. Locus of a point in a plane equidistant from a fixed point is called:**

- (a) radius
- (b) circle
- (c) circumference
- (d) diameter

**12. The symbol for a triangle is denoted by:**

- (a)  $\angle$
- (b)  $\Delta$
- (c)  $\perp$
- (d)  $\square$

**13. A complete circle is divided into:**

- (a) 90 degree
- (b) 180 degree
- (c) 270 degree
- (d) 360 degree

**14. Through how many non-collinear points, a circle can pass?**

- (a) one
- (b) two
- (c) three
- (d) None

**15. The vertex of central angle is at:**

- (a) circumference
- (b) center
- (c) any point of radius
- (d) any point of diameter

**16. The line segment joining the centre and any point of circle is called:**

- (a) circumference
- (b) radial segment
- (c) chord
- (d) diameters

**17. The length of boundary traced by a moving point in a circular path is called:**

- (a) circumference
- (b) radial segment
- (c) chord
- (d) diameter

**18. The line segment joining any two points of circle is called:**

- (a) circumference
- (b) radial segment
- (c) chord
- (d) diameter

**19. The central chord of circle is its:**

- (a) circumference
- (b) radial segment
- (c) chord
- (d) diameter

**20. The largest chord of a circle is its:**

- (a) circumference
- (b) radial segment
- (c) chord
- (d) diameter

**21. A circle of radius 4cm has a chord few cm away from its centre, which of the following length of chord may be?**

- (a) 6cm
- (b) 8cm
- (c) 10cm
- (d) 12cm

**22.  $\pi$  is the ratio of: (a) radius and diameter**

- (b) diameter and circumference
- (c) circumference and diameter
- (d) circumference and radius

**23.  $\pi \approx \frac{22}{7}$  is an ..... number. 10309049**

- (a) rational
- (b) irrational
- (c) natural
- (d) prime



24. If radius of a circle is “r”, then its diameter is:
- (a)  $r^2$  (b)  $2 + r$   
(c)  $2r$  (d)  $r - 2$

25. If central chord of a circle is 12cm, then its radius is:
- (a) 6cm (b) 8cm  
(c) 12cm (d) 24cm

(Answer key)

5.	a								
6.	b	7.	c	8.	b	9.	a	10.	c
11.	b	12.	b	13.	d	14.	c	15.	b
16.	b	17.	a	18.	c	19.	d	20.	d
21.	a	22.	c	23.	b	24.	c	25.	a

# Unit-10

## TANGENT TO A CIRCLE

### THEOREM 1:

#### Home Work+ Class Work

If a line is drawn perpendicular to a radial segment of a circle at its outer end point, it is tangent to the circle at that point.

#### Given:

A circle with centre  $O$  and  $\overline{OC}$  is the radial segment.  
 $\overline{AB}$  is perpendicular to  $\overline{OC}$  at its outer end  $C$ .

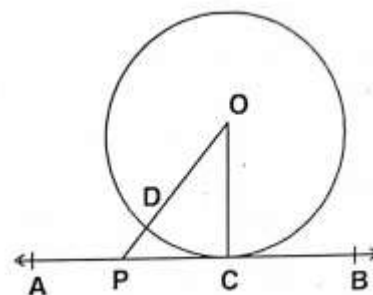
#### To Prove:

$\overline{AB}$  is a tangent to the circle.

#### Construction:

Take any point  $P$  other than  $C$  on  $\overline{AB}$ . Join  $O$  with  $P$

#### Proof:



Statements	Reasons
$\triangle OCP$ $m\angle OCP = 90^\circ$ And $m\angle OPC < 90^\circ$ $m\overline{OP} > m\overline{OC}$	$\overline{AB} \perp \overline{OC}$ (given) Acute angle of right angled triangle. Greater angle has the greater side opposite to it.
$\therefore P$ is the point outside the circle. Similarly, every point on $\overline{AB}$ except $C$ lies outside the circle. Hence $\overline{AB}$ intersect the circle at one point $C$ only. $\overline{AB}$ is a tangent to the circle one point only.	$\overline{OC}$ is the radial segment.

**THEOREM 3**

**Two tangents drawn to a circle from a point outside it, are equal in length.**

**Given:**

Two tangents  $\overline{PA}$  and  $\overline{PB}$  are drawn from an external point  $P$  to the circle with center  $O$ .

**To Prove:**

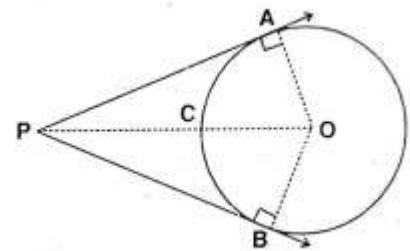
$$m\overline{PA} = m\overline{PB}.$$

**Construction:**

Join  $O$  with  $A, B$  and  $P$ .

So that we have  $\angle \text{right } \Delta^s OAP$  and  $OBP$ .

**Proof:**

**Statements**

In  $\angle \text{rt } \Delta^s OAP \leftrightarrow OBP$

$$m\angle OAP = m\angle OBP = 90^\circ$$

$$\text{hyp. } \overline{OP} = \text{hyp. } \overline{OP}$$

$$m\overline{OA} = m\overline{OB}$$

$$\therefore \Delta OAP \cong OBP$$

$$\text{Hence, } m\overline{PA} = m\overline{PB}$$

**Reasons**

$\overline{OH} \perp \overline{AB}$  By theorem 3

Radii  $\perp$  to the tangents  $\overline{PA}$  and  $\overline{PB}$

Common

Radii of the same circle.

In  $\angle \text{rt } \Delta^s H.S \cong H.S$

**MISCELLANEOUS EXERCISE -10**

5. A line which has two points in common with a circle is called:  
 (a) sine of a circle  
 (b) cosine of a circle  
 (c) tangent of a circle  
 (d) secant of a circle
6. A line which has only one point in common with a circle is called:  
 (a) sine of a circle  
 (b) cosine of a circle  
 (c) tangent of a circle  
 (d) secant of a circle
7. Two tangents drawn to a circle from a point outside it are .....in length.  
 (a) half (b) equal  
 (c) double (d) triple
8. A circle has only one:  
 (a) secant (b) chord  
 (c) diameter (d) centre
9. A tangent line intersects the circle at:  
 (a) three points (b) two points  
 (c) single point (d) no point at all
10. Tangents drawn at the ends of diameter of a circle are..... to each other.  
 (a) parallel (b) non-parallel  
 (c) collinear (d) perpendicular
11. The distance between the centres of two congruent touching circles externally is:  
 (a) of zero length  
 (b) the radius of each circle  
 (c) the diameter of each circle  
 (d) twice the diameter of each circle
12. In the adjacent circular figure with centre  $O$  and radius 5cm. The length of the chord intercepted at 4cm away from the centre of this circle is:  
 (a) 4cm  
 (b) 6cm  
 (c) 7cm  
 (d) 9cm
13. In the adjoining figure there is a circle with centre  $O$ . If  $\overline{DC} \parallel$  diameter  $\overline{AB}$  and  $m\angle AOC = 120^\circ$ , then  $m\angle ACD$  is: 10310033  
 (a)  $40^\circ$   
 (b)  $30^\circ$   
 (c)  $50^\circ$   
 (d)  $60^\circ$

(Answer key)

5.	d	6.	c	7.	b	8.	d	9.	c
10	a	11	c	12	b	13	b		

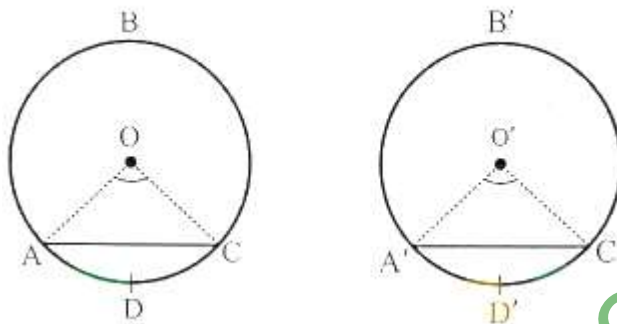
# Unit-11

## CHORDS AND ARCS

### THEOREM 1

#### Home Work+ Class Work

If two arcs of a circle (or of congruent circles) are congruent then the corresponding chords are equal.



#### Given:

$ABCD$  and  $A'B'C'D'$  are two equal circles with center  $O$  and  $O'$  respectively. So that  $m\widehat{ADC} = m\widehat{A'D'C'}$

#### To Prove:

$$m\overline{AC} = m\overline{A'C'}$$

#### Construction:

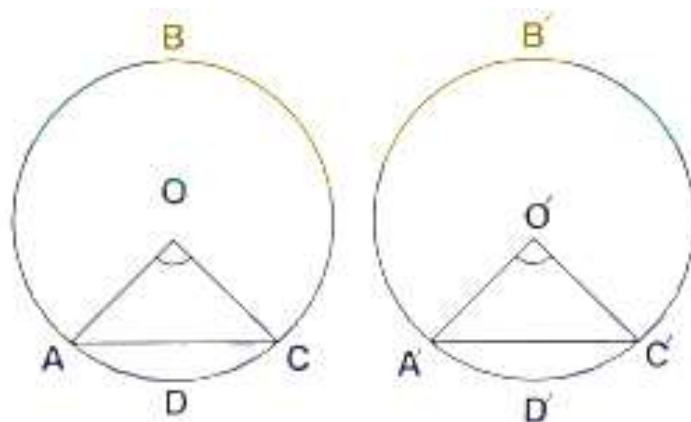
Join  $O$  with  $A$ ,  $O$  with  $C$  and  $O'$  with  $A'$ ,  $O'$  with  $C'$ .  
So that we can form  $\triangle OAC$  and  $\triangle O'A'C'$ .

#### Proof:

Statements	Reasons
In two equal circles $ABCD$ and $A'B'C'D'$ with centers $O$ and $O'$ respectively.	Given
$m\widehat{ADC} = m\widehat{A'D'C'}$	Given
$m\angle AOC = m\angle A'O'C'$	Central angle subtended by equal arcs of the equal circles.
Now in $\triangle AOC \leftrightarrow \triangle A'O'C'$	
$m\overline{OA} = m\overline{O'A'}$	Radii of equal circles.
$m\angle AOC = m\angle A'O'C'$	Already Proved.
$m\overline{OC} = m\overline{O'C'}$	Radii of the equal circles
$\triangle AOC \cong \triangle A'O'C'$	S.A.S $\cong$ S.A.S
And in particular $m\overline{AC} = m\overline{A'C'}$	
Similarly we can prove the theorem in the same circle.	

# THEOREM 4

If the angles subtended by two chords of a circle (or congruent circles) at the centre (corresponding centres) are equal, the chords are equal.



## Given:

$ABCD$  and  $A'B'C'D'$  are two congruent circles with center  $O$  and  $O'$  respectively.  $\overline{AC}$  and  $\overline{A'C'}$  are chords of circles  $ABCD$  and  $A'B'C'D'$  respectively and  $m\angle AOC = m\angle A'O'C'$ .

## To Prove:

$$m\overline{AC} = m\overline{A'C'}$$

## Proof:

Statements	Reasons
In $\triangle OAC \leftrightarrow \triangle O'A'C'$	
$m\overline{OA} = m\overline{O'A'}$	Radii of the congruent circles.
$m\angle AOC = m\angle A'O'C'$	Given
$m\overline{OC} = m\overline{O'C'}$	Radii of the congruent circles.
$\triangle OAC \cong \triangle O'A'C'$	S.A.S $\cong$ S.A.S
Hence $m\overline{AC} = m\overline{A'C'}$	

## MISCELLANEOUS EXERCISE – 11

### Q.1 Multiple Choice Questions

Four possible answers are given for the following questions.

1. A 4 cm long chord subtends central angle of  $60^\circ$ . The radial segment of this, circle:
  - (a) 1cm                      (b) 2cm
  - (c) 3cm                      (d) 4cm
2. The length of a chord and the radial segment of a circle are congruent, the central angle made by the chord will be:
  - (a)  $30^\circ$                       (b)  $45^\circ$
  - (c)  $60^\circ$                       (d)  $75^\circ$
3. Out of two congruent arcs of a circle, if one arc makes a central angle of  $30^\circ$  then the other arc will subtend the central angle of:
  - (a)  $15^\circ$                       (b)  $30^\circ$
  - (c)  $45^\circ$                       (d)  $60^\circ$
4. An arc subtends a central angle of  $40^\circ$  then the corresponding chord will subtended a central angle of:
  - (a)  $20^\circ$                       (b)  $40^\circ$
  - (c)  $60^\circ$                       (d)  $80^\circ$
5. A pair of chords of a circle subtending two congruent central angles is:
  - (a) congruent                      (b) incongruent
  - (c) over lapping                      (d) parallel
6. If an arc of a circle subtends a central angle of  $60^\circ$ , then the corresponding chord of the arc will make the central angle of:)
  - (a)  $20^\circ$                       (b)  $40^\circ$
  - (c)  $60^\circ$                       (d)  $80^\circ$
7. The semi circumference and the diameter of a circle both subtend a central angle of
  - (a)  $90^\circ$                       (b)  $180^\circ$                       10311018
  - (c)  $270^\circ$                       (d)  $360^\circ$
8. The chord length of a circle subtending a central angle of  $180^\circ$  is always:
  - (a) less than radial segment
  - (b) equal to the radial segment
  - (c) double of the radial segment
  - (d) none of these
9. If a chord of a circle subtends a central angle of  $60^\circ$ , then the length of the chord and the radial segment are:
  - (a) congruent                      (b) incongruent
  - (c) parallel                      (d) perpendicular
10. The arcs opposite to incongruent central angles of a circle are always:
  - (a) Congruent                      (b) incongruent
  - (c) parallel                      (d) perpendicular

### (Answer key)

1.	d	2.	c	3.	b	4.	b	5.	a
6.	c	7.	b	8.	c	9.	a	10.	b

# Unit-12

## ANGLE IN A SEGMENT OF A CIRCLE

### THEOREM 1

**Home Work+ Class Work:**

The measure of a central angle of a minor arc of a circle, is double that of the angle subtended by the corresponding major arc.

**Given:**

$\widehat{AC}$  is an arc of a circle with centre O.

Whereas  $\angle AOC$  is central angle.

And  $\angle ABC$  is circum angle.

**To prove:**

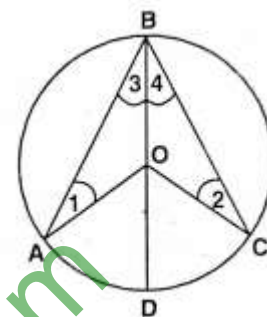
$$m\angle AOC = 2m\angle ABC$$

**Construction:**

Join B with O and produce it to meet the circle at D.

Write angles  $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5$  and  $\angle 6$  as shown in figure.

**Proof:**



Statements	Reasons
As $m\angle 1 = m\angle 3 \rightarrow (i)$	Angles opposites to equal sides in $\triangle OAB$
And $m\angle 2 = m\angle 4 \rightarrow (ii)$	Angles opposites to equal sides in $\triangle OBC$
Now $m\angle 5 = m\angle 1 + m\angle 3 \rightarrow (iii)$	External angle is the sum of internal opposite angles.
Similarly $m\angle 6 = m\angle 2 + m\angle 4 \rightarrow (iv)$	
Again $m\angle 5 = m\angle 3 + m\angle 3 = 2m\angle 3 \rightarrow (v)$	Using (i) and (ii)
And $m\angle 6 = m\angle 4 + m\angle 4 = 2m\angle 4 \rightarrow (vi)$	Using (ii) and (iv)
Then from figure	
$\Rightarrow m\angle 5 + m\angle 6 = 2m\angle 3 + 2m\angle 4$	Adding (v) and (vi)
$\Rightarrow m\angle AOC = 2(m\angle 3 + m\angle 4) = 2m\angle ABC$	

**THEOREM 2**

Any two angles in the same segment of a circle are equal.

**Given:**

$\angle ACB$  and  $\angle ADB$  are the circumangles

In the same segment of a circle with Centre O.

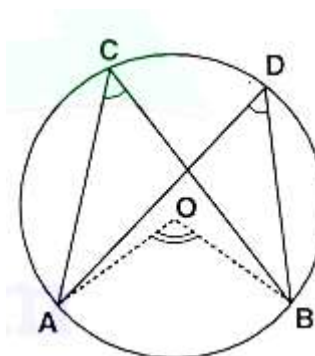
**To prove:**

$$m\angle ACB = m\angle ADB$$

**Construction:**

Join O with A and O with B.

So that  $\angle AOB$  is the central angle.

**Proof:****Statement****Reasons**

Standing on the same arc AB of a circle.

$\angle AOB$  is the central angle whereas

$\angle ACB$  and  $\angle ADB$  are circum angles

$$\therefore m\angle AOB = 2m\angle ACB$$

$$\text{And } m\angle AOB = 2m\angle ADB$$

$$\Rightarrow 2m\angle ACB = 2m\angle ADB$$

$$\text{Hence, } m\angle ACB = m\angle ADB$$

Construction

Given

By theorem 1

By theorem 1

Using (i) and (ii)



# Unit-13

## PRACTICAL GEOMETRY-CIRCLES

### Exercise 13.1

#### Home Work+ Class Work

**Question No.1** Divide an arc of any length

(i) *in to equal parts.*

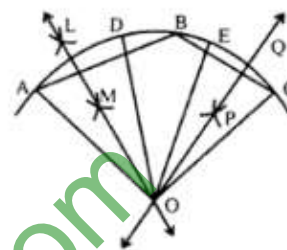
(ii) *into four equal parts.*

Solution:

(i) **in to three equal parts.**

Steps of construction:

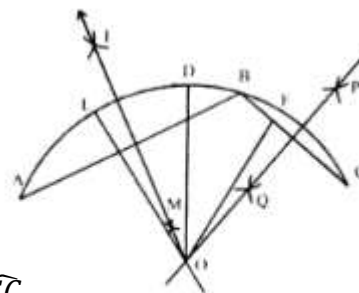
1. Draw an arc  $\widehat{ABC}$ .
2. Join A with B and B and C
3. Draw  $\overline{LM}$  and  $\overline{PQ}$  right bisectors of AB and BC respectively.  $\overline{LM}$  and  $\overline{PQ}$  intersect at point O.
4. Divide the arc ABC in three equal parts. Such that  $\widehat{AD} = \widehat{DE} = \widehat{EC}$



(ii) **into four equal parts.**

Solution:

1. Draw on arc  $\widehat{ABC}$
2. Join A with B and B and C.
3. Divide the arc ABC in four parts. Such that  $\widehat{AE} = \widehat{ED} = \widehat{DF} = \widehat{FC}$
4. Draw  $\overline{LM}$  and  $\overline{PQ}$  right bisector of  $\overline{AB}$  and  $\overline{AC}$  respectively.  $\overline{LM}$  and  $\overline{PQ}$  intersect at point O.

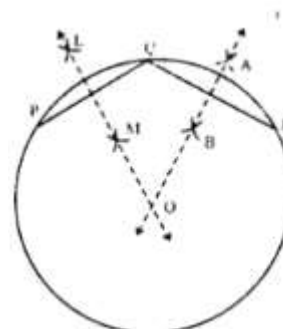


**Question No.4** for an arc two perpendicular bisectors of the chords  $\overline{PQ}$  and  $\overline{QR}$  of this arc, construct a circle through P, Q and R.

Solution:

Steps of construction:

1. Draw an arc  $\widehat{ABC}$
2. Join p with Q and Q with R
3. Draw  $\overline{LM}$  and  $\overline{PQ}$  right bisector of  $\overline{PQ}$  and  $\overline{QR}$  respectively.  $\overline{LM}$  and  $\overline{PQ}$  intersect at point O.
4. O is the required centre of an arc ABC.
5. Draw a circle with radius  $\overline{OP} = \overline{OQ} = \overline{OR}$  having centre at O. which is required circle.



## Exercise 13.2

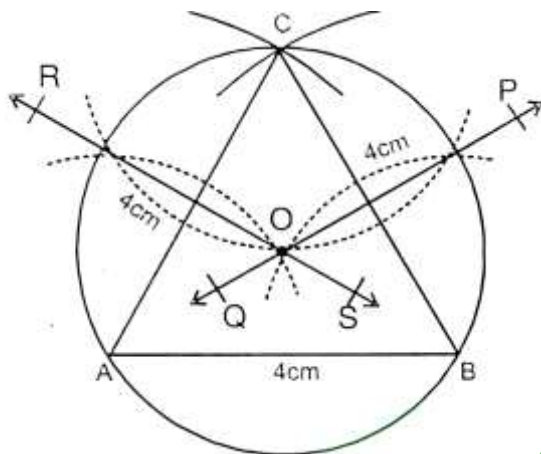
### Question No.4

**Circumscribe a circle about an equilateral triangle ABC with each side of length 4cm**

**Solution:**

**Data:**

$$m\overline{AB} = m\overline{BC} = 4\text{cm}$$



### Steps of construction:

1. We construct equilateral triangles ABC with each side 4cm long.
2. We draw right bisectors  $\overleftrightarrow{PQ}$  and  $\overleftrightarrow{RS}$  of side  $\overline{BC}$  and  $\overline{AC}$ , respectively intersecting each other at point O.
3. Taking O as centre and radius equal to  $m\overline{OA}$  or  $m\overline{OB}$  or  $m\overline{OC}$ , we draw a circle passing through the points A, B and C.
4. This is our required circum circle whose radius is measured to be 2.3 cm

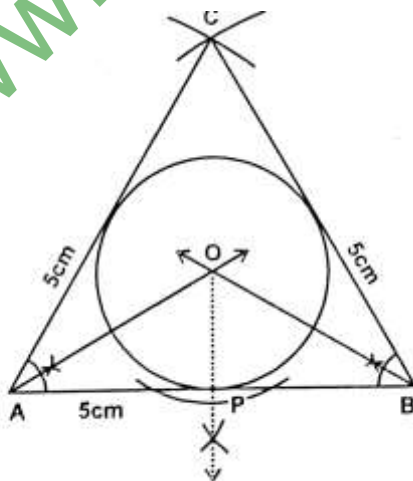
### Question No.5

**Inscribe a circle in an equilateral triangle ABC with each side of length 5cm.**

**Solution:**

**Data:**

$$m\overline{AB} = m\overline{BC} = m\overline{CA} = 5\text{cm}$$

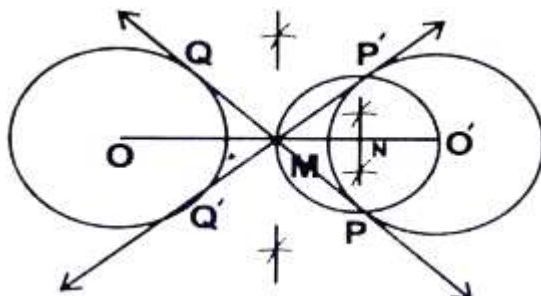


1. We construct equilateral triangles ABC with each side 5cm long.
2. We draw bisectors of  $\angle A$  and  $\angle B$  intersecting each other at point 'O'.
3. From point O, we draw  $\overline{OP} \perp$  to  $\overline{AB}$ .
4. Taking 'O' as Centre and radius equal to  $\overline{OP}$ , we draw a circle, touching three sides of a triangle internally.
5. This is the required in-circle whose radius is measured to be 1.4 cm.

### Exercise 13.3

**Question No.6** Draw two equal circle of each radius 2.4cm. if the distance between their centres is 6cm, then draw their transverse tangents.

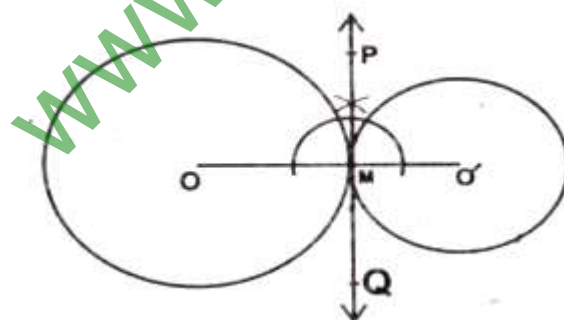
**Solution:**



Steps of construction:

1. Draw  $m\overline{OO'} = 7\text{cm}$
  2. Draw two circles of 2.4 cm radius on O and  $O'$
  3. Find M, the midpoint of  $OO'$
  4. Find N, the mid-point of  $\overline{MO'}$
  5. Draw a circle with center at N and of radius  $\overline{NO'}$  this circles intersects the circle at p and  $P'$
  6. Join  $P'$  with M and produce towards M, it touch the second circle at  $Q'$
  7. Join P with M and produce toward M.  $\overline{PM}$  produce touches the second circle at Q.
- $\overline{PQ}$  and  $\overline{P'Q'}$  are the required tangents.

**Question No.9** Draw two common tangents to two touching circles of radii 2.5 cm and 3.5 cm.

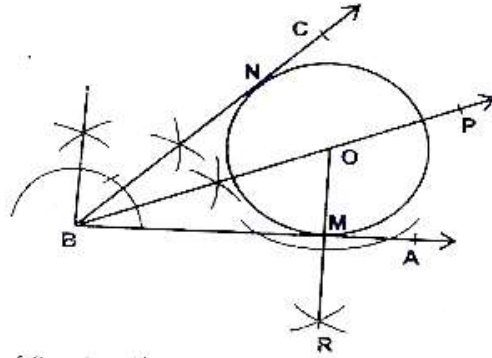


**Steps of construction:**

1. Draw a line segment  $\overline{OO'}$  of measure  $2.5 + 3.5 = 6.0\text{cm}$
  2. Take O as centre and draw a circle with radius  $m\overline{OM} = 3.5\text{cm}$
  3. Take  $O'$  as centre and draw a circle with radius 2.5 cm. these circles touch each other at point M
  4. Draw  $\overline{PQ} \perp \overline{OO'}$
- Result  
 $\overline{PQ}$  is the required common tangents.

**Question No.11** Draw a circle which touches both the arms of angles (i)  $45^\circ$  (ii)  $60^\circ$

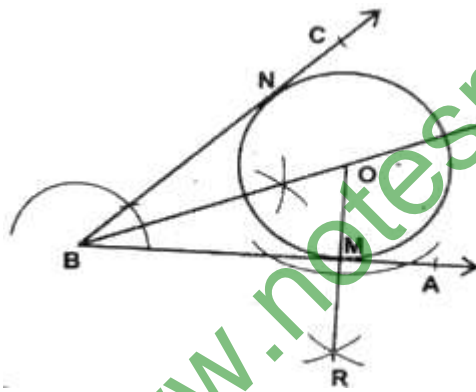
**Solution: (i)**



Steps of construction:

1. Draw an angle  $\angle ABC$  of  $45^\circ$
2. Draw  $\overrightarrow{BP}$  bisector of angle  $\angle ABC$ .
3. Take any point O on  $\overrightarrow{BP}$ .
4. Drop  $\overline{OM} \perp \overline{BA}$ .
5. Take O as centre and draw a circle with radius  $m\overline{OM}$ .  
This circle touch arm  $\overrightarrow{BC}$  at N and  $\overrightarrow{BA}$  touch at M also.

**(ii) solution:**



**Steps of construction:**

1. Draw an angle  $\angle ABC$  of  $60^\circ$
2. Draw  $\overrightarrow{BP}$  bisector of angle  $\angle ABC$ .
3. Take any point O on  $\overrightarrow{BP}$ .
4. Drop  $\overline{OM} \perp \overline{BA}$ .
5. Take O as centre and draw a circle with radius  $m\overline{OM}$ .  
This circle touch arm  $\overrightarrow{BC}$  at N and  $\overrightarrow{BA}$  touch at M also.

## MISCELLANEOUS EXERCISE – 13

### Q-1 Multiple Choice Questions

Three possible answers are given for the following questions. Tick (✓) the correct answer.

1. The circumference of a circle is called:
  - (a) chord
  - (b) segment
  - (c) boundary
  - (d) point
2. A line intersecting a circle is called:
  - (a) tangent
  - (b) secant
  - (c) chord
  - (d) diameter
3. The portion of a circle between two radii and an arc is called:
  - (a) sector
  - (b) segment
  - (c) chord
  - (d) interior
4. Angle inscribed in a semi-circle is:
  - (a)  $\frac{\pi}{2}$
  - (b)  $\frac{\pi}{3}$
  - (c)  $\frac{\pi}{4}$
  - (d)  $\pi$
5. The length of the diameter of a circle is how many times the radius of the circle?
  - (a) 1
  - (b) 2
  - (c) 3
  - (d) 4
6. The tangent and radius of a circle at the point of contact are:
  - (a) parallel
  - (b) not perpendicular
  - (c) perpendicular
  - (d) collinear
7. Circles having three points in common
  - (a) overlapping
  - (b) collinear
  - (c) not coincide
  - (d) non-concentric
8. If two circles touch each other, their centre and point of contact are:
  - (a) coincident
  - (b) non collinear
  - (c) collinear
  - (d) non co planer
9. The measure of the external angle of a regular hexagon is:
  - (a)  $\frac{\pi}{3}$
  - (b)  $\frac{\pi}{4}$
  - (c)  $\frac{\pi}{6}$
  - (d)  $\pi$
10. If the in-centre and circum-centre of a triangle coincide, the triangle is:
  - (a) an isosceles
  - (b) a right triangle
  - (c) an equilateral
  - (d) a scalene triangle
11. The measure of the external angle of a regular octagon is:
  - (a)  $\frac{\pi}{4}$
  - (b)  $\frac{\pi}{6}$
  - (c)  $\frac{\pi}{8}$
  - (d)  $\pi$
12. Tangents drawn at the end points of the diameter of a circle are:
  - (a) parallel
  - (b) perpendicular
  - (c) Intersecting
  - (d) non co planer
13. The lengths of two transverse tangents to a pair of circles are:
  - (a) unequal
  - (b) equal
  - (c) overlapping
  - (d) double of each other
14. How many tangents can be drawn from a point outside the circle?
  - (a) 1
  - (b) 2
  - (c) 3
  - (d) none
15. If the distance between the centre of two circles is equal to the sum of their radii, then the circles will:
  - (a) intersect
  - (b) do not intersect
  - (c) touch each other externally
  - (d) touch each other internally
16. If the two circles touch externally, then the distance between their centre is equal to the:
  - (a) difference of their radii
  - (b) sum of their radii
  - (c) product of their radii
  - (d) division of their radii
17. How many common tangents can be drawn for two touching circles?
  - (a) 1
  - (b) 2
  - (c) 3
  - (d) 4
18. How many common tangents can be drawn for two disjoint circles?
  - (a) 1
  - (b) 2
  - (c) 3
  - (d) 4

19. How many common tangents can be drawn for two intersecting circles?<sup>1</sup>  
 (a) 1 (b) 2  
 (c) 3 (d) 4
20. The word geometry is derived from two \_\_\_\_ words Geo and Metron.  
 (a) English (b) Latin  
 (c) Greek (d) Chinese
21. Euclid was a \_\_\_\_ mathematician.  
 (a) English (b) Latin  
 (c) Greek (d) Chinese
22. The circle passing through vertices of a triangle is called:  
 (a) circum – circle (b) in-circle  
 (c) escribed circle (d) right circle
23. The circle which touches the three sides of a triangle is called:  
 (a) circum – circle (b) in-circle  
 (c) escribed circle (d) right circle
- The circle touching one side of the triangle externally and two produced
24. The circle touching one side of the triangle externally and two produced sides internally is called:  
 (a) circum – circle (b) in-circle  
 (c) escribed circle (d) right circle
25. Tangent is a line touching a circle at:  
 (a) no point (b) one point  
 (c) two points (d) infinite points
26. Two circles of different radii can touch each other at:  
 (a) no point (b) one point  
 (c) two points (d) infinite points
27. Two circles of same radii can touch each other at:  
 (a) no point (b) one point  
 (c) two points (d) infinite points

**(Answer key)**

1.	c	2.	b	3.	a	4.	a	5.	b
6.	c	7.	a	8.	c	9.	a	10.	c
11.	a	12.	a	13.	b	14.	b	15.	c
16.	b	17.	c	18.	d	19.	b	20.	c
21.	c	22.	a	23.	b	24.	c	25.	b
26.	b		26	d					

Compiled By

M. Amin (MSc. Mathematics)

(0336-7965065)