GOVT. Model HIGH SCHOOL 343 GB

Solved Exercise Important Theory +MCQ's

According To Smart Syllabus

MATHEMATICS

(SCIENCE GROUP)

9

Fully Solved Homework +Class Work

ACCELERATED
LEARNING
PROGRAMME
(ALP)

تسریع التعلّم پروگرام

برائے

سیکنڈری کلاسز

پنجاب کریکولم اینڈ ٹیکسٹ بک بورڈ، لاہور

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ACCELERATED
L E A R N I N G PROGRAMME

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Contents

Unit-1

[MATRICS AND DETERMINANTS]

Introduction:

The matrices and determinants are used in the field of Mathematics, Physics, Statistics, Electronics and other branches of science. The Matrices have played a very important role in this age of computer science. The idea of matrices was given by the Arthur Cayley, an English Mathematician of 19th century, who first developed, *Theory of Matrices"* "in 1858.

Matrix:

"An arrangement of different elements in the rows and columns, within square brackets is called Matrix".

e.g $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$.

The real numbers used in the formation of the matrix are called entries or elements of the matrix. The matrices are denoted by the capital letters $A, B, C, D, \ldots, M, N$ *etc*. of the English alphabets.

Rows and Columns of a Matrix:

In a matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, the entries presented in the horizontal way are called rows.

In a matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, the entries presented in the vertical way are called columns.

Order of a Matrix:

Order of Matrix tells us about no of rows and columns.

$\textit{Order of a matrix} = \textit{no. of rows} \times \textit{no. of columns}.$

If a matrix A has m rows and n column then its order is

$$O(A) = m \times n \text{ or } m - by - n.$$

For example

$A = \begin{bmatrix} 1 & 2 & 3 \\ 9 & 7 & 6 \\ 4 & 6 & 8 \end{bmatrix}$ *has order* $3 - by - 3$ *or* 3×3.

Equal matrices:

"Two matrices are said to be equal if

i. **The order of matrix A =The order of Matrix B**
ii. **Their corresponding elements are equal. Thus $A = B$."**

Example:

$$A = \begin{bmatrix} 1 & 2 \\ -5 & 7 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 1+1 \\ -5 & 5+2 \end{bmatrix}$$

are equal matrices.

Exercise 1.1

Home Work:

Question No.1 Find the order of the following matrices.

$$G = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$$

Solution.

$$\textbf{Order of } G = O(G) = 3 - by - 3$$

$$H = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix}$$

Solution.

$$\textbf{Order of } H = O(H) = 2 - by - 3$$

Class Work:

Question.3. Find the values of a, b, c and d which satisfy the matrix equation.

$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$$

Solution.

Given

$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$$

By the definition of equal matrices, we have

$$a + c = 0 \rightarrow (i), a + 2b = -7 \rightarrow (ii),$$
$$c - 1 = 3 \rightarrow (iii), 4d - 6 = 2d \rightarrow (iv)$$

From (iii), we have

$$c = 3 + 1 = 4$$

From (iv), we have

$$4d - 6 = 2d$$
$$4d - 2d = 6$$
$$2d = 6$$
$$d = \frac{6}{2}$$
$$d = 3$$

Using value of $c = 4$ *in* (i), *we have*

$$a + 4 = 0$$
$$a = -4$$

Using value of $a = -4$ *in* (ii), *we have*

$$-4 + 2b = -7$$
$$2b = -7 + 4$$
$$2b = -3$$
$$b = -\frac{3}{2}$$

Hence $a = -4$, $b = -\frac{3}{2}$, $c = 4$ *and* $d = 3$.

Types of Matrices:

Row matrix:

"A matrix having single row is called Row Matrix."

Example:

$M = [1 \quad 2 \quad 3]$ *is a row matrix of order* $1 - by - 3$.

Column matrix:

A matrix having single column is called column Matrix.

Example:

$$M = \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix} \text{ is a column matrix of order } 3-by-1.$$

Rectangular matrix:

A matrix in which number of rows is not equal to number of columns is called rectangular Matrix.

Example:

$$\begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix} \text{ and } \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 6 \end{bmatrix} \text{ are rectangular matrices.}$$

Square matrix:

"A matrix in which number of rows is equal to the number of columns then matrix is called square matrix."

Example:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 9 & 7 & 6 \\ 4 & 6 & 8 \end{bmatrix} \text{ has order } 3-by-3.$$

Null or Zero Matrix:

"A matrix whose each element is zero, is called a null or zero matrix. It is denoted by O."

Examples:

$$[0] \; , \; [0 \quad 0] \; , \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ are null matrices.}$$

Transpose of a Matrix:

"A matrix obtained by changing the rows into columns or columns into rows of a matrix is called transpose of that matrix. If A is a matrix, then its transpose matrix is denoted by A^t."

Example:

$$\text{If } A = \begin{bmatrix} 1 & 2 & 3 \\ 9 & 7 & 6 \\ 4 & 6 & 8 \end{bmatrix} \text{ then } A^t = \begin{bmatrix} 1 & 9 & 4 \\ 2 & 7 & 6 \\ 3 & 6 & 8 \end{bmatrix}$$

$$\text{If } B = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 9 & 4 \end{bmatrix} \text{ then } B^t = \begin{bmatrix} 1 & 1 \\ 3 & 9 \\ 2 & 4 \end{bmatrix}$$

If a matrix B is of order 2-by-3 then order its transpose matrix B^t is 3-by-2.

Negative of a Matrix:

"Let A be a matrix. Then its negative, $-A$ is obtained by changing the signs of all the entries of A."

Example:

$$\text{If } A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} \text{, then } -A = \begin{bmatrix} -1 & 2 \\ -3 & -4 \end{bmatrix}.$$

Symmetric matrix:

"Let A be the square matrix, if $A^t = A$ then A is called symmetric matrix."

Example:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 4 & 0 \end{bmatrix} \text{ is a square matrix then}$$

$$A^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 4 & 0 \end{bmatrix} = A.$$

$$\text{Thus } A \text{ is symmteric matrix.}$$

Skew-symmetric matrix:

"Let A be the square matrix, if $A^t = -A$ then A is called skew symmetric matrix."

Example:

$$A = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} \text{ is a square matrix then}$$

$$A^t = \begin{bmatrix} 0 & -2 & -3 \\ 2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = -A.$$

$$\text{Thus } A \text{ is a skew}-\text{symmteric matrix.}$$

Diagonal matrix:

"A square matrix A is called a diagonal matrix if at least any one of the entries of its diagonal is not zero and non-diagonal entries are zero."

Example:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

are called diagonal matrices.

Scalar Matrix:

"A diagonal matrix having same elements in principle diagonal except 1 or 0 is called scalar matrix."

Example:

$$A = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \text{ are Scalar matrices.}$$

Unit Matrix or Identity Matrix:

A diagonal matrix is called identity matrix if all diagonal entries are 1. It is denoted by I.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ are identity matrices.}$$

Exercise 1.2

Home Work:

Question.1. From the following matrices, identify unit matrices, row matrices, column matrices and null matrices.

$$A=\begin{bmatrix}0&0\\0&0\end{bmatrix},\quad B=\begin{bmatrix}2&3&4\end{bmatrix},\quad C=\begin{bmatrix}4\\0\\6\end{bmatrix}$$

$$D=\begin{bmatrix}1&0\\0&1\end{bmatrix},\quad E=[0],\quad F=\begin{bmatrix}5\\6\\7\end{bmatrix}$$

Solution.

Identity Matrices: D

Row Matrices: B and E.

Column Matrices: C, E and F.

Null Matrices: A and E.

Question.2. From the following matrices, identify (a) Square matrices, (b) Rectangular matrices, (c) Row matrices, (d) Column matrices, (e) Identity Matrices, (f) Null matrices.

(i). $\begin{bmatrix}-8&2&7\\12&0&4\end{bmatrix}$ (ii). $\begin{bmatrix}3\\0\\1\end{bmatrix}$ (iii). $\begin{bmatrix}6&-4\\3&-2\end{bmatrix}$

(iv). $\begin{bmatrix}1&0\\0&1\end{bmatrix}$ (v). $\begin{bmatrix}1&2\\3&4\\5&6\end{bmatrix}$

(vi). $\begin{bmatrix}3&10&-1\end{bmatrix}$ (vii). $\begin{bmatrix}1\\0\\0\end{bmatrix}$ (viii). $\begin{bmatrix}1&2&3\\-1&2&0\\0&0&1\end{bmatrix}$ (ix). $\begin{bmatrix}0&0\\0&0\\0&0\end{bmatrix}$

Solution.

(a). Square Matrices: (iii). (iv). (viii).

(b). Rectangular Matrices: (i). (ii). (v).

(c). Row Matrices: (vi).

(d). (ii). (vii).

(e). (iv).

(f). (ix).

Question.3. From the following matrices, identify Diagonal matrices, Scalar matrices and Unit (identity) matrices.

$$A=\begin{bmatrix}4&0\\0&4\end{bmatrix},\quad B=\begin{bmatrix}2&0\\0&-1\end{bmatrix},\quad C=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$D=\begin{bmatrix}3&0\\0&0\end{bmatrix},\quad E=\begin{bmatrix}5-3&0\\0&1+1\end{bmatrix}$$

Solution.

Diagonal Matrices: A, B, C, D, E.

Scalar Matrices: A, C, E.

Unit Matrices: C.

Question.4. Find the negative of matrices A, B, C, D and E when:

Class Work:

$$A=\begin{bmatrix}1\\0\\-1\end{bmatrix}$$

Solution.

$$-A=\begin{bmatrix}-1\\0\\1\end{bmatrix}$$

Home Work:

$$D=\begin{bmatrix}2&3\\-4&5\end{bmatrix}$$

Solution.

$$-D=\begin{bmatrix}-2&-3\\4&-5\end{bmatrix}$$

Question.5. Find the transpose of the following matrices:

Class Work:

$$B=\begin{bmatrix}5&1&-6\end{bmatrix}$$

$$A^t=\begin{bmatrix}5\\1\\-6\end{bmatrix}$$

Home Work:

$$C=\begin{bmatrix}1&2\\2&-1\\3&0\end{bmatrix}$$

Solution.

$$C^t=\begin{bmatrix}1&2&3\\2&-1&0\end{bmatrix}$$

Home Work:

$$E=\begin{bmatrix}2&3\\-4&5\end{bmatrix}$$

Solution.

$$E^t=\begin{bmatrix}2&-4\\3&5\end{bmatrix}$$

Question.6. Verify that if $A=\begin{bmatrix}1&2\\0&1\end{bmatrix}$, $B=\begin{bmatrix}1&1\\2&0\end{bmatrix}$, then

Class Work (i)

$$(A^t)^t=A$$

Solution.

Given

$$A=\begin{bmatrix}1&2\\0&1\end{bmatrix}$$

$$A^t=\begin{bmatrix}1&0\\2&1\end{bmatrix}$$

$$(A^t)^t=\begin{bmatrix}1&2\\0&1\end{bmatrix}=A$$

$$(A^t)^t=A$$

Hence Proved.

Home Work: (ii)

$$(B^t)^t=B$$

Solution.

Given

$$B=\begin{bmatrix}1&1\\2&0\end{bmatrix}$$

$$B^t = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$
$$(B^t)^t = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} = B$$
$$(B^t)^t = B$$

Hence Proved

Addition of matrices:

"Let A and B be any two matrices of same order then A and B are comfortable for addition." Addition of A and B, Written as $A+B$ is obtained by adding the entries of the matrix A to the corresponding entries of the matrix B."

Example:

$$Let\ A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 6 \end{bmatrix} \quad and\ B = \begin{bmatrix} -2 & 3 & 4 \\ 1 & 2 & 3 \end{bmatrix}\ are\ comfortable\ for\ addition.$$
$$A+B = \begin{bmatrix} 2-2 & 3+3 & 0+4 \\ 1+1 & 0+2 & 6+3 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 4 \\ 2 & 2 & 9 \end{bmatrix}$$

Subtraction of matrices:

Let A and B be any two matrices of same order then A and B are comfortable for Subtraction. Subtraction of A and B, Written as $A-B$ is obtained by subtracting the entries of the matrix A to the corresponding entries of the matrix B.

Example:

$$Let\ A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 6 \end{bmatrix} \quad and\ B = \begin{bmatrix} -2 & 3 & 4 \\ 1 & 2 & 3 \end{bmatrix}$$
are comfortable for Subtraction.
$$A-B = \begin{bmatrix} 2+2 & 3-3 & 0-4 \\ 1-1 & 0-2 & 6-3 \end{bmatrix} = \begin{bmatrix} 4 & 0 & -4 \\ 0 & -2 & 3 \end{bmatrix}$$

Multiplication of a Matrix by a Real Number:

Let A be any matrix and the real number k be a scalar. Then the scalar multiplication of matrix A with k is obtained by multiplying each entry of matrix A with k. It is denoted by kA.

Example:

$$Let\ A = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 6 \end{bmatrix} \quad then\ kA = \begin{bmatrix} 2k & 3k & 0 \\ 1k & 0 & 6k \end{bmatrix}$$

Commutative Law for Addition.

If A and B are two matrices of the same order, Then $A+B = B+A$ *is* called commutative law under addition.

$$A+B = B+A$$

Associative Law for Addition:

If A, B and C are three matrices of the same order, Then $(A+B)+C = A+(B+C)$ *is* Called Associative law under addition.

$$(A+B)+C = A+(B+C)$$

Additive Identity of a Matrix:

If A and B are two matrices of same order and $A+B = A = B+A$

Then matrix B is called additive identity of matrix A. For any matrix A and zero matrix of same order, O is called additive identity of A as

$$A+O = A = O+A.$$

Additive Inverse of a Matrix:

If A and B are two matrices of same order and $A+B = O = B+A$

Then matrix B is called additive inverse of matrix A. "Additive inverse of any matrix A is obtained by changing to negative of the symbols (entries) of each non zero entry of A."

Exercise 1.3

Question.1. which of the following matrices are comfortable for addition?

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 1 & -2 \end{bmatrix}$$
$$D = \begin{bmatrix} 2+1 \\ 3 \end{bmatrix}, \quad E = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix},$$
$$F = \begin{bmatrix} 3 & 2 \\ 1+1 & -4 \\ 3+2 & 2+1 \end{bmatrix}$$

Solution. Class Work+ Home Work

Since order of $A\ and\ E\ are$ same so they are comfortable for addition.

Also order of $B\ and\ D\ are$ same so they are comfortable for addition.

Also order of $C\ and\ F\ are$ same so they are comfortable for addition.

Question.2. Find the additive inverse of the following matrices:

Class Work:

$$B = \begin{bmatrix} 1 & 0 & -1 \\ 2 & -1 & 3 \\ 3 & -2 & 1 \end{bmatrix}$$

Solution.

$$Additive\ inverse\ of\ B = -B = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 1 & -3 \\ -3 & 2 & -1 \end{bmatrix}$$

Home Work:

$$C = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

Solution.

$$Additive\ inverse\ of\ C = -C = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

Home Work:

$$F = \begin{bmatrix} \sqrt{3} & 1 \\ -1 & \sqrt{2} \end{bmatrix}$$

Solution.

$$Additive\ inverse\ of\ F = -F = \begin{bmatrix} -\sqrt{3} & -1 \\ 1 & -\sqrt{2} \end{bmatrix}$$

Question.3.If $A = \begin{bmatrix} -1 & 2 \\ -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, C = [1 \quad -1 \quad 2], D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$

Then find,

Class Work: (ii)

$$B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

Solution.

$$B + \begin{bmatrix}-2\\3\end{bmatrix} = \begin{bmatrix}1\\-1\end{bmatrix} + \begin{bmatrix}-2\\3\end{bmatrix}$$
$$= \begin{bmatrix}1-2\\-1+3\end{bmatrix}$$
$$= \begin{bmatrix}-1\\2\end{bmatrix}$$

Answer.

Home Work: (iv)

$$\boldsymbol{D} + \begin{bmatrix}\mathbf{0} & \mathbf{1} & \mathbf{0}\\ \mathbf{2} & \mathbf{0} & \mathbf{1}\end{bmatrix}$$

Solution.

$$D + \begin{bmatrix}0 & 1 & 0\\2 & 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 2 & 3\\-1 & 0 & 2\end{bmatrix} + \begin{bmatrix}0 & 1 & 0\\2 & 0 & 1\end{bmatrix}$$
$$= \begin{bmatrix}1+0 & 2+1 & 3+0\\-1+2 & 0+0 & 2+1\end{bmatrix}$$
$$= \begin{bmatrix}1 & 3 & 3\\1 & 0 & 3\end{bmatrix}$$

Answer.

Home Work: (v)

$$(-\mathbf{1})\boldsymbol{B}$$

Solution.

$$(-1)B = (-1)\begin{bmatrix}1\\-1\end{bmatrix}$$
$$= \begin{bmatrix}-1\\1\end{bmatrix}$$

Question.4. perform the indicated operations and simplify the following

Class Work: (ii)

$$\begin{bmatrix}\mathbf{1} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{bmatrix} + \left(\begin{bmatrix}\mathbf{0} & \mathbf{2}\\ \mathbf{3} & \mathbf{0}\end{bmatrix} - \begin{bmatrix}\mathbf{1} & \mathbf{1}\\ \mathbf{1} & \mathbf{0}\end{bmatrix}\right)$$

Solution.

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} + \left(\begin{bmatrix}0 & 2\\3 & 0\end{bmatrix} - \begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}\right)$$
$$= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} + \left(\begin{bmatrix}0-1 & 2-1\\3-1 & 0-1\end{bmatrix}\right)$$
$$= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} + \begin{bmatrix}-1 & 1\\2 & -1\end{bmatrix}$$
$$= \begin{bmatrix}1-1 & 0+1\\0+2 & 1-1\end{bmatrix}$$
$$= \begin{bmatrix}0 & 1\\2 & 0\end{bmatrix}$$

Answer.

Home Work: (iv)

$$\begin{bmatrix}\mathbf{1} & \mathbf{2} & \mathbf{3}\\ -\mathbf{1} & -\mathbf{1} & -\mathbf{1}\\ \mathbf{0} & \mathbf{1} & \mathbf{2}\end{bmatrix} + \begin{bmatrix}\mathbf{1} & \mathbf{1} & \mathbf{1}\\ \mathbf{2} & \mathbf{2} & \mathbf{2}\\ \mathbf{3} & \mathbf{3} & \mathbf{3}\end{bmatrix}$$

Solution.

$$\begin{bmatrix}1 & 2 & 3\\-1 & -1 & -1\\0 & 1 & 2\end{bmatrix} + \begin{bmatrix}1 & 1 & 1\\2 & 2 & 2\\3 & 3 & 3\end{bmatrix}$$
$$= \begin{bmatrix}1+1 & 2+1 & 3+1\\-1+2 & -1+2 & -1+2\\0+3 & 1+3 & 2+3\end{bmatrix}$$
$$= \begin{bmatrix}2 & 3 & 4\\1 & 1 & 1\\3 & 4 & 5\end{bmatrix}$$

Answer.

Question5.For the matrices

$$\boldsymbol{A} = \begin{bmatrix}\mathbf{1} & \mathbf{2} & \mathbf{3}\\ \mathbf{2} & \mathbf{3} & \mathbf{1}\\ \mathbf{1} & -\mathbf{1} & \mathbf{0}\end{bmatrix}, \boldsymbol{B} = \begin{bmatrix}\mathbf{1} & -\mathbf{1} & \mathbf{1}\\ \mathbf{2} & -\mathbf{2} & \mathbf{2}\\ \mathbf{3} & \mathbf{1} & \mathbf{3}\end{bmatrix},$$

$\boldsymbol{C} = \begin{bmatrix}-\mathbf{1} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & -\mathbf{2} & \mathbf{3}\\ \mathbf{1} & \mathbf{1} & \mathbf{2}\end{bmatrix}$, **Verify the following rules:**

Home Work: (vi)

$$\mathbf{2}\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{A} + (\boldsymbol{A} + \boldsymbol{B})$$

Solution.

$L.H.S = 2A + B$

$$L.H.S = 2\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$
$$L.H.S = \begin{bmatrix}2 & 4 & 6\\4 & 6 & 2\\2 & -2 & 0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}$$
$$L.H.S = \begin{bmatrix}2+1 & 4-1 & 6+1\\4+2 & 6-2 & 2+2\\2+3 & -2+1 & 0+3\end{bmatrix}$$
$$L.H.S = \begin{bmatrix}3 & 3 & 7\\6 & 4 & 4\\5 & -1 & 3\end{bmatrix}$$

$R.H.S = A + (A + B)$

$$L.H.S = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \left(\begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right)$$
$$R.H.S = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \left(\begin{bmatrix}1+1 & 2-1 & 3+1\\2+2 & 3-2 & 1+2\\1!+3 & -1+1 & 0+3\end{bmatrix}\right)$$
$$R.H.S = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix} + \begin{bmatrix}2 & 1 & 4\\4 & 1 & 3\\4 & 0 & 3\end{bmatrix}$$
$$R.H.S = \begin{bmatrix}1+2 & 2+1 & 3+4\\2+4 & 3+1 & 1+3\\1+4 & -1+0 & 0+3\end{bmatrix}$$
$$R.H.S = \begin{bmatrix}3 & 3 & 7\\6 & 4 & 4\\5 & -1 & 3\end{bmatrix}$$

Hence Proved $L.H.S = R.H.S.$

Home Work: (vii)

$$(\boldsymbol{C} - \boldsymbol{B}) - \boldsymbol{A} = (\boldsymbol{C} - \boldsymbol{A}) - \boldsymbol{B}$$

Solution.

$L.H.S = (C - B) - A$

$$L.H.S = \left(\begin{bmatrix}-1 & 0 & 0\\0 & -2 & 3\\1 & 1 & 2\end{bmatrix} - \begin{bmatrix}1 & -1 & 1\\2 & -2 & 2\\3 & 1 & 3\end{bmatrix}\right) - \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$
$$L.H.S = \left(\begin{bmatrix}-1-1 & 0+1 & 0-1\\0-2 & -2+2 & 3-2\\1-3 & 1-1 & 2-3\end{bmatrix}\right) - \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$
$$L.H.S = \begin{bmatrix}-2 & 1 & -1\\-2 & 0 & 1\\-2 & 0 & -1\end{bmatrix} - \begin{bmatrix}1 & 2 & 3\\2 & 3 & 1\\1 & -1 & 0\end{bmatrix}$$
$$L.H.S = \begin{bmatrix}-2-1 & 1-2 & -1-3\\-2-2 & 0-3 & 1-1\\-2-1 & 0+1 & -1-0\end{bmatrix}$$
$$L.H.S = \begin{bmatrix}-3 & -1 & -4\\-4 & -3 & 0\\-3 & 1 & -1\end{bmatrix}$$

$R.H.S = (C - A) - B$

$$R.H.S=\left(\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}-\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}\right)-\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}$$

$$R.H.S=\left(\begin{bmatrix}-1-1&0+1&0-1\\0-2&-2+2&3-2\\1-3&1-1&2-3\end{bmatrix}\right)-\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}$$

$$R.H.S=\begin{bmatrix}-2&1&-1\\-2&0&1\\-2&0&-1\end{bmatrix}-\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}$$

$$R.H.S=\begin{bmatrix}-2-1&1-2&-1-3\\-2-2&0-3&1-1\\-2-1&0+1&-1-1\end{bmatrix}$$

$$R.H.S=\begin{bmatrix}-3&-1&-4\\-4&-3&0\\-3&1&-1\end{bmatrix}$$

Hence Proved $L.H.S=R.H.S.$

Home Work: (viii)

$$(\boldsymbol{A}+\boldsymbol{B})+\boldsymbol{C}=\boldsymbol{A}+(\boldsymbol{B}+\boldsymbol{C})$$

Solution.

$$L.H.S=(A+B)+C$$

$$L.H.S=\left(\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}\right)+\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}$$

$$L.H.S=\begin{bmatrix}1+1&2-1&3+1\\2+2&3-2&1+2\\1+3&-1+1&0+3\end{bmatrix}+\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}$$

$$L.H.S=\begin{bmatrix}2&1&4\\4&1&3\\4&0&3\end{bmatrix}+\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}$$

$$L.H.S=\begin{bmatrix}2-1&1+0&4+0\\4+0&1-2&3+6\\4+1&0+1&3+2\end{bmatrix}$$

$$L.H.S=\begin{bmatrix}1&1&4\\4&-1&9\\5&1&5\end{bmatrix}$$

$$R.H.S=A+(B+C)$$

$$R.H.S=\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\left(\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}+\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}\right)$$

$$R.H.S=\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\left(\begin{bmatrix}1-1&-1+0&1+0\\2+0&-2-2&2+3\\3+1&1+1&3+2\end{bmatrix}\right)$$

$$R.H.S=\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}0&-1&1\\2&-4&5\\4&2&5\end{bmatrix}$$

$$R.H.S=\begin{bmatrix}1+0&2-1&3+1\\2+2&3-4&1+5\\1+4&-1+2&0+5\end{bmatrix}$$

$$R.H.S=\begin{bmatrix}1&1&4\\4&-1&9\\5&1&5\end{bmatrix}$$

Hence Proved $L.H.S=R.H.S.$

Home Work: (ix)

$$\boldsymbol{A}+(\boldsymbol{B}-\boldsymbol{C})=(\boldsymbol{A}-\boldsymbol{C})+\boldsymbol{B}$$

Solution.

$$L.H.S=A+(B-C)$$

$$L.H.S=\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\left(\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}-\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}\right)$$

$$L.H.S=\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\left(\begin{bmatrix}1+1&-1-0&1-0\\2-0&-2+2&2-3\\3-1&1-1&3-2\end{bmatrix}\right)$$

$$L.H.S=\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}2&-1&1\\2&0&-1\\2&0&1\end{bmatrix}$$

$$L.H.S=\begin{bmatrix}1+2&2-1&3+1\\2+2&3+0&1-1\\1+2&-1+0&0+1\end{bmatrix}$$

$$L.H.S=\begin{bmatrix}3&1&4\\4&3&0\\3&-1&1\end{bmatrix}$$

$$R.H.S=(A-C)+B$$

$$R.H.S=\left(\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}-\begin{bmatrix}-1&0&0\\0&-2&3\\1&1&2\end{bmatrix}\right)+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$R.H.S=\left(\begin{bmatrix}1+1&2-0&3-0\\2-0&3+2&1-3\\1-1&-1-1&0-2\end{bmatrix}\right)+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$R.H.S=\begin{bmatrix}2&2&3\\2&5&-2\\0&-2&-2\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$R.H.S=\begin{bmatrix}2+1&2-1&3+1\\2+2&5-2&-2+2\\0+3&-2+1&-2+3\end{bmatrix}$$

$$R.H.S=\begin{bmatrix}3&1&4\\4&3&0\\3&-1&1\end{bmatrix}$$

Hence Proved. $\boldsymbol{L.H.S=R.H.S.}$

Home Work: (x)

$$\boldsymbol{2A+2B=2(A+B)}$$

Solution.

$$L.H.S=2A+2B$$

$$L.H.S=2\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+2\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}$$

$$L.H.S=\begin{bmatrix}2&4&6\\4&6&2\\2&-2&0\end{bmatrix}+\begin{bmatrix}2&-2&2\\4&-4&4\\6&2&6\end{bmatrix}$$

$$L.H.S=\begin{bmatrix}2+2&4-2&6+2\\4+4&6-4&2+4\\2+6&-2+2&0+6\end{bmatrix}$$

$$L.H.S=\begin{bmatrix}4&2&8\\8&4&6\\7&0&6\end{bmatrix}$$

$$R.H.S=2(A+B)$$

$$L.H.S=2\left(\begin{bmatrix}1&2&3\\2&3&1\\1&-1&0\end{bmatrix}+\begin{bmatrix}1&-1&1\\2&-2&2\\3&1&3\end{bmatrix}\right)$$

$$R.H.S = 2\left(\begin{bmatrix} 1+1 & 2-1 & 3+1 \\ 2+2 & 3-2 & 1+2 \\ 1!+3 & -1+1 & 0+3 \end{bmatrix}\right)$$

$$R.H.S = 2\begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

$$R.H.S = \begin{bmatrix} 4 & 2 & 8 \\ 8 & 4 & 6 \\ 7 & 0 & 6 \end{bmatrix}$$

Hence Proved $L.H.S = R.H.S.$

Question.6. If $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ **and** $B = \begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$, *find*

Class Work: (i)

$$3A - 2B$$

Solution.

$$3A - 2B = 3\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} - 2\begin{bmatrix} 0 & 7 \\ -3 & 8 \end{bmatrix}$$

$$3A - 2B = \begin{bmatrix} 3 & -6 \\ 9 & 12 \end{bmatrix} - \begin{bmatrix} 0 & 14 \\ -6 & 16 \end{bmatrix}$$

$$3A - 2B = \begin{bmatrix} 3-0 & -6-14 \\ 9+6 & 12-16 \end{bmatrix}$$

$$3A - 2B = \begin{bmatrix} 3 & -20 \\ 15 & -4 \end{bmatrix}$$

Answer.

Multiplication of Matrices:

Two matrices A and B are conformable for multiplication if

No of col of A =No. Of Rows of B

Exercise 1.4

Question No.1 Which of the following product matrices is conformable for multiplication?.

Class Work: (i)

$$\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

Solution:

Conformable for multiplication because

No of col of 1st Matrix= 2 =No. Of Rows of 2nd Matrix

Home Work: (ii)

$$\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

Solution:

Conformable for multiplication because

No of col of 1st Matrix= 2 =No. Of Rows of 2nd Matrix

Home Work: (iii)

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$$

Solution:

Not conformable for multiplication because

No of col of 1st Matrix= $1 \neq 2$ =No. Of Rows of 2nd Matrix

Home Work: (iv)

$$\begin{bmatrix} 1 & 2 \\ 0 & -1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$$

Solution:

Conformable for multiplication because

No of col of 1st Matrix= 2 =No. Of Rows of 2nd Matrix

Class Work: (v)

$$\begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -2 & 3 \end{bmatrix}$$

Solution:

Conformable for multiplication because

No of col of 1st Matrix= 3 =No. Of Rows of 2nd Matrix

Question No.4 Multiply the following matrices.

Class Work: (a)

$$\begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$$

Solution: $\begin{bmatrix} 2 & 3 \\ 1 & 1 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$

$$= \begin{bmatrix} (2)(2)+(3)(3) & (2)(-1)+(3)(0) \\ (1)(2)+(1)(3) & (1)(-1)+(1)(0) \\ (0)(2)+(-2)(3) & (0)(-1)+(-2)(0) \end{bmatrix}$$

$$= \begin{bmatrix} 4+9 & -2+0 \\ 2+3 & -1+0 \\ 0-6 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & -2 \\ 5 & -1 \\ -6 & 0 \end{bmatrix}$$

Home Work: (d)

$$\begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} 2 & -\frac{5}{2} \\ -4 & 4 \end{bmatrix}$$

Solution: $\begin{bmatrix} 8 & 5 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} 2 & -\frac{5}{2} \\ -4 & 4 \end{bmatrix}$

$$= \begin{bmatrix} (8)(2)+(5)(-4) & (8)\left(-\frac{5}{2}\right)+(5)(4) \\ (6)(2)+(4)(-4) & (6)\left(-\frac{5}{2}\right)+(4)(4) \end{bmatrix}$$

$$= \begin{bmatrix} 16-20 & -20+20 \\ 12-16 & -15+16 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 0 \\ -4 & 1 \end{bmatrix}$$

Home Work: (e)

$$\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Solution: $\begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} (-1)(0)+(2)(0) & (-1)(0)+(2)(0) \\ (1)(0)+(1)(0) & (1)(0)+(1)(0) \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Question No.5 Let $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$ **and** $C = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, **Verify that**

Class Work: (ii)

$$A(BC) = (AB)C$$

Sol:

$L.H.S = A(BC)$

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\right)$$
$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\left(\begin{bmatrix} (1)(2)+(2)(1) & (1)(1)+(2)(3) \\ (-3)(2)+(-5)(1) & (-3)(1)+(-5)(3) \end{bmatrix}\right)$$
$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 2+2 & 1+6 \\ -6-5 & -3-15 \end{bmatrix}$$
$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 4 & 7 \\ -11 & -18 \end{bmatrix}$$
$$= \begin{bmatrix} (-1)(4)+(3)(-11) & (-1)(7)+(3)(-18) \\ (2)(4)+(0)(-11) & (2)(7)+(0)(-18) \end{bmatrix}$$
$$= \begin{bmatrix} -4-33 & -7-54 \\ 8+0 & 14+0 \end{bmatrix}$$
$$= \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix} \rightarrow (1)$$

$R.H.S = (AB)C$

$$= \left(\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\right)\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$
$$= \left(\begin{bmatrix} (-1)(1)+(3)(-3) & (-1)(2)+(3)(-5) \\ (2)(1)+(0)(-3) & (2)(2)+(0)(-5) \end{bmatrix}\right)\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} (-10)(2)+(-17)(1) & (-10)(1)+(-17)(3) \\ (2)(2)+(4)(1) & (2)(1)+(4)(3) \end{bmatrix}$$
$$= \begin{bmatrix} -20-17 & -10-51 \\ 4+4 & 2+12 \end{bmatrix}$$
$$= \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix} \ldots(2)$$

From (1) and (2), we have

$A(BC) = (AB)C$

Home Work: (iv)

$$A(B-C) = AB - AC$$

Solution

$L.H.S = A(BC)$

$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\right)$$
$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1-2 & 2-1 \\ -3-1 & -5-3 \end{bmatrix}$$
$$= \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ -4 & -8 \end{bmatrix}$$
$$= \begin{bmatrix} (-1)(-1)+(3)(-4) & (-1)(1)+(3)(-8) \\ (2)(-1)+(0)(-4) & (2)(1)+(0)(-8) \end{bmatrix}$$
$$= \begin{bmatrix} 1-12 & -1-24 \\ -2+0 & 2+0 \end{bmatrix}$$
$$= \begin{bmatrix} -11 & -25 \\ -2 & 2 \end{bmatrix} \ldots(1)$$

$R.H.S = AB - BC$

$$= \left(\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\right) - \left(\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\right)$$
$$= \begin{bmatrix} (-1)(1)+(3)(-3) & (-1)(2)+(3)(-5) \\ (2)(1)+(0)(-3) & (2)(2)+(0)(-5) \end{bmatrix}$$
$$- \begin{bmatrix} (-1)(2)+(3)(1) & (-1)(1)+(3)(3) \\ (2)(2)+(0)(1) & (2)(1)+(0)(3) \end{bmatrix}$$
$$= \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix} - \begin{bmatrix} -2+3 & -1+9 \\ 4+0 & 2+0 \end{bmatrix}$$
$$= \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 8 \\ 4 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} -10-1 & -17-8 \\ 2-4 & 4-2 \end{bmatrix}$$
$$= \begin{bmatrix} -11 & -25 \\ -2 & 2 \end{bmatrix} \rightarrow (2)$$

From (1) and (2), we have

$A(B-C) = AB - AC$

Question No.6 For the matrices $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$ **and** $C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$, **verify that**

Home Work: (ii)

$$(BC)^t = C^tB^t$$

Solution: $L.H.S = (BC)^t$

First we find BC

$$BC = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$$
$$= \begin{bmatrix} (1)(-2)+(2)(3) & (1)(6)+(2)(-9) \\ (-3)(-2)+(-5)(3) & (-3)(6)+(-5)(-9) \end{bmatrix}$$
$$= \begin{bmatrix} -2+6 & 6-18 \\ 6-15 & -18+45 \end{bmatrix}$$
$$= \begin{bmatrix} 4 & -12 \\ -9 & 27 \end{bmatrix} \rightarrow (2)$$

Taking transpose on both side

$$(AB)^t = \begin{bmatrix} 4 & -12 \\ -9 & 27 \end{bmatrix}^t$$
$$(AB)^t = \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix} \rightarrow (1)$$

$: R.H.S = C^tB^t$

$$= \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}^t \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}^t$$
$$= \begin{bmatrix} -2 & 3 \\ 6 & -9 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$$
$$= \begin{bmatrix} (-2)(1)+(3)(2) & (-2)(-3)+(3)(-5) \\ (6)(1)+(-9)(2) & (6)(-3)+(-9)(-5) \end{bmatrix}$$
$$= \begin{bmatrix} -2+6 & 6-15 \\ 6-18 & -18+45 \end{bmatrix}$$
$$C^tB^t = \begin{bmatrix} 4 & -9 \\ -12 & 27 \end{bmatrix} \rightarrow (2)$$

From (1) and (2), we have

$$(BC)^t = C^tB^t$$

Determinant of 2×2 matrix:

Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be 2×2 square matrix, the determinant of A is denoted by $|A|$ or detA

And given as

$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$
$= (a)(d) - (b)(c)$
$= ad - bc$

For example, $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

$|A| = \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix}$
$= (-1)(0) - (1)(2)$
$= 0 - 2 = -2$

Singular and Non-singular matrices:

Singular matrix:

A square matrix A is called Singular matrix if its determinant is zero i.e. $|A| = 0$

For example, $A = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$

$|A| = \begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix}$
$= (3)(2) - (3)(2)$
$= 6 - 6 = 0$

Non-Singular matrix:

A square matrix A is called Non-Singular matrix if its determinant is not zero i.e. $|A| \neq 0$

For example, $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

$|A| = \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix}$
$= (-1)(0) - (1)(2)$
$= 0 - 2 = -2 \neq 0$

Adjoint of Matrix A:

"Adjoint of a square matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is obtained by interchanging the diagonal entries and changing the sign of other entries."

For example, $A = \begin{bmatrix} -1 & 4 \\ 2 & 0 \end{bmatrix}$

Exercise 1.5

Question No.1 Find the determinant of the following matrices.

Class Work: (ii)

$$B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$$

Sol:

$|B| = \begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix}$
$= (1)(-2) - (3)(2)$
$= -2 - 6 = -8$

Question No.2

Find which of the following matrices are singular or non-singular?

Class Work: (i)

$$A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$$

Solution:

$|A| = \begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix}$
$= (3)(4) - (6)(2)$
$= 12 - 12 = 0$

Hence, matrix A is singular matrix.

Home Work: (ii)

$$B = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$$

Solution:

$|B| = \begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix}$
$= (4)(2) - (1)(3)$
$= 8 - 3 = 5$

Which is not zero and hence, matrix A is Non-singular matrix.

Home Work: (iii)

$$C = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$$

Solution:

$|C| = \begin{vmatrix} 7 & -9 \\ 3 & 5 \end{vmatrix}$
$= (7)(5) - (-9)(3)$
$= 35 + 27 = 62$

Which is not zero and hence, matrix A is Non-singular matrix.

Home Work: (iv)

$$D = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$$

Solution:

$|D| = \begin{vmatrix} 5 & -10 \\ -2 & 4 \end{vmatrix}$
$= (5)(4) - (-10)(-2)$
$= 20 - 20 = 0$

Hence, matrix A is singular matrix.

Question No.3

Find the multiplicative inverse (if exists) of each:

Class Work: (i)

$$A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$$

Sol: First we find the determinant of A as

$|A| = \begin{vmatrix} -1 & 3 \\ 2 & 0 \end{vmatrix}$
$= (-1)(0) - (3)(2)$
$= 0 - 6 = -6$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA = \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix}$

As

$A^{-1} = \dfrac{1}{|A|} AdjA$

Putting values

$$A^{-1}=\frac{1}{-6}\begin{bmatrix}0 & -3\\ -2 & -1\end{bmatrix}=\begin{bmatrix}\frac{0}{-6} & \frac{-3}{-6}\\ \frac{-2}{-6} & \frac{-1}{-6}\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}0 & \frac{1}{2}\\ \frac{1}{3} & \frac{1}{6}\end{bmatrix}$$

Home Work: (iii)

$$\boldsymbol{C}=\begin{bmatrix}\mathbf{-2} & \mathbf{6}\\ \mathbf{3} & \mathbf{-9}\end{bmatrix}$$

Sol: First we find the determinant of C as

$|C|=\begin{vmatrix}-2 & 6\\ 3 & -9\end{vmatrix}$

$=(-2)(-9)-(3)(6)$

$=18-18=0$

Which is zero and hence, matrix C is singular matrix and C^{-1} does not exist.

Home Work: (iv)

$$\boldsymbol{D}=\begin{bmatrix}\frac{1}{2} & \frac{3}{4}\\ \mathbf{1} & \mathbf{2}\end{bmatrix}$$

Sol: First we find the determinant of D as

$|D|=\begin{vmatrix}\frac{1}{2} & \frac{3}{4}\\ 1 & 2\end{vmatrix}$

$=\left(\frac{1}{2}\right)(2)-\left(\frac{3}{4}\right)(1)=1-\frac{3}{4}$

$=\frac{4-3}{4}=\frac{1}{4}$

Which is not zero and hence, matrix D is Non-singular matrix and D^{-1} exist.

Now, $AdjD=\begin{bmatrix}2 & -\frac{3}{4}\\ -1 & \frac{1}{2}\end{bmatrix}$

As

$D^{-1}=\frac{1}{|D|}AdjD$

Putting values

$D^{-1}=\frac{1}{\frac{1}{4}}\begin{bmatrix}2 & -\frac{3}{4}\\ -1 & \frac{1}{2}\end{bmatrix}=\frac{4}{1}\begin{bmatrix}2 & -\frac{3}{4}\\ -1 & \frac{1}{2}\end{bmatrix}$

$D^{-1}=\begin{bmatrix}8 & -3\\ -4 & 2\end{bmatrix}$

Question No.6 If $\boldsymbol{A}=\begin{bmatrix}\mathbf{4} & \mathbf{0}\\ \mathbf{-1} & \mathbf{2}\end{bmatrix}$, $\boldsymbol{B}=\begin{bmatrix}\mathbf{-4} & \mathbf{-2}\\ \mathbf{1} & \mathbf{-1}\end{bmatrix}$ and $\boldsymbol{D}=\begin{bmatrix}\mathbf{3} & \mathbf{1}\\ \mathbf{-2} & \mathbf{2}\end{bmatrix}$, then verify that

Class Work: (i)

$(\boldsymbol{AB})^{-1}=\boldsymbol{B}^{-1}\boldsymbol{A}^{-1}$

Solution: L.H.S=$(\boldsymbol{AB})^{-1}$

First we find

$AB=\begin{bmatrix}4 & 0\\ -1 & 2\end{bmatrix}\begin{bmatrix}-4 & -2\\ 1 & -1\end{bmatrix}$

$=\begin{bmatrix}(4)(-4)+(0)(1) & (4)(-2)+(0)(-1)\\ (-1)(-4)+(2)(1) & (-1)(-2)+(2)(-1)\end{bmatrix}$

$=\begin{bmatrix}-16+0 & -8+0\\ 4+2 & 2-2\end{bmatrix}$

$=\begin{bmatrix}-16 & -8\\ 6 & 0\end{bmatrix}$

Now, we find the its determinant

$|AB|=\begin{vmatrix}-16 & -8\\ 6 & 0\end{vmatrix}$

$=(-16)(0)-(-8)(6)$

$=0-(-48)=48$

Which is not zero and hence, matrix AB is Non-singular matrix and $(AB)^{-1}$ exist.

Now, $AdjAB=\begin{bmatrix}0 & 8\\ -6 & -16\end{bmatrix}$

As

$(AB)^{-1}=\frac{1}{|AB|}AdjAB$

Putting values

$L.H.S=(AB)^{-1}=\frac{1}{48}\begin{bmatrix}0 & 8\\ -6 & -16\end{bmatrix}$...(1)

$R.H.S=B^{-1}A^{-1}$

First, we find B^{-1} and A^{-1}

$|A|=\begin{vmatrix}4 & 0\\ -1 & 2\end{vmatrix}$

$=(4)(2)-(0)(-1)$

$=8-0=8$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA=\begin{bmatrix}2 & 0\\ 1 & 4\end{bmatrix}$

$A^{-1}=\frac{1}{|A|}AdjA$

Putting values

$A^{-1}=\frac{1}{8}\begin{bmatrix}2 & 0\\ 1 & 4\end{bmatrix}$

Also, $|B|=\begin{vmatrix}-4 & -2\\ 1 & -1\end{vmatrix}$

$=(-4)(-1)-(-2)(1)$

$=4+2=6$

Which is not zero and hence, matrix B is Non-singular matrix and B^{-1} exist.

Now, $AdjB=\begin{bmatrix}-1 & 2\\ -`1 & -4\end{bmatrix}$

$B^{-1}=\frac{1}{|B|}AdjB$

Putting values

$B^{-1}=\frac{1}{6}\begin{bmatrix}-1 & 2\\ -`1 & -4\end{bmatrix}$

$R.H.S=B^{-1}A^{-1}$

$=\frac{1}{6}\begin{bmatrix}-1 & 2\\ -`1 & -4\end{bmatrix}\frac{1}{8}\begin{bmatrix}2 & 0\\ 1 & 4\end{bmatrix}=\frac{1}{8\times 6}\begin{bmatrix}-1 & 2\\ -`1 & -4\end{bmatrix}\begin{bmatrix}2 & 0\\ 1 & 4\end{bmatrix}$

$= \frac{1}{48}\begin{bmatrix} (-1)(2)+(2)(1) & (-1)(0)+(2)(4) \\ (-1)(2)+(-4)(1) & (-1)(0)+(-4)(4) \end{bmatrix}$

$= \frac{1}{48}\begin{bmatrix} -2+2 & 0+8 \\ -2-4 & 0-16 \end{bmatrix}$

$= \frac{1}{48}\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$...(2)

From (1) and (2), we have

$\boldsymbol{(AB)^{-1} = B^{-1}\,A^{-1}}$

Home Work: (ii)

$\boldsymbol{(DA)^{-1} = A^{-1}D^{-1}}$

Solution: L.H.S=$(DA)^{-1}$

First we find

$DA = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} (3)(4)+(1)(-1) & (1)(0)+(1)(2) \\ (-2)(4)+(2)(-1) & (-2)(0)+(2)(2) \end{bmatrix}$

$= \begin{bmatrix} 12-1 & 0+2 \\ -8-2 & 0-4 \end{bmatrix}$

$= \begin{bmatrix} 11 & 2 \\ -10 & 4 \end{bmatrix}$

Now, we find the its determinant

$|DA| = \begin{vmatrix} 11 & 2 \\ -10 & 4 \end{vmatrix}$

$= (11)(4) - (2)(-10)$

$= 44 + 20 = 64$

Which is not zero and hence, matrix DA is Non-singular matrix and $(DA)^{-1}$ exist.

Now, $AdjDA = \begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}$

$(DA)^{-1} = \frac{1}{|DA|}AdjDA$

Putting values

$L.H.S = (DA)^{-1} = \frac{1}{64}\begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}$...(1)

$R.H.S = A^{-1}D^{-1}$

First, we find D^{-1} and A^{-1}

$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix}$

$= (4)(2) - (0)(-1)$

$= 8 - 0 = 8$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}AdjA$

Putting values

$A^{-1} = \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$

Also, $|D| = \begin{vmatrix} 3 & 1 \\ -2 & 2 \end{vmatrix}$

$= (3)(2) - (1)(-2)$

$= 6 + 2 = 8$

Which is not zero and hence, matrix D is Non-singular matrix and D^{-1} exist.

Now, $AdjD = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$

$D^{-1} = \frac{1}{|D|}AdjD$

Putting values

$D^{-1} = \frac{1}{8}\begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$

$R.H.S = A^{-1}D^{-1}$

$= \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}\frac{1}{8}\begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix} = \frac{1}{8\times 8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$

$= \frac{1}{64}\begin{bmatrix} (2)(2)+(0)(-2) & (2)(-1)+(0)(3) \\ (1)(2)+(4)(-2) & (1)(-1)+(4)(3) \end{bmatrix}$

$= \frac{1}{64}\begin{bmatrix} 4+0 & -2+0 \\ 2+8 & -1+12 \end{bmatrix}$

$= \frac{1}{64}\begin{bmatrix} 4 & -2 \\ 10 & 11 \end{bmatrix}$...(2)

From (1) and (2), we have

$(DA)^{-1} = A^{-1}\,D^{-1}$

Exercise 1.6

Question No.1 Use matrices, to solve the following system of linear equations by:

(a). the matrix inverse method

(b). the Cramer's rule

Home Work: (ii)

$\boldsymbol{2x - 2y = 4\,;\quad 3x + 2y = 6}$

Solution: (a). the matrix inverse method

In matrix form

$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$AX = B \Rightarrow X = A^{-1}B \rightarrow$(1)

Where$A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

Now, we find A^{-1} using

$A^{-1} = \frac{1}{|A|}AdjA \rightarrow$(2)

$|A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$

$= (2)(2) - (-2)(3)$

$= 4 + 6 = 10$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA = \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$

Putting values in eq. (2), we have

$A^{-1} = \frac{1}{|A|}AdjA$

$\Rightarrow A^{-1} = \frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$

Now, put values in eq. (1)

$X = A^{-1}B$

$\Rightarrow \mathrm{X} = \frac{1}{10}\begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 6 \end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix} (2)(4)+(2)(6) \\ (-3)(4)+(2)(6) \end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix} 8+12 \\ -12+12 \end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix} 20 \\ 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$\Rightarrow \quad x = 2, y = 0$

(b). the Cramer's rule

In matrix form

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

Where $A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$, $A_x = \begin{bmatrix} 4 & -2 \\ 6 & 2 \end{bmatrix}$ and

$$A_y = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$$

First of all we find $|A|, |A_x| and\ |A_y|$

$A = \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix}$

$= (2)(2) - (-2)(3)$

$|A| = 4 + 6 = 10$

Which is non-zero, so solution exists and

$A_x = \begin{bmatrix} 4 & -2 \\ 6 & 2 \end{bmatrix}$

$\Rightarrow |A_x| = \begin{vmatrix} 4 & -2 \\ 6 & 2 \end{vmatrix}$

$|A_x| = (4)(2) - (-2)(6)$

$|A_x| = 8 + 12 = 20$

Also,

$A_y = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$

$\Rightarrow |A_y| = \begin{vmatrix} 2 & 4 \\ 3 & 6 \end{vmatrix}$

$= (2)(6) - (4)(3)$

$|A_y| = 12 - 12 = 0$

Now

$x = \frac{|A_x|}{|A|} \Rightarrow x = \frac{20}{10} = 2$

And $y = \frac{|A_y|}{|A|} \Rightarrow y = \frac{0}{10} = 0$

Hence, $x = 2$ and $y = 0$

Class Work: (iii)

$$\mathbf{4x + 2y = 8\,; \quad 3x - y = -1}$$

Solution: (a). the matrix inverse method

In matrix form

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B \rightarrow (1)$

Where $A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$

Now, we find A^{-1} using

$A^{-1} = \frac{1}{|A|}AdjA$...(2)

$|A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$

$= (4)(-1) - (2)(3)$

$= -4 - 6 = -10$

Which is not zero and hence, matrix A is Non-singular matrix and A^{-1} exist.

Now, $AdjA = \begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}$

Putting values in eq. (2), we have

$A^{-1} = \frac{1}{|A|}AdjA$

$\Rightarrow A^{-1} = \frac{1}{-10}\begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}$

Now, put values in eq. (1)

$X = A^{-1}B$

$\Rightarrow X = \frac{1}{-10}\begin{bmatrix} -1 & -2 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 8 \\ -1 \end{bmatrix}$

$= \frac{1}{-10}\begin{bmatrix} (-1)(8) + (-2)(-1) \\ (-3)(8) + (4)(-1) \end{bmatrix}$

$= \frac{1}{-10}\begin{bmatrix} -8+2 \\ -24-4 \end{bmatrix}$

$= \frac{1}{-10}\begin{bmatrix} -6 \\ -28 \end{bmatrix} = \begin{bmatrix} \frac{-6}{-10} \\ \frac{-28}{-10} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{5} \\ \frac{7}{5} \end{bmatrix}$

$\Rightarrow \quad x = \frac{3}{5}, y = \frac{7}{5}$

(b). the Cramer's rule

In matrix form

$$\begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$$

Where $A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$, $A_x = \begin{bmatrix} 8 & 2 \\ -1 & -1 \end{bmatrix}$ and

$$A_y = \begin{bmatrix} 4 & 8 \\ 3 & -1 \end{bmatrix}$$

First of all we find $|A|, |A_x| and\ |A_y|$

$A = \begin{bmatrix} 4 & 2 \\ 3 & -1 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 4 & 2 \\ 3 & -1 \end{vmatrix}$

$= (4)(-1) - (2)(3)$

$= -4 - 6 = -10$

Which is non-zero, so solution exists and

$A_x = \begin{bmatrix} 8 & 2 \\ -1 & -1 \end{bmatrix}$

$\Rightarrow |A_x| = \begin{vmatrix} 8 & 2 \\ -1 & -1 \end{vmatrix}$

$|A_x| = (8)(-1) - (2)(-1)$

$|A_x| = -8 + 2 = -6$

Also,

$A_y = \begin{bmatrix} 4 & 8 \\ 3 & -1 \end{bmatrix}$

$\Rightarrow |A_y| = \begin{vmatrix} 4 & 8 \\ 3 & -1 \end{vmatrix}$

$= (4)(-1) - (8)(3)$

$|A_y| = -4 - 24 = -28$

Now

$x = \frac{|A_x|}{|A|} \Rightarrow x = \frac{-6}{-10} = \frac{3}{5}$

And $y = \frac{|A_y|}{|A|} \Rightarrow y = \frac{-28}{-10} = \frac{7}{5}$

Hence, $x = \frac{3}{5}$ and $y = \frac{7}{5}$

Home Work: (v)

$$3x - 2y = 4\,; \quad -6x + 4y = 7$$

Sol: . the matrix inverse method

In matrix form

$$\begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

$AX = B \Rightarrow X = A^{-1}B$**…(1)**

Where $A = \begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}$ **and** $B = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$

Now, we find A^{-1} **using**

$A^{-1} = \frac{1}{|A|} AdjA$**…(2)**

$|A| = \begin{vmatrix} 3 & -2 \\ -6 & 4 \end{vmatrix}$

$= (3)(4) - (-2)(-6)$

$= 12 - 12 = 0$

Which is zero and hence, matrix A is singular matrix and A^{-1} does not exist. No solution possible.

Home Work:

Question No.3 two sides of a rectangular differ by 3.5cm. find the dimensions of the rectangular if its perimeter is 67cm.

Let required sides of rectangle are x *and* y.

According to first condition.

$$x - y = 3.5 \rightarrow (i)$$

According to 2nd condition

perimeter $= 67$

$$2(x + y) = 67$$

$$\Rightarrow x + y = 33.5 \rightarrow (ii)$$

In matric form

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3.5 \\ 33.5 \end{bmatrix}$$

We have

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix};\; A_x = \begin{bmatrix} 3.5 & -1 \\ 33.5 & 1 \end{bmatrix}$$

$$A_y = \begin{bmatrix} 1 & 3.5 \\ 1 & 33.5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$$

$$= 1(1) - 1(-1)$$

$$= 1 + 1 = 2 \neq 0$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{\begin{vmatrix} 3.5 & -1 \\ 33.5 & 1 \end{vmatrix}}{2}$$

$$= \frac{3.5(1) - 33.5(-1)}{2}$$

$$= \frac{3.5 + 33.5}{2}$$

$$= \frac{37}{2} = 18.5$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{\begin{vmatrix} 1 & 3.5 \\ 1 & 33.5 \end{vmatrix}}{2}$$

$$= \frac{1(33.5) - 1(3.5)}{2}$$

$$= \frac{33.5 - 3.5}{2}$$

$$\frac{30}{2} = 15$$

$$\Rightarrow x = 18.5 \quad , \quad y = 15$$

Class work:

Question No.4 the third angle of an isosceles triangle is 16^0 less than the sum of the two equal angles. Find three angles of the triangle.

Let third angle of triangle $= y$

And two equal angles of triangle $= x$

We know that

$$x + x + y = 180^0$$

$$2x + y = 180^0 \rightarrow (i)$$

According to given condition.

$$y = 2x - 16$$

$$2x - y = 16$$

In matrix form

$$\begin{bmatrix} 2 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 180 \\ 16 \end{bmatrix}$$

$$AX = B$$

$$\Rightarrow X = A^{-1}B$$

Now

$$A = \begin{bmatrix} 2 & -1 \\ 2 & 1 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 2 & 1 \\ 2 & -1 \end{vmatrix}$$

$$|A| = 2(-1) - 2(1)$$

$$= -2 - 2$$

$$= -4 \neq 0$$

$$AdjA = \begin{bmatrix} -1 & -1 \\ -2 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times AdjA$$

$$= \frac{1}{-4}\begin{bmatrix} -1 & -1 \\ -2 & 2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix}$$

$$\Rightarrow X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} 180 \\ 16 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 1(180) + 1(16) \\ 1(180) + (-2)(16) \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 180 + 16 \\ 360 - 32 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 196 \\ 328 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 49 \\ 82 \end{bmatrix}$$

Hence $x = 49^0, y = 82^0$

Required angles are $49^0, 49^0, 82^0$

Review Exercise -1

Home Work:

1. The order of matrix [2 1] is …

(a) 2-by-1 (b) 1-by-2
(c) 1-by-1 (d) 2-by-2

2. $\begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$ **is called ……. Matrix.**

(a) zero (b) unit
(c) scalar (d) singular

3. Which is order of a square matrix?

(a) 2-by-2 (b) 1-by-2
(c) 2-by-1 (d) 3-by-2

4. Which is order of a rectangular matrix?

(a) 2-by-2 (b) 4-by-4
(c) 2-by-1 (d) 3-by-3

5. Order of transpose of $\begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 3 & 2 \end{bmatrix}$ **is …**

(a) 3-by-2 (b) 2-by-3
(c) 1-by-3 (d) 3-by-1

6. Adjoint of $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$ **is ………**

(a) $\begin{bmatrix} -1 & -2 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$

7. If $\begin{vmatrix} 2 & 6 \\ 3 & x \end{vmatrix} = 0$**, then** x **is equal to**

(a) 9 (b) –6
(c) 6 (d) –9

8. Product of [x y] $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ **is ……..**

(a) $[2x+y]$ (b) $[x-2y]$
(c) $[2x-y]$ (d) $[x+2y]$

9. If $X + \begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

then X is equal to………

(a) $\begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2 \\ 2 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix}$

Additional MCQ

10. The idea of a matrices was given by:__

(a) Arthur Cayley (b) Leonard Euler
(c) Henry Briggs (d) John Napier

11. If $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$ **then –A = ______**

(a) $\begin{bmatrix} -1 & 2 \\ -3 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ -3 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 2 \\ 3 & -4 \end{bmatrix}$

12. A square matrix is symmetric if ___

(a) $A^t = A$ (b) $A^{-1} = A$
(c) $(A^t)^t = -A^t$ (d) $A^t = -A$

13. A square matrix is skew-symmetric if:

(a) $A^t = -A$ (b) $A^{-1} = -A$
(c) $(A)^t = -A^t$ (d) $A^t = A$

14. A square matrix A is called singular if

(a) $|A| \neq 0$ (b) $|A| = 0$
(c) $A = 0$ (d) $A^t = 0$

15. A square matrix A is called nonsingular if :

(a) $|A| = 0$ (b) $A = 0$
(c) $|A| \neq 0$ (d) $A^t = 0$

16. $(AB)^{-1}$ **= ____**

(a) $A^{-1}B^{-1}$ (b) $B^{-1}A^{-1}$
(c) BA (d) AB

17. Additive inverse of $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$ **is ____**

(a) $\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}$

18. if A is a matrix then its transpose denoted by :

(a) A^{-1} (b) A^t
(c) -A (d) $(A^t)^t$

19. Which of the following is singular matrix?

a) $\begin{bmatrix}1 & 4\\2 & 7\end{bmatrix}$ (b) $\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$

(c) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ (d) $\begin{bmatrix}1 & 2\\3 & 6\end{bmatrix}$

20. If $A=\begin{bmatrix}a & b\\c & d\end{bmatrix}$ then the det. A is:

(a) ad – bc (b) bc – ad
(c) ad + bc (d) bc + ad

Answer key

1.	b	2.	c	3.	a	4.	c	5.	b
6	a	7	a	8	c	9	d	10	a
11	a	12	a	13	a	4	b	15	c
16	b	17	a	18	b	19	d	20	a

Question No.3

$$if\ \begin{bmatrix}a+3 & 4\\6 & b-1\end{bmatrix}=\begin{bmatrix}-3 & 4\\6 & 2\end{bmatrix}\ then\ find\ a\ and\ b$$

Solution:

By Comparing the corresponding elements we, get

$$a+3=-3$$
$$a=-3-3=6$$
$$a=-6\quad Answer.$$

And

$$b-1=2$$
$$b=2+1$$
$$b=3\ Answer.$$

Question No.5

Find the value of X, if $\begin{bmatrix}2 & 1\\3 & -3\end{bmatrix}+X=\begin{bmatrix}4 & -2\\-1 & -2\end{bmatrix}$

Solution:

$$\begin{bmatrix}2 & 1\\3 & -3\end{bmatrix}+X=\begin{bmatrix}4 & -2\\-1 & -2\end{bmatrix}$$
$$X=\begin{bmatrix}4 & -2\\-1 & -2\end{bmatrix}-\begin{bmatrix}2 & 1\\3 & -3\end{bmatrix}$$
$$X=\begin{bmatrix}4-2 & -2-1\\-1-3 & -2+3\end{bmatrix}$$
$$X=\begin{bmatrix}2 & -3\\-4 & 1\end{bmatrix}Answer$$

Question No7

$$if\ A\begin{bmatrix}3 & 2\\1 & -1\end{bmatrix}, B\begin{bmatrix}2 & 4\\-3 & -5\end{bmatrix}\ then\ verify\ that$$
$$(ii)(AB)^{-1}=B^{-1}A^{-1}$$

Solution:

$$A=\begin{bmatrix}3 & 2\\1 & -1\end{bmatrix}$$
$$|A|=\begin{vmatrix}3 & 2\\1 & -1\end{vmatrix}$$
$$|A|=3\times(-1)-1\times 2=-3-2=-5\neq o$$
$$A^{-1}=\frac{AdjA}{|A|}$$
$$AdjA=\begin{bmatrix}-1 & 2\\1 & 3\end{bmatrix}$$
$$A^{-1}=\frac{\begin{bmatrix}-1 & 2\\1 & 3\end{bmatrix}}{-5}$$
$$=\begin{bmatrix}\frac{1}{5} & \frac{2}{3}\\\frac{1}{5} & -\frac{3}{5}\end{bmatrix}$$
$$B=\begin{bmatrix}2 & 4\\-3 & -5\end{bmatrix}$$
$$|B|=2\times(-5)-4\times(-3)=-10+12=2\neq 0$$
$$B^{-1}=\frac{AdjB}{|B|}$$
$$=\frac{1}{2}\begin{bmatrix}-5 & -4\\3 & 2\end{bmatrix}=\begin{bmatrix}-\frac{5}{2} & -2\\\frac{3}{2} & 1\end{bmatrix}$$
$$AB=\begin{bmatrix}3 & 2\\1 & -1\end{bmatrix}\begin{bmatrix}2 & 4\\-3 & -5\end{bmatrix}$$
$$=\begin{bmatrix}3\times 2+2\times(-3) & 3\times 4+2\times 2(-5)\\1\times 2+(-1)\times(-3) & 1\times 4+(-1)\times(-5)\end{bmatrix}$$
$$=\begin{bmatrix}6-6 & 12-10\\2+3 & 4+5\end{bmatrix}$$
$$=\begin{bmatrix}0 & 2\\5 & 9\end{bmatrix}$$
$$|AB|=0\times 9-2\times 5=0-10=-10\neq 0$$
$$(AB)^{-1}=\frac{AdjAB}{|AB|}$$
$$=\frac{1}{10}\begin{bmatrix}9 & -2\\-5 & 0\end{bmatrix}$$
$$(AB)^{-1}=\begin{bmatrix}-\frac{9}{10} & \frac{1}{5}\\\frac{1}{2} & 0\end{bmatrix}\to(i)$$

Now

$$R.H.S=B^{-1}A^{-1}$$
$$A=\begin{bmatrix}3 & 2\\1 & -1\end{bmatrix}$$
$$|A|=3(-1)(2)=-3-2=-5\neq 0$$
$$AdjA=\begin{bmatrix}-1 & -2\\-1 & 3\end{bmatrix}$$
$$A^{-1}=\frac{1}{|A|}AdjA=\frac{1}{-5}\begin{bmatrix}-1 & -2\\-1 & 3\end{bmatrix}$$
$$B=\begin{bmatrix}2 & 4\\-3 & -5\end{bmatrix}$$
$$|B|=2(-5)-(-3)(4)$$
$$=-10+12=2\neq 0$$
$$B^{-1}=\frac{1}{|B|}AdjB$$
$$=\frac{1}{2}\begin{bmatrix}-5 & -4\\3 & 2\end{bmatrix}$$
$$B^{-1}A^{-1}=\left(-\frac{1}{5}\right)\left(\frac{1}{2}\right)\begin{bmatrix}-5 & -4\\3 & 2\end{bmatrix}\begin{bmatrix}-1 & -2\\-1 & 3\end{bmatrix}$$
$$=\frac{1}{10}\begin{bmatrix}-5(-1)+(-4)(-1) & -5(-2)+(-4)(3)\\3(-1)+2(-1) & 3(-1)+2(-1)\end{bmatrix}$$
$$=-\frac{1}{10}\begin{bmatrix}5+4 & 10-12\\-3-2 & -6+6\end{bmatrix}=\frac{1}{10}\begin{bmatrix}9 & -2\\-5 & 0\end{bmatrix}$$
$$=\begin{bmatrix}-\frac{9}{10} & \frac{-2}{-10}\\\frac{-5}{-10} & \frac{0}{-10}\end{bmatrix}$$
$$=\begin{bmatrix}-\frac{9}{10} & \frac{1}{5}\\\frac{1}{2} & 0\end{bmatrix}$$
$$L.H.S=R.H.S$$

Hence $(AB)^{-1}=B^{-1}A^{-1}$

Unit-2

REAL AND COMPLEX NUMBERS

Rational number:

A number which can be written in the form of $\frac{p}{q}$ $, where\ p, q \in Z \wedge \neq 0$ is called a rational number.

e.g. $\frac{3}{4}, \frac{22}{7}, \frac{2}{6}$.

Irrational number:

A real number which cannot be written in the form of $\frac{p}{q}$ $, where\ p, q \in Z \wedge \neq 0$ is called an irrational number.

e.g. $\sqrt{2}$, $\sqrt{5}$

Real number:

The field of all rational and irrational numbers is called the real numbers, or simply the "reals," and denoted $\mathbb{R}$.

Terminating decimal:

A decimal which has only a finite number of digits in its decimal part, is called terminating decimal.

e.g. 202.04, 0.25, 0.5 example of terminating decimal.

Recurring decimal:

A decimal in which one or more digits repeats indefinitely is called recurring decimal or periodic decimal.

e.g. 0.33333 , 21.134134 … …

Exercise 2.1

Question.1. Identify which of the following are rational and irrational numbers

Class Work:

(i). $\sqrt{3}$

Solution.

Is an irrational number.

Home Work:

(ii). $\frac{1}{6}$

Solution.

Is a rational number.

Class Work:

(iii). π

Solution.

Is an irrational number.

Home Work:

(iv). $\frac{15}{7}$

Solution.

Is a rational number.

Home Work:

(v). 7.25

Solution.

Is a rational number.

Home Work:

(vi). $\sqrt{29}$

Solution.

Is an irrational number.

Question.2. Convert the following fractions into decimal fraction.

Class Work:

(iii) $\frac{57}{8}$

Solution.

$$7.125$$

Home Work:

(iv) $\frac{205}{18}$

Solution.

$$11.3889$$

Home Work:

(vi) $\frac{25}{38}$

Solution.

$$0.65789$$

Question.3. Which of the following statements are true and which are false?

Class Work:

(ii). π *is an irrational number.*

Solution.

True.

Home Work:

(iii). $\frac{1}{9}$ *is a terminating fraction.*

Solution.

False.

Home Work:

(v). $\frac{4}{5}$ *is a recurring fraction.*

Solution.

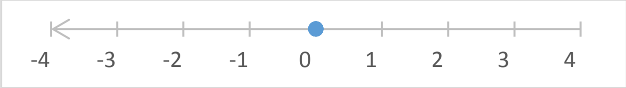
False.

Question.4. Represent the following numbers on the number line

Class Work:

(i) $\frac{2}{3}$

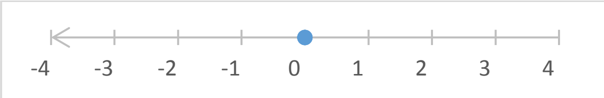
Solution.



Home Work:

(ii). $-\frac{4}{5}$

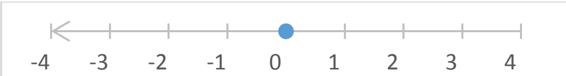
Solution.



Home Work:

(iv). $-2\frac{5}{8}$

Solution.



***Question*.6.** **Express the following recurring decimals as the rational number $\frac{p}{q}$, where p, q are integers and $q \neq 0$.**

Class Work:

(ii). $0.\overline{13}$

Solution.

Let

$$x = 0.\overline{13}$$

That is

$$x = 0.13131313 \ldots \rightarrow (i)$$

Only two digits 13 is being repeated, multiply by 100 on both sides of (i), we have

$$100x = (0.13131313 \ldots) \times 100$$

$$100x = 13.13131313 \ldots \rightarrow (ii)$$

Subtracting (i) *from* (ii), *we have*

$$100x - x = 13.13131313 \ldots . -0.13131313 \ldots$$

$$99x = 13$$

$$x = \frac{13}{99}$$

$$0.\overline{13} = \frac{13}{99}$$

Answer.

Home Work:

(iii). $0.\overline{67}$

Solution.

Let

$$x = 0.\overline{67}$$

That is

$$x = 0.67676767 \ldots \rightarrow (i)$$

Only two digits 67 is being repeated, multiply by 100 on both sides of (i), we have

$$100x = (0.67676767 \ldots) \times 100$$

$$100x = 67.67676767 \ldots \rightarrow (ii)$$

Subtracting (i) *from* (ii), *we have*

$$100x - x = 67.67676767 \ldots . -0.67676767 \ldots$$

$$99x = 67$$

$$x = \frac{67}{99}$$

$$0.\overline{67} = \frac{67}{99}$$

Answer.

Radicals and Radicands:

If n is a positive integer greater than 1 and a is a real number , then any real number x such that $x^n = a$ is called the nth root of a , *and in symbols is written as*

$$x = \sqrt[n]{a} \quad or\ x = (a)^{\frac{1}{n}}$$

And $\sqrt[n]{a}$ is called radical, the symbol $\sqrt{\quad}$ is called the radical sign , n is called the index of the radical and the real number a under the radical sign is called the radicand or base.

Radical and Exponent Form:

$$x = \sqrt[n]{a}\ is\ called\ redical\ form\ and\ a = x^{\frac{1}{n}}\ is\ called\ exponent\ form.$$

Some Properties of Radicals:

(i). $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$

(ii). $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(iii). $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}$

(iv). $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

(v). $\sqrt[n]{a^n} = a$

Exercise 2.3

Question.1. Write each radical expression in exponential notation and each exponential expression in radical notation. Do not simplify.

Class Work:

(i). $\sqrt[3]{-64} = (-64)^{\frac{1}{3}}$

Solution.

$$\sqrt[3]{-64} = (-64)^{\frac{1}{3}}$$

Home Work:

(iii). $-7^{\frac{1}{3}}$

Solution.

$$-7^{\frac{1}{3}} = -\sqrt[3]{7}$$

Question.2. Tell whether the following statements are true or false?

Class Work:

(i). $5^{\frac{1}{5}} = \sqrt{5}$

Solution.

$$False\ because\ 5^{\frac{1}{5}} = \sqrt[5]{5}\ is\ true.$$

Home Work:

(ii). $2^{\frac{2}{3}} = \sqrt[3]{4}$

Solution.

$$True\ because\ 2^{\frac{2}{3}} = \sqrt[3]{2^2} = \sqrt[3]{4}\ is\ true.$$

Question.3. Simplify the following radical expressions.

Class Work:

(ii). $\sqrt[4]{32}$

Solution.

$$\sqrt[4]{32} = \sqrt[4]{2^4 \times 2}$$
$$= (2^4 \times 2)^{\frac{1}{4}}$$
$$= (2^4)^{\frac{1}{4}} \times 2^{\frac{1}{4}}$$
$$= 2 \times \sqrt[4]{2}$$
$$= 2\sqrt[4]{2}$$

Answer.

Home Work:

(iii). $\sqrt[5]{\frac{3}{32}}$

Solution.

$$\sqrt[5]{\frac{3}{32}} = \left(\frac{3}{32}\right)^{\frac{1}{5}}$$
$$= \left(\frac{3}{2^5}\right)^{\frac{1}{5}}$$
$$= \frac{3^{\frac{1}{5}}}{2^{5\times\frac{1}{5}}}$$
$$= \frac{\sqrt[5]{3}}{2}$$

Answer.

Base and Exponents:

In the exponential form

a^n ($read\ as\ a\ to\ the\ nth\ power$) we call $''a''$ as the base and $''n''$ as the exponent or power.

Laws of Exponents:

If $a, b \in R\ and\ m\ , n\ are\ positive\ integers, then$

(i). $a^m \,.\, a^n = a^{m+n}$

(ii). $(a^m)^n = a^{mn}$

(iii). $(ab)^n = a^n b^n$

(iv). $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

(v). $\frac{a^m}{a^n} = a^{m-n}$

(vi). $a^0 = 1\ , where\ a \neq 0$

(vii). $a^{-n} = \frac{1}{a^n}\ , where\ a \neq 0$

Exercise 2.4

Question.1. Use laws of exponents to simplify

(i). $\frac{(243)^{-\frac{2}{3}}(32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}}$

Solution.

$$\frac{(243)^{-\frac{2}{3}}(32)^{-\frac{1}{5}}}{\sqrt{(196)^{-1}}} = \frac{(3^5)^{-\frac{2}{3}}(2^5)^{-\frac{1}{5}}}{(14^2)^{-1\times\frac{1}{2}}}$$
$$= \frac{3^{-\frac{10}{3}}\,2^{-1}}{14^{-1}}$$
$$= \frac{3^{-\frac{10}{3}}\,2^{-1}}{(2\times 7)^{-1}}$$
$$= \frac{3^{-\frac{10}{3}}\,2^{-1}}{2^{-1}\times 7^{-1}}$$
$$= \frac{3^{-\frac{10}{3}}}{7^{-1}}$$
$$= \frac{7}{3^{\frac{9+1}{3}}}$$
$$= \frac{7}{3^{\frac{9}{3}+\frac{1}{3}}}$$
$$= \frac{7}{3^{\frac{9}{3}} \times 3^{\frac{1}{3}}}$$

$$=\frac{7}{3^3\times\sqrt[3]{3}}$$
$$=\frac{7}{27\sqrt[3]{3}}$$

Answer.

Home Work:

(iv). $\frac{(81)^n.3^5-(3)^{4n-1}(243)}{(9)^{2n}.3^3}$

Solution.

$$\frac{(81)^n.3^5-(3)^{4n-1}(243)}{(9)^{2n}.3^3}$$
$$=\frac{(3^4)^n.3^5-(3)^{4n-1}(3)^5}{(3^2)^{2n}.3^3}$$
$$=\frac{(3)^{4n}.3^5-(3)^{4n-1}(3)^5}{(3)^{4n}.3^3}$$
$$=\frac{3^{4n+5}-3^{4n-1+5}}{3^{4n+3}}$$
$$=\frac{3^{4n+5}-3^{4n+4}}{3^{4n+3}}$$
$$=\frac{3^{4n+4}(3^1-1)}{3^{4n+3}}$$
$$=3^{4n+4-4n-3}(2)$$
$$=3^1(2)$$
$$=6$$

Question.3. Simplify

Class Work:

ii). $\sqrt{\frac{(216)^{\frac{2}{3}}(25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}}$

Solution.

$$\sqrt{\frac{(216)^{\frac{2}{3}}(25)^{\frac{1}{2}}}{(0.04)^{-\frac{1}{2}}}}=\sqrt{\frac{(6^3)^{\frac{2}{3}}(5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{-\frac{1}{2}}}}$$
$$=\sqrt{\frac{6^2.5^1}{\left(\frac{1}{25}\right)^{-\frac{1}{2}}}}$$
$$=\sqrt{\frac{6^2.5^1}{(25)^{\frac{1}{2}}}}$$
$$=\sqrt{\frac{6^2.5}{(5^2)^{\frac{1}{2}}}}$$
$$=\sqrt{\frac{6^2.5}{5}}$$
$$=\sqrt{36}$$
$$=6$$

Answer.

Home Work:

(iii). $5^{2^3}\div(5^2)^3$

Solution.

$$5^{2^3}\div(5^2)^3=\frac{5^8}{5^6}$$
$$=5^{8-6}$$
$$=5^2$$
$$=25$$

Answer.

Home Work:

(iv). $(x^3)^2\div x^{3^2}$

Solution.

$$(x^3)^2\div x^{3^2}=\frac{x^6}{x^8}$$
$$=\frac{1}{x^{8-6}}$$
$$=\frac{1}{x^2}$$

Answer.

Complex Numbers:

The numbers of the form $x+iy$, where $x,y\in\Re$,are called **complex numbers,** here x is called **real part** and y is called **imaginary part** of the complex number.

Remarks:

1. Every real number is a complex number with 0 as its imaginary part.

Conjugate Complex Numbers:

$if\ Z=a+ib\ be\ a\ complex\ number\ then\ \bar{Z}=a-ib$ is the conjugate of the complex number $Z=a+ib$.

Remarks:

1. A real number is self-Conjugate.

Equality of Two Complex Numbers:

Two complex numbers $a+bi\ and\ c+di\ are\ said\ to\ be\ equal\ if\ a=c\ and\ b=d.$

That is

$$a+ib=c+id\quad\Rightarrow a=b\ and\ c=d.$$

Exercise 2.5

Question.1. Evaluate

Class Work:

(ii). i^{50}

Solution.

$$i^{50} = (i^2)^{25}$$
$$= (-1)^{25}$$
$$= -1$$

Answer.

Home Work:

(iv). $(-i)^8$

Solution.

$$(-i)^8 = i^8$$
$$= (i^2)^4$$
$$= (-1)^4$$
$$= 1$$

Answer.

Home Work:

(vi). i^{27}

Solution.

$$i^{27} = i^{26}.i$$
$$= (i^2)^{13}.i$$
$$= (-1)^{13}.i$$
$$= (-1).i$$
$$= -1$$

Answer.

Question.2. Write the conjugate of the following numbers.

(iii). $-i$

Class Work:

Solution.

$$Suppose\ \ Z = -i$$
$$\bar{Z} = \overline{-i} = +i$$

Answer.

Home Work:

(v). $-4 - i$

Solution.

$$Suppose\ \ Z = -4 - i$$
$$\bar{Z} = \overline{-4 - i} = -4 + i$$

Answer.

Home Work:

(v). $i - 3$

Solution:

$$Suppose\ Z = i - 3$$
$$\bar{Z} = \overline{i - 3}$$
$$= -i - 3$$

Question.3. Write the real and imaginary part of the following numbers.

Class Work:

(ii). $-1 + 2i$

Solution.

$$Suppose\ \ Z = -1 + 2i$$
$$Re(Z) = -1 \quad , \quad Im(Z) = 2$$

Answer.

Home Work:

Question.4. Find the value of x and y if

$$x + iy + 1 = 4 - 3i$$

Solution.

Given that

$$x + 1 + iy = 4 - 3i$$

Separating real and imaginary parts

$$x + 1 = 4 \quad , \quad y = -3$$
$$x = 4 - 1 \quad , \quad y = -3$$
$$x = 3 \quad , \quad y = -3$$

Answer.

Operations on Complex Numbers:

The symbols *a,b,c,d,k*, where used, represent real numbers

Addition of Two Complex Numbers:

$$(a + ib) + (c + id) = (a + b) + i(c + d).$$

Scalar Multiplication:

$$k(a + ib) = ka + ikb.$$

Subtraction of Two Complex Numbers:

$$(a + ib) - (c + id) = (a - b) + i(c - d).$$

Multiplication of Two Complex Numbers:

$$(a + ib)(c + id) = (ac - bd) + i(ad + bc).$$

Division of two Complex Numbers:

$$\frac{(a + ib)}{(c + id)} = \frac{ac - bd}{c^2 + d^2} + i\frac{bc - ad}{c^2 + d^2}$$

Exercise 2.6

Question.1. Identify the following statements as true or false.

Class Work:

(v). Difference of a complex number $z = a + bi$ and its conjugate is a real number.

Solution.

$$False\ because\ Z - \overline{Z} = (a + bi) - (a - bi)$$
$$= a + bi - a + bi = 2bi$$

Home Work:

(vi). If $(a-1) - (b+3)i = 5 + 8i$ then $a = 6$ and $b = -11.$

Solution.

True because Comparing real and imaginary parts in given equation

$$a - 1 = 5 \quad , \quad -(b+3) = 8$$
$$a = 5 + 1 \quad , \quad b + 3 = -8$$
$$a = 6 \quad , \quad b = -8 - 3$$
$$a = 6 \quad , \quad b = -11$$

Question.2. Express each complex number in the standard form $a + bi$ where 'a' and 'b' are real numbers.

Class Work:

(iii). $-1(-3+5i) - (4+9i)$

Solution.

$$-1(-3+5i) - (4+9i) = 3 - 5i - 4 - 9i$$
$$= -1 - 14i$$

Answer.

Home Work:

(iv). $2i^2 + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25}$

Solution.

$$2i^2 + 6i^3 + 3i^{16} - 6i^{19} + 4i^{25}$$
$$= 2(-1) + 6i^2 i + 3i^{16} - 6i^{18} i + 4i^{24} i$$
$$= 2(-1) + 6(-1)i + 3(i^2)^8 - 6(i^2)^9 i + 4(i^2)^{12} i$$
$$= -2 - 6i + 3(-1)^8 - 6(-1)^9 i + 4(-1)^{12} i$$
$$= -2 - 6i + 3(1) - 6(-1)i + 4(1)i$$
$$= -2 - 6i + 3 + 6i + 4i$$
$$= 1 + 4i$$

Question.3. Simplify and write your answer in the form $a + bi$.

Class Work:

(i). $(-7+3i)(-3+2i)$

Solution.

$$(-7+3i)(-3+2i) = 21 - 14i - 9i + 6i^2$$
$$= 21 - 14i - 9i - 6$$
$$= 15 - 23i$$

Answer.

Home Work:

(iv). $(2-3i)(\overline{3-2i})$

Solution.

$$(2-3i)(\overline{3-2i}) = (2-3i)(3+2i)$$
$$= 2(3+2i) - 3i(3+2i)$$
$$= 6 + 4i - 9i - 6i^2$$
$$= 6 - 5i + 6$$
$$= 12 - 5i$$

Answer.

Class Work:

Question.4. Simplify and write your answer in the form of $a + bi$.

(iii). $\frac{9-7i}{3+i}$

Solution.

$$\frac{9-7i}{3+i} = \frac{9-7i}{3+i} \times \frac{3-i}{3-i}$$
$$= \frac{9(3-i) - 7i(3-i)}{3^2 - i^2}$$
$$= \frac{27 - 9i - 21i + 7i^2}{9+1}$$
$$= \frac{27 - 30i - 7}{10}$$
$$= \frac{20 - 30i}{10}$$
$$= \frac{20}{10} - \frac{30}{10}i$$
$$= 2 - 3i$$

Answer.

Class Work:

Question.6. If $z = 2 + 3i$, $w = 5 - 4i$, *show that*

(ii). $\overline{z - w} = \overline{z} - \overline{w}$

Solution.

$$L.H.S = \overline{z - w}$$
$$L.H.S = \overline{(2+3i) - (5-4i)}$$
$$L.H.S = \overline{2 + 3i - 5 + 4i}$$
$$L.H.S = \overline{-3 + 7i}$$
$$L.H.S = -3 - 7i \;---\; (1)$$
$$R.H.S = \overline{z} - \overline{w}$$
$$R.H.S = \overline{(2+3i)} - \overline{(5-4i)}$$
$$R.H.S = (2-3i) - (5+4i)$$
$$R.H.S = 2 - 3i - 5 - 4i$$
$$R.H.S = -3 - 7i \;---\; (2)$$

From (1) and (2), we have

$$L.H.S = R.H.S$$

Hence Proved.

(vi). $\frac{1}{2i}(z-\bar{z})$ *is a imaginary part of z.*

Solution.

$$\frac{1}{2i}(z-\bar{z}) = \frac{1}{2i}\left((2+3i)+(\overline{2+3i})\right)$$
$$= \frac{1}{2i}\left((2+3i)-(2-3i)\right)$$
$$= \frac{1}{2i}(2+3i-2+3i)$$
$$= \frac{1}{2i}(6i)$$
$$= 3 \text{ which is imaginary part of } z.$$

Hence Proved.

Class Work:

Question.7. Solve the following equations for real x *and* y.

(i). $(2-3i)(x+iy) = 4+i$

Solution. Given that

$$(2-3i)(x+iy) = 4+i$$
$$2(x+iy)-3i(x+iy) = 4+i$$
$$2x+2iy-3ix-3i^2y = 4+i$$
$$2x+2iy-3ix+3y = 4+i$$
$$2x+3y+(2y-3x)i = 4+i$$

Comparing real and imaginary parts, we have

$$2x+3y = 4 --- (i) \ , \quad 2y-3x = 1 --- (ii)$$

$3\times(i)+2\times(ii)$, *we have*

$$3(2x+3y)+2(2y-3x) = 3(4)+2(1)$$
$$6x+9y+4y-6x = 12+2$$
$$13y = 14$$
$$y = \frac{14}{13}$$

Using value of y *in equation* (i), *we have*

$$2x+3\left(\frac{14}{13}\right) = 4$$
$$2x+\frac{42}{13} = 4$$
$$2x = 4-\frac{42}{13}$$
$$2x = \frac{4\times 13-42}{13}$$
$$2x = \frac{52-42}{13}$$
$$2x = \frac{10}{13}$$
$$x = \frac{10}{13\times 2}$$
$$x = \frac{5}{13}$$

Hence required $x = \frac{5}{13}$ *and* $y = \frac{14}{13}$.

Home Work:

(ii). $(3-2i)(x+iy) = 2(x-2yi)+2i-1$

Solution. Given that

$$(3-2i)(x+iy) = 2(x-2yi)+2i-1$$
$$3(x+iy)-2i(x+iy) = 2x-4yi+2i-1$$
$$3x+3iy-2ix-2i^2y = 2x-1+2i-4yi$$
$$3x+3iy-2ix+2y = 2x-1+(2-4y)i$$
$$3x+2y+(3y-2x)i = 2x-1+(2-4y)i$$

Comparing real and imaginary parts, we have

$$3x+2y = 2x-1 \ ,(3y-2x) = 2-4y$$
$$3x-2x+2y = -1 \ , -2x+3y+4y = 2$$
$$x+2y = -1 --- (i) \ , -2x+7y = 2 --- (ii)$$

$2\times(i)+(ii)$, *we have*

$$2(x+2y)+(-2x+7y) = 2(-1)+2$$
$$2x+4y-2x+7y = -2+2$$
$$11y = 0$$
$$y = 0$$

Using value of y *in equation* (i), *we have*

$$x+2(0) = -1$$
$$x = -1$$

Hence required $x = -1$ *and* $y = 0$.

Review Exercise 2

Home Work+ Class Work

Question No.1 Multiple Choice Questions. Choose the correct answer.

1. $\left(27x^{-1}\right)^{\frac{-2}{3}} =$ _____

(a) $\frac{\sqrt[3]{x^2}}{9}$ (b) $\frac{\sqrt{x^3}}{9}$

(c) $\frac{\sqrt[3]{x^2}}{8}$ (d) $\frac{\sqrt{x^3}}{8}$

2. Write $\sqrt[7]{x}$ **in exponential form**

(a) x (b) x^7

(c) $x^{\frac{1}{7}}$ (d) $x^{\frac{7}{2}}$

3. Write $4^{\frac{2}{3}}$ **with radical sign….**

(a) $\sqrt[3]{4^2}$ (b) $\sqrt{4^3}$

(c) $\sqrt[2]{4^3}$ (d) $\sqrt{4^6}$

4. In $\sqrt[3]{35}$ the radicand is

(a) 3 (b) $\frac{1}{3}$
(c) 35 (d) None of these

5. $\left(\frac{25}{16}\right)^{\frac{-1}{2}}$ = ____

(a) $\frac{5}{4}$ (b) $\frac{4}{5}$
(c) $\frac{-5}{4}$ (d) $\frac{-4}{5}$

6. The conjugate of $5 + 4i$ is ____
(a) $-5 + 4i$ (b) $-5 - 4i$
(c) $5 - 4i$ (d) $5 + 4i$

7. The value of i^9 is ____
(a) 1 (b) -1
(c) i (d) $-i$

8. Every real number is ____
(a) A positive integer
(b) A rational number
(c) A negative integer
(d) A complex number

9. Real part of 2ab $(i + i^2)$ is ____
(a) 2ab (b) -2ab
(c) 2ab$i$ (d) -2ab$i$

10. Imaginary part of $-i(3i+2)$ is_
(a) -2 (b) 2
(c) 3 (d) -3

11. Which of the following sets have the closure property w.r.t. addition____
(a) {0} (b) {0, –1}
(c) {0, 1} (d) $\left\{1, \sqrt{2}, \frac{1}{2}\right\}$

12. Name the property of real numbers used in $\left(\frac{-\sqrt{5}}{2}\right) \times 1 = \frac{-\sqrt{5}}{2}$
(a) Additive identity
(b) Additive Inverse
(c) Multiplicative identity
(d) Multiplicative Inverse

13. If x, y, z $\in$ R z < 0 then x<y$\Rightarrow$
(a) x z < y z (b) x z > y z
(c) x z = y z (d) none of these

14. If a, b $\in$ R then only one of a = b or a < b or a > b holds is called…
(a) Trichotomy property
(b) Transitive property
(c) Additive property
(d) Multiplicative property

15. A non-terminating, non-recurring decimal represents:
(a) A natural number
(b) A rational number
(c) An irrational number
(d) A prime number

Additional MCQ

16. The union of the set of rational numbers and irrational numbers is known as set of ___

(a) Rational number (b) Irrational
(c) Real number (d) Whole number

17. $\sqrt{3}.\sqrt{3}$ is a ___ number.
(a) Rational (b) Irrational
(c) Real (d) None

18. $\sqrt[n]{ab}$ = ____
(a) $\sqrt[n]{a}\,\sqrt[n]{b}$ (b) $\sqrt{a}\,\sqrt{b}$
(c) $\sqrt[n]{a}\,\sqrt{b}$ (d) $\sqrt{a}\,\sqrt[n]{b}$

19. $\sqrt[5]{-8}$ = - - - - - -
(a) $(-8)^{1/5}$ (b) $(-8)^5$
(c) (-8) (d) $(8)^{\frac{1}{5}}$

20. The value of i^{10} is:
(a) -1 (b) 1
(c) $-i$ (d) i

21. The conjugate of $2 + 3i$ is ___
(a) $2 - 3i$ (b) $-2 - 3i$
(c) $-2 + 3i$ (d) $2 + 3i$

22. Real part of $\left(-1+\sqrt{-2}\right)^2$ is:
(a) -1 (b) $-2\sqrt{2}$
(c) 1 (d) $2\sqrt{2}$

23. Imaginary part of $\left(-1+\sqrt{-2}\right)^2$ is
(a) -1 (b) $-2\sqrt{2}$
(c) 1 (d) $2\sqrt{2}$

24. $\frac{P}{q}$ is a/an………number
(a) irrational (b) rational
(c) natural (d) whole

25. The value of i (iota) is______

(a) $\sqrt{-1}$ (b) -1
(c) $+1$ (d) $(-1)^2$

26. In $-2+3i$, 3 is called ________
(a) imaginary part (b) real part
(c) negative part (d) complex number

27. The set of natural numbers is……
(a) {0,1,2,3….} (b) {2,4,6….}
(c) {1,2,3…..} (d) {2,3,5,7…}

28. π, e, $\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$ are called…
(a) irrational numbers
(b) rational number
(c) natural numbers (d) real number

29. If $x+iy+1=4-3i$, then
(a) $x=4,\quad y=-3$
(b) $x=3,\quad y=3$
(c) $x=3,\quad y=-3$
(d) $x=5,\quad y=-3$

30. $\frac{p}{q}$ form of $0.\bar{3}$ is ________.
(a) $\frac{3}{10}$ (b) $\frac{1}{3}$
(c) 0.33 (d) $\frac{10}{3}$

(Answer key)

1	a	2	c	3	a	4	c	5	b
6	c	7	c	8	d	9	b	10	a
11	a	12	c	13	b	14	a	15	c
16	c	17	c	18	a	19	a	20	a
21	a	22	a	23	b	24	a	25	a
26	a	27	c	28	a	29	c	30	b

Question No.3

(ii) $\sqrt{25x^{10n}y^{8m}}$

$$\sqrt{25x^{10n}y^{8m}}$$

Solution:

$$\sqrt{25x^{10n}y^{8m}}$$
$$(25x^{10n}y^{8m})^{\frac{1}{2}}$$
$$=(5^2)^{\frac{1}{2}}(x^{10n})^{\frac{1}{2}}(y^{8m})^{\frac{1}{2}}=5x^{5n}y^{4m}$$

(iv) $\frac{(32x^{-6}y^{-4}z)^{\frac{2}{5}}}{625x^4yz^{-4}}$

Solution:

$$\frac{(32x^{-6}y^{-4}z)^{\frac{2}{5}}}{625x^4yz^{-4}}=\left(\left(\frac{2^5}{5^5}\right)x^{-6}x^{-4}y^{-4}.z^1z^{-4}\right)^{\frac{2}{5}}$$
$$\left(\frac{2^5}{5^5}.x^{-6-4}y^{-4-1}z^{1+4}\right)^{\frac{2}{5}}=\left(\frac{2^5}{5^5}x^{-10}y^{-5}z^5\right)^{\frac{2}{5}}$$
$$=\frac{(2^5)^{\frac{2}{5}}}{(5^5)^{\frac{2}{5}}}(x^{-10})^{\frac{2}{5}}(y^{-5})^{\frac{2}{5}}(z^{-5})^{\frac{2}{5}}=\frac{2^2}{5^2}(x)^{-4}(y)^{-2}(z^2$$
$$=\frac{4z^2}{25x^4y^2}$$

Question No.4 Simplify

$$\sqrt{\frac{(216)^{\frac{2}{3}}\times(25)^{\frac{1}{2}}}{(0.04)^{-\frac{3}{2}}}}$$
$$=\left[\frac{(2^3\times3^3)^{\frac{2}{3}}\times(5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{-\frac{3}{2}}}\right]^{\frac{1}{2}}$$
$$=\left[\frac{(2^3)^{\frac{2}{3}}\times(3^3)^{\frac{2}{3}}\times5}{\left(\frac{100}{4}\right)^{\frac{3}{2}}}\right]^{\frac{1}{2}}$$
$$=\left[\frac{2^2\times3^2\times5}{(25)^{\frac{3}{2}}}\right]=\left[\frac{2^2\times3^2\times5}{(5^2)^{\frac{3}{2}}}\right]^{\frac{1}{2}}$$
$$=\left[\frac{2^2\times3^2\times5}{5^3}\right]^{\frac{1}{2}}=\left[\frac{2^2\times3^2}{5^2}\right]^{\frac{1}{2}}$$
$$=\frac{(2^2)^{\frac{1}{2}}\times(3^2)^{\frac{1}{2}}}{(5^2)^{\frac{1}{2}}}=\frac{2\times3}{5}=\frac{6}{5}$$

Question No.5
Simplify:

$$\left(\frac{a^p}{a^q}\right)^{p+q}.\left(\frac{a^q}{a^r}\right)\div5(a^p.a^r)^{p-r}.a\neq0$$
$$=(a^{p-q})^{p+q}.(a^{q-r})^{q+r}\div5(a^{p+r})^{p-r}$$
$$=ap^2-q^2.a^{q^2-r^2}\div5a^{p^2-r^2}$$
$$=\frac{a^{p^2-q^2}.a^{q^2-r^2}}{5a^{p^2-r^2}}$$
$$=\frac{a^{p^2-q^2+q^2-r^2-p^2+r^2}}{5}$$
$$=\frac{a^0}{5}=\frac{1}{5}$$

Unit-3

[LOGARITHMAS]

Scientific Notation:

A number written in the form $a \times 10^n$, *where* $1 \le a \le 10$ and n is an integer, is called the scientific notation.

Example:

Write each of the following ordinary numbers in scientific notation.

Solution:

(i) 30600= 3.06×10^4

$(move\ decimal\ point\ four\ places\ to\ the\ left)$

(ii) $0.000058 = 5.8 \times 10^{-5}$

$(move\ dicemal\ point\ five\ places\ to\ the\ right)$

Example:

Change each of the following numbers from scientific notation to ordinary notation.

Solution:

(i) $6.35 \times 10^6 = 6350000$

$(Move\ the\ decimal\ point\ six\ places\ to\ the\ right)$

(ii) $7.61 \times 10^{-4} = 0.000761$

$(Move\ the\ decimal\ point\ four\ places\ to\ the\ left)$

Exercise 3.1

Question.1. Express each of the following numbers in scientific notation.

Class Work: (iv). 416.9

Solution.

$\mathbf{416.9 = 4 \wedge 16.9}$

$$\mathbf{416.9 = 4.169 \times 10^2}$$

Answer.

Home Work: (v). $83,000$

Solution.

$\mathbf{83,000 = 8 \wedge 3,000.}$

$$\mathbf{83,000 = 8.3 \times 10^4}$$

Answer.

Question.2. Express the following numbers in ordinary notation.

(ii). 5.06×10^{10}

Solution.

$$\mathbf{5.06 \times 10^{10} = 5.06 \times 10,000,000,000}$$
$$\mathbf{= 50,600,000,000.}$$

Answer.

Logarithm of real numbers:

$if\ a^x = y\ then\ x\ is\ called\ the\ logarithm\ of\ y\ to\ the\ base$ "a" $and\ its\ written\ as\ \log_a y = x\ where\ a > 0, a \ne 1\ and\ y > 0$

$i.e\ the\ logarithm\ of\ a\ number\ y\ to\ the\ base$ "a" $is\ the\ index\ x\ of\ the\ power\ to\ which\ a\ must\ be\ raised\ to\ get\ that\ number\ y.$

$the\ relation\ a^x = y\ and\ \log_a y = x\ are\ equivalen$

When one relation is given, it can be converted into the other. Thus

$$a^x = y \iff \log_a y = x$$

Example: find $log_4 2$ $i.e.\ find\ log\ of\ 2\ to\ the$ Base 4.

Solution:

Let $\log_4 2 = x$

$then\ its\ exponenenial\ form\ is\ 4^x = 2$

$i.e\ \ 2^{2x} = 2^1 \Rightarrow 2x = 1$

$\because x = \dfrac{1}{2} \Rightarrow log_4 2 = \dfrac{1}{2}$

Deductions from Definition of logarithm

1. Since $a^0 = 1,\ log_a 1 = 0$
2. $Since\ a^1 = a\ , log_a\ a = 1$

Common logarithm:

If the base of logarithm is taken as 10 then logarithm is called Common Logarithm.

Characteristic:

The integral part of the logarithm of any number is called the characteristic.

Mantissa: the fractional part of the logarithm of a number is called the mantissa. Mantissa is always positive.

Example: find the mantissa of the logarithm of 43.254

Solution:

Rounding off 43.254 $we\ consider\ only\ the\ four$ Significant digits4325.

(i) We first locate the row corresponding to 43 in the log tables and
(ii) Proceed horizontally till we reach the column corresponding to 2. The number at the intersection is 6355.
(iii) Again proceeding horizontally till the mean difference column corresponding to 5 intersects this row. We get the number 5 at the intersection.
(iv) Adding the two numbers 6355 and 5 we get .6360 as the mantissa of the logarithm of 43.25

Example:

Find the mantissa of the logarithm of 0.002347

Solution:

Here also, we consider only the four Signiant digits 2347 We first locate the row corresponding to 23 in the logarithm tables and proceeding to 4 the resulting number 3692. The number at the intersection of this row and the mean difference column corresponding to 7 is 13. Hence the sum of 3692 and 13 gives the mantissa of the logarithm of 0.0023476 as o.3705

Example:

1. **Find log 278.23**
2. **Log0.07058**

Solution:

1. $278.22\ can\ be\ rounded\ off\ as\ 278.22$

The characteristic is 2 and the mantissa, using log tables, is .4443

$$\because log278.23 = 2.4443$$

2. The characteristic of $log 0.07058$ is -2 which is written as $\bar{2}$ by convention.

Using log tables the mantissa is .8487, so that

$$log 0.07058 = \bar{2}.8487$$

Example:

Find the numbers whose logarithms are

(i) 1.3247

(ii) $\bar{2}.1324$

Solution:

(i) 1.3247

antilog1.3247 = 21.12

(ii) $\bar{2}.1324$

$$antilog(\bar{2}.1324) \text{ is } 0.01356$$

Exercise 3.2

Question.2. If $log 31.09 = 1.4926$, find the values of the following

(iii). $log 0.003109$

Solution.

$$Characteristics = -3$$
$$Mantisa = 0.4926$$
$$Log(0.003109) = \bar{3}.4926$$

Answer.

(iv). $log 0.3109$

Solution.

$$Characteristics = -1$$
$$Mantisa = 0.4926$$
$$Log(0.3109) = \bar{1}.4926$$

Answer.

Question.4. what replacement for the unknown in each of the following will make the statement true?

(ii). $log_a 6 = 0.5$

Solution.

$$log_a 6 = 0.5$$

Exponential Form

$$a^{0.5} = 6$$
$$a^{\frac{1}{2}} = 6$$

Squaring on both sides , we have

$$\left(a^{\frac{1}{2}}\right)^2 = 6^2$$
$$a = 36.$$

Question.6. Evaluate the value of "x" from the following statements.

(i). $log_2 x = 5$

Solution.

$$log_2 x = 5$$

Exponential Form

$$2^5 = x$$
$$x = 2^5$$
$$x = 2 \times 2 \times 2 \times 2 \times 2$$

$x = 32$Answer.

(iv). $log_x 64 = 2$

Solution.

$$log_x 64 = 2$$

Exponential Form

$$(x)^2 = 64$$

Taking square root on both sides

$$\sqrt{x^2} = \sqrt{64}$$
$$x = 8$$

Answer.

(v). $log_3 x = 4$

Solution.

$$log_3 x = 4$$

Exponential Form

$$3^4 = x$$
$$x = 3 \times 3 \times 3 \times 3$$
$$x = 81$$

Answer.

Laws of Logarithm

(i) $\log_a(mn) = \log_a m + \log_a n$

(ii) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

(iii) $\log_a m^n = n log_a m$

(iv) $\log_a n = \log_b n \times \log_a b$

$$or = \frac{\log_b n}{\log_b a}$$

Note:

(i) $log_a(mn) \neq log_a m \times log_a n$

(ii) $log_a m + log_a n \neq log_a(m+n)$

(iii) $log_a(mnp) = log_a m + log_a n + log_n p + ...$

Exercise 3.3

Which of the following into sum of difference.

(iii) $\log\left(\frac{21\times5}{8}\right)$

Sol: $\log\left(\frac{21\times5}{8}\right) = \log 21 + \log 5 - \log 8$

(v) $\log\frac{(22)^{\frac{1}{3}}}{5^3}$

Sol: $\log\frac{(22)^{\frac{1}{3}}}{5^3} = \log(22)^{\frac{1}{3}} - \log 5^3$

$$\log\frac{(22)^{\frac{1}{3}}}{5^3} = \frac{1}{3}\log 22 - 3\log 5$$

(vi) $\log\left(\frac{25\times47}{29}\right)$

Sol: $\log\left(\frac{25\times47}{29}\right) = \log 25 + \log 47 - \log 29$

Question Write the following in the single logarithm.

(ii) $\log 25 - 2\log 3$

Sol: $\log 2 5 - 2\log 3 = \log 25 - \log 3^2$

$$= \log\frac{25}{3^2}$$

(iv) $\log 5 + \log 6 - \log 2$

Sol: $\log 5 + \log 6 - \log 2 = \log\left(\frac{5\times6}{2}\right)$

Q#4) calculate the following:

(i). $\log_3 2 \times \log_2 81$

Sol: $\log_3 2 \times \log_2 81$

(using $\log_a n = \frac{\log_b n}{\log_b a}$)

$$\log_3 2\times\log_2 81=\frac{\log 2}{\log 3}\times\frac{\log 81}{\log 2}$$
$$=\frac{\log 3^4}{\log 3}$$
$$=\frac{4\ \log 3}{\log 3}$$
$$=4$$

(i). $\log_5 3\times\log_3 25$

Sol:). $\log_5 3\times\log_3 25$

(using $\log_a n=\frac{\log_b n}{\log_b a}$)

$$\log_5 3\times\log_3 25=\frac{\log 3}{\log 5}\times\frac{\log 25}{\log 3}$$
$$=\frac{\log 5^2}{\log 5}$$
$$=\frac{2\ \log 5}{\log 5}$$
$$=2$$

Q#5) If $\log 2=0.3010$, $\log 3=0.4171$, and $\log 5=0.6990$, then find the values of the following:

(iii). $\log\sqrt{3\frac{1}{3}}$

Sol: $\log\sqrt{3\frac{1}{3}}=\log\sqrt{\frac{10}{3}}=\log\left(\frac{10}{3}\right)^{\frac{1}{2}}$
$$=\frac{1}{2}\log\frac{10}{3}$$
$$=\frac{1}{2}(\log 10-\log 3)$$
$$=\frac{1}{2}(\log(2\times 5)-\log 3)$$
$$=\frac{1}{2}(\log 2+\log 5-\log 3)$$
$$=\frac{1}{2}(0.3010+0.6990-0.4171)$$
$$=\frac{1}{2}(0.5229)$$
$$=0.2615$$

(iv). $\log\frac{8}{3}$

Sol: $\log\frac{8}{3}=\log 8-\log 3=\log 2^3-log3$
$$=3\ \log 2-\log 3=3\ (0.3010)-0.4171$$
$$=0.9030-0.4171$$
$$=0.4259$$

Application of logarithm

Exercise 3.4

1. Using log tables to find the value of.

(i) 0.8176×13.64

Sol: Let $x=0.8176\times 13.64$

Taking log on both sides

$$\log x=\log(0.8176\times 13.64)$$
$$=\log 0.8176+\log 13.64$$

(In $\log 0.8176$, the ch. Is $\bar{1}$ we find the log(8.176) which is 0.9125, so combine both that is $\log 0.8176=\bar{1}+0.9125=\bar{1}.9125$)

$$=\bar{1}.9125+1.1348$$
$$=-1+0.9125+1.1348$$
$$=-0.0875+1.1348$$

$$\log x=1.0473$$

Taking anti-log on both sides, we have

$$x=Antilog(1.0473)$$
$$x=11.15$$

(iii) $\frac{0.678\times 9.01}{0.0234}$

Sol: Let $x=\frac{0.678\times 9.01}{0.0234}$

Taking log on both sides

$$\log x=\log\left(\frac{0.678\times 9.01}{0.0234}\right)$$
$$=\log(0.678)+\log(9.01)-\log(0.0234)$$
$$=\bar{1}.8312+0.9547-\bar{2}.3692$$
$$=(-1+0.83120)+0.9547-(-2+0.3692)$$
$$=(-0.1688)+0.9547-(-1.6308)$$
$$=-0.1688+0.9547+1.6308$$
$$=2.4163$$
$$\log x=2.4163$$

Taking anti-log on both side, we have

$$x=Antilog(2.4163)$$
$$x=261$$

(v) $\frac{(1.23)(0.6975)}{(0.0075)(1278)}$

Sol: Let $x=\frac{(1.23)(0.6975)}{(0.0075)(1278)}$

Taking log on both sides

$$\log x=\log\left(\frac{(1.23)(0.6975)}{(0.0075)(1278)}\right)$$
$$=\log(1.23)+\log(0.6975)-\log(0.0075)-\log(1279)$$
$$=0.0899+\bar{1}.8435-\bar{3}.8751-3.1065$$
$$=0.0899+(-1+0.8435)-(-3+0.8751-3.1065$$
$$=0.0899+(-0.1565)-(-2.1249)-3.1065$$
$$=0.0899-0.1565+2.1249-3.1065$$
$$\log x=-1.0482$$

Adding and subtracting 2 on R.H.S

$$\log x=-2+2-1.0482$$
$$\log x=\bar{2}+(2-1.0482)$$
$$\log x=\bar{2}+(0.9518)$$
$$\log x=\bar{2}.9518$$

Taking anti-log on both side, we have

$$x=Antilog(\bar{2}.9518)$$

(Here $Antilog(0.9518)=8.50$ but Ch. $\bar{2}$ indicates that point will move two digits to left side)

$$x=0.0850$$

(viii) $\frac{(438)^3\sqrt{0.056}}{(388)^4}$

Sol: Let $x=\frac{(438)^3\sqrt{0.056}}{(388)^4}$

Taking log on both sides

$$\log x=\log\left(\frac{(438)^3\sqrt{0.056}}{(388)^4}\right)$$
$$=\log(438)^3+\log(0.056)^{\frac{1}{2}}-\log(388)^4$$
$$=3\log(438)+\frac{1}{2}\log(0.056)-4\log(388)$$
$$=3(2.6415)+\frac{1}{2}(\bar{2}.7482)-4(2.5888)$$

$= 7.9245 + \frac{1}{2}(-2 + 0.7482) - 10.3552$

$= 7.9245 + \frac{1}{2}(-1.2518) - 10.3552$

$= 7.9245 - 0.6259 - 10.3552$

$\log x = -3.0566$

Adding and subtracting 4 on R.H.S

$\log x = -4 + 4 - 3.0566$

$\log x = \bar{4} + (4 - 3.0566)$

$\log x = \bar{4} + (0.9434)$

$\log x = \bar{4}.9434$

Taking anti-log on both side, we have

$x = Antilog(\bar{4}.9434)$

(Here $Antilog(0.9434) = 8.778$ but Ch. $\bar{4}$ indicates that point will move four digits to left side)

$x = 0.0008778$

Q#4) If $A = \pi r^2$, find A, when $\pi = \frac{22}{7}$ and $r = 15$

Sol: $A = \pi r^2$

Taking log on both sides

$\log(A) = \log(\pi r^2)$

$\log(A) = \log\left(\frac{22r^2}{7}\right)$

$\log A = \log 22 + \log(r)^2 - \log 7$

$\log A = \log 22 + 2\log r - \log 7$

Putting values

$\log A = \log 22 + 2\log 15 - \log 7$

$\log A = 1.3424 + 2(1.1761) - 0.8451$

$\log A = 1.3424 + 2.3522 - 0.8451$

$\log A = 2.8495$

Taking anti-log on both side, we have

$A = Antilog(2.8495)$

$A = 707.1$ Sq. units

Review Exercise-3

Q.1 Multiple Choice Questions. Choose the correct answer.

1. If $a^x = n$, then _____

(a) $a = \log_x n$ (b) $x = \log_n a$
(c) $x = \log_a n$ (d) $a = \log_n x$

2. The relation of $y = \log_z x$ implies

(a) $x^y = z$ (b) $z^y = x$
(c) $x^z = y$ (d) $y^z = x$

3. The logarithm of unity to any base is

(a) 1 (b) 10
(c) e (d) 0

4. The logarithm of any number to itself as base is___

(a) 1 (b) 0
(c) –1 (d) 10

5. log e = ____ where e ≈ 2. 718

(a) 0 (b) 0.4343
(c) ∞ (d) 1

6. The value of $\log\left(\frac{p}{q}\right)$ is ___

(a) log p –log q (b) $\frac{\log p}{\log q}$
(c) log p + log q (d) log q – log p

7. Logp – logq is same as:

(a) $\log\left(\frac{q}{p}\right)$ (b) $\log(p-q)$
(c) $\frac{\log p}{\log q}$ (d) $\log\frac{p}{q}$

8. $\log m^n$ can be written as

(a) $(\log m)^n$ (b) m log n
(c) n log m (d) log (m n)

9. $\log_b a \times \log_c b$ can be written as___

(a) $\log_c a$ (b) $\log_a c$
(c) $\log_a b$ (d) $\log_b c$

10. $\log_y x$ will be equal to___

(a) $\frac{\log_z x}{\log_y z}$ (b) $\frac{\log_x z}{\log_y z}$
(c) $\frac{\log_z x}{\log_z y}$ (d) $\frac{\log_z y}{\log_z x}$

Additional MCQ

11. For common logarithm, the base is_

(a) 2 (b) 10
(c) e (d) 1

12. For natural logarithm, the base is__

(a) 10 (b) e
(c) 2 (d) 1

13. The integral part of the common logarithm of a number is called the_

(a) Characteristic (b) Mantissa
(c) Logarithm (d) None

14. The decimal part of the common logarithm of a number is called the _____:

(a) Characteristic (b) Mantissa
(c) Logarithm (d) None

15. If x = log y, then y is called the _______ of x.

(a) Antilogarithm (b) Logarithm
(c) Characteristic (d) None

16. 30600 in scientific notation is __

(a) 3.06×10^4 (b) $3.006\ 10^4$
(c) 30.6×10^4 (d) 306×10^4

17. 6.35×10^6 in ordinary notation is___

(a) 6350000 (b) 635000
(c) 6350 (d) 63500

18. A number written in the form $a \times 10^n$, where $1 \le a < 10$ and n is an integer is called ____
(a) Scientific notation (b) Ordinary notation
(c) Logarithm notation (d) None

19. Common logarithm is also known as ______ logarithm.
(a) natural (b) simple
(c) scientific (d) decadic

20. $\log_a m + \log_a n$ is same as:
(a) $\log_a(m+n)$ (b) $\log_a m \times n$
(c) $\log_a m \times \log_a n$ (d) $\log_a \frac{m}{n}$

21. John Napier prepared the logarithms tables to the base _______.
(a) 0 (b) 1
(c) 10 (d) e

22. $\log_2^3$ in common logarithm is written as __________.
(a) $\frac{\log 3}{\log 2}$ (b) $\frac{\log 2}{\log 3}$
(c) $\frac{\log 3}{2}$ (d) $\log 2^3$

23. $\log_e 10 =$ _______
(a) 2.3026 (b) 0.4343
(c) e^{10} (d) 10

24. If $\log_2^x = 5$ then x is:
(a) 25 (b) 32
(c) 10 (d) 2^{5x}

(Answer key)

1	c	2	b	3	d	4	a	5	b	6	a	7	d	8	c
9	a	10	c	11	b	12	b	13	a	14	b	15	a	16	a
17	a	18	a	19	d	20	d	21	d	22	a	23	a	24	b

Question No.3 Find the value of x in the following.

(i) $log_3 x = 5$

Solution: $log_3 x = 5$

$$\Rightarrow x = 3^5 \Rightarrow x = 243$$

(ii) $log_4 256 = x$

Solution: $log_4 256 = x$

$$4^x = 256 \Rightarrow 4^x = 4^4$$
$$\Rightarrow x = 4$$

(iii) $log_{625} 5 = \frac{1}{4}x$

Solution:

$$log_{625} 5 = \frac{1}{4}x \Rightarrow 625^{\frac{1}{4}x} = 5 \Rightarrow (5^4)^{\frac{1}{4}x} = 5$$
$$\Rightarrow 5^x = 5^1$$

(iv) $log_{64} x = -\frac{2}{3}$

solution:

$$log_{64} x = -\frac{2}{3} \Rightarrow x = 64^{-\frac{2}{3}}$$
$$x = 4^{3\left(-\frac{2}{3}\right)} \Rightarrow x = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

Question No.5

if $log 2 = 0.3010, log 3 = 0.4771$ and $log 5 = 0.6990$, then find the valves of the following.

(ii) $log \frac{16}{15}$

Solution:

$$log \frac{16}{15}$$
$$= log16 - log15$$
$$= log2^4 - log(3 \times 5)$$
$$= 4log2 - [log3 - log5]$$
$$= 4log2 - log3 - log5$$
$$= 4(0.3010) - 0.4771 - 0.6990$$
$$= 1.2040 - 0.4771 - 0.6990$$
$$= 0.0279$$

Question No.6

(i) $\sqrt[3]{25.47}$

Solution:

$$let\ y = \sqrt[3]{25.47}$$
$$logy = \log(25.47)^{\frac{1}{3}}$$
$$logy = \frac{1}{3}\log(25.47)$$
$$logy = \frac{1}{3}(1.4060)$$
$$logy = 0.4687$$
$$y = Antilog(0.4687)$$
$$y = 2.942$$

(iii)

$$\frac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$$

Solution:

$$let\ \ y = \frac{(8.97)^3 \times (3.95)^2}{\sqrt[3]{15.37}}$$
$$logy = \log(8.97)^3 + \log(3.95)^2 - \log(15.37)^{\frac{1}{3}}$$
$$logy = 2.8584 + 1.1932 - 0.3956$$
$$logy = 3.6560$$
$$y = Antilog(3.6560)$$
$$y = Antilog(3.6560)$$
$$y = 4528.98 \Rightarrow 4529$$

Unit-4

[ALGEBRAIC EXPRESSION AND ALGEBRAIC FORMULA]

Algebraic Expressions

Algebra is a generalization of arithmetic. Recall that when operations of addition and subtraction are applied to algebraic terms, we obtain an algebraic expression. For instance, $5x^2 - 3x + \frac{2}{\sqrt{x}}, 3xy + \frac{3}{x}$ $(x \neq 0)$ are algebraic expressions.

Polynomials

it is a polynomial A polynomial in the variable x is an algebraic expression of the form
$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots +$
$a_1 x + a_0, \quad a_n \neq 0$ ...... (i)
where n, the highest power of x, is a non-negative integer called the degree of the polynomial and each coefficient a_n, is a real number. The coefficient a_n of the highest power of x is called the *leading coefficient* of the polynomial. $2x^4y^2 + x^2y^2 + 8x$ is a polynomial in two variables x and y having degree 6 (4+2=6).

Rational Expression

The quotient $\frac{p(x)}{q(x)}$ of two polynomials, $p(x)$ and $q(x)$, where $q(x)$
is a non-zero polynomial, is called a rational expression.
For example, $\frac{2x+5}{5x-1}$, $5x- \neq 0$ is a rational expression.
Note:
Every polynomial $p(x)$ can be regarded as a rational expression, since we can write $p(x)$ as $\frac{p(x)}{1}$. Thus, every polynomial is a rational expression, but every rational expression need not be a polynomial.
Algebraic formulas
(i). $(a+b)^2 = a^2 + b^2 + 2ab$
(ii). $(a-b)^2 = a^2 + b^2 - 2ab$
(iii). $x^2 - y^2 = (x-y)(x+y)$
(iv) $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$
(v) $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$
(vi) $\left(x+\frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x+\frac{1}{x}\right)$
(vii) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$
(viii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

Exercise 4.1

Question No.1) Identify whether the following algebraic expressions are polynomials (Yes or No).
(i). $3x^2 + \frac{1}{x} - 5$
Sol: No, it is not a polynomial because it contains the term $\frac{1}{x}$.
(ii). $3x^3 - 4x^2 - x\sqrt{x} + 3$
Sol: No, it is not a polynomial because it contains the term $x\sqrt{x}$.
Question No.2 State whether each of the following expressions is a rational expression or not.
(i). $\frac{3\sqrt{x}}{3\sqrt{x}+5}$
Sol: It is not rational expression.
(iii). $\frac{x^2+6x+9}{x^2-9}$
Sol: It a rational expression.
Q#3) Reduce the following rational expressions to the lowest form.
(i). $\frac{120\, x^2\, y^3\, z^5}{30\, x^3 y\, z^2}$
Sol: $\frac{120\, x^2\, y^3\, z^5}{30\, x^3 y\, z^2} = 4x^{2-3}\, y^{3-1}\, z^{5-2}$
$= 4x^{-1}\, y^2\, z^3$
$= \frac{4\, y^2\, z^3}{x}$
Sol: $\frac{(x+y)^2-4xy}{(x-y)^2} = \frac{x^2+y^2+2xy-4xy}{x^2+y^2-2xy}$
$== \frac{x^2+y^2-2xy}{x^2+y^2-2xy} = 1$

(iv). $\frac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)}$
Sol: $\frac{(x^3-y^3)(x^2-2xy+y^2)}{(x-y)(x^2+xy+y^2)} = \frac{(x^3-y^3)(x^2-2xy+y^2)}{(x^3-y^3)}$
$= (x-y)^2$
Q#4) Evaluate (a). $\frac{x^3y-2z}{xz}$ for
(b). $\frac{x^2y^3-5z^4}{xyz}$ for $x = 4, y = -2$ *and* $z = -1$
Sol: As given $\frac{x^2y^3-5z^4}{xyz}$
Putt $x = 4, y = -2$ *and* $z = -1$ in above
$\frac{x^2y^3 - 5z^4}{xyz} = \frac{(4)^2(-2)^3 - 5(-1)^4}{(4)(-2)(-1)}$
$\frac{(16)(-8) - 5(1)}{8} = \frac{-128-5}{8}$
$= \frac{-133}{8} = -16\frac{5}{8}$
#6) Perform the indicated operation and simplify.
(i). $(x^2 - 49).\frac{5x+2}{x+7}$
Sol: $(x^2 - 49).\frac{5x+2}{x+7} = (x^2 - 7^2).\frac{5x+2}{x+7}$
$= (x-7)(x+7).\frac{5x+2}{x+7}$
$= (x-7)(5x+2)$

(iii). $\frac{x^6-y^6}{x^2-y^2} \div (x^4+x^2y^2+y^4)$

Sol: $\frac{x^6-y^6}{x^2-y^2} \div (x^4+x^2y^2+y^4)$

$$=\frac{(x^2)^3-(y^2)^3}{(x-y)(x+y)} \times \frac{1}{(x^4+x^2y^2+y^4)}$$

$$=\frac{(x^2-y^2)((x^2)^2+(x^2)(y^2)+(y^2)^2)}{(x-y)(x+y)} \times \frac{1}{(x^4+x^2y^2+y^4)}$$

$$=\frac{(x-y)(x+y)(+x^2y^2+y^4)}{(x-y)(x+y)} \times \frac{1}{(x^4+x^2y^2+y^4)}$$

$$=1$$

(v). $\frac{x^2+xy}{y(x+y)} \cdot \frac{x^2+xy}{y(x+y)} \div \frac{x^2-x}{xy-2y}$

Sol: $\frac{x^2+xy}{y(x+y)} \cdot \frac{y^2+xy}{y(x+y)} \div \frac{x^2-x}{xy-2y}$

$$=\frac{x(x+y)}{y(x+y)} \cdot \frac{x(x+y)}{y(x+y)} \times \frac{xy-2y}{x^2-x}$$

$$=\frac{x}{y} \cdot \frac{x}{y} \times \frac{y(x-2)}{x(x-1)}$$

$$=\frac{x}{y} \times \frac{(x-2)}{(x-1)}$$

$$=\frac{x(x-2)}{y(x-1)}$$

Algebraic formulas

(i). $(a+b)^2+(a-b)^2=2(a^2+b^2)$

(ii). $(a+b)^2-(a-b)^2=4ab$

(iii). $(a+b+c)^2=a^2+b^2+c^2+2(ab+ac+ca)$

(iv) $(x+y)^3=x^3+y^3+3xy(x+y)$

(v) $(x-y)^3=x^3-y^3-3xy(x-y)$

(vi) $\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$

(vii) $x^3+y^3=(x+y)(x^2-xy+y^2)$

(viii) $x^3-y^3=(x-y)(x^2+xy+y^2)$

Exercise 4.2

Q#3) If $m+n+p=10$ and $mn+np+mp=27$, then find the value of $m^2+n^2+p^2$

Solution: As given $m+n+p=10$ and $mn+np+mp=27$

We find $m^2+n^2+p^2$=?

Using the identity

$(m+n+p)^2=m^2+n^2+p^2+2(mn+np+mp)$

Put values

$$(10)^2=m^2+n^2+p^2+2(27)$$
$$100=m^2+n^2+p^2+54$$
$$100-54=m^2+n^2+p^2$$
$$m^2+n^2+p^2=46$$

Which is required.

Q#5) If $x^2+y^2+z^2=78$ and $xy+yz+zx=59$, then find the value of $x+y+z$

Solution: As given $x+y+z=12$ and$x^2+y^2+z^2=64$

We find $xy+yz+zx$ =?

Using the identity

$(x+y+z)^2=x^2+y^2+z^2+2(xy+yz+zx)$

Put values

$$(12)^2=64+2(xy+yz+zx)$$
$$144=64+2(xy+yz+zx)$$
$$144-64=2(xy+yz+zx)$$
$$80=2(xy+yz+zx)$$
$$\frac{80}{2}=(xy+yz+zx)$$
$$xy+yz+zx=40$$

Which is required.

Q#8) If $x-y=4$ and $xy=21$, then find the value of x^3-y^3

Solution: As given $x-y=4$ and $xy=21$

We find x^3-y^3 =?

Using the identity

$(x-y)^3=x^3-y^3-3xy(x-y)$

Put values

$$(4)^3=x^3-y^3-3(21)(4)$$
$$64=x^3+y^3-252$$
$$64+252=x^3+y^3$$
$$x^3+y^3=316$$

Which is required value.

Q#11) If $x-\frac{1}{x}=7$ then find the value of $x^3-\frac{1}{x^3}$

Sol: As given $x-\frac{1}{x}=7$

We find $x^3-\frac{1}{x^3}$ =?

Using the identity

$$\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)$$

Put values

$$(7)^3=x^3-\frac{1}{x^3}-3(7)$$
$$343=x^3-\frac{1}{x^3}-21$$
$$343+21=x^3-\frac{1}{x^3}$$
$$x^3-\frac{1}{x^3}=364$$

Q#13) If $\left(5x-\frac{1}{5x}\right)=6$ then find the value of $125x^3-\frac{1}{125x^3}$

Solution: As given $5x-\frac{1}{5x}=6$

We find $125x^3-\frac{1}{125x^3}$ =?

Using the identity

$$\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)$$

It becomes

$$\left(5x-\frac{1}{5x}\right)^3=(3x)^3-\frac{1}{(3x)^3}-\left(5x-\frac{1}{5x}\right)$$
$$\left(5x-\frac{1}{5x}\right)^3=125x^3-\frac{1}{125x^3}+\left(5x-\frac{1}{5x}\right)$$
$$(6)^3=125x^3-\frac{1}{125x^3}-3(6)$$
$$216=125x^3-\frac{1}{125x^3}-18$$
$$216+18=125x^3-\frac{1}{125x^3}$$
$$125x^3-\frac{1}{125x^3}=2$$

Surd:

An irrational radical with rational radicand is called a surd.

That is $\sqrt[n]{a}$ surd if a is rational and $\sqrt[n]{a}$ is irrational.

For example, $\sqrt{2}, \sqrt[4]{5}, \sqrt{10}$

Also, $\sqrt{\pi}$ is not a surd because π is not rational.

$\sqrt{10+\sqrt{2}}$ is not a surd because $10+\sqrt{2}$ is not a rational number.

Exercise 4.3

1. Express each of the following surd in the simplest foam.

(ii) $3\sqrt{162}$

Solution: $3\sqrt{162} = 3\sqrt{2\times 3\times 3\times 3\times 3}$

$= 3\sqrt{2\times 3^2\times 3^2}$

$= 3\times 3\times 3\sqrt{2}$

$= 27\sqrt{2}$

(iv) $\sqrt[5]{96x^6y^7z^8}$

Solution: $\sqrt[5]{96x^6y^7z^8}$

$= \sqrt[5]{2\times 2\times 2\times 2\times 2\times 3\times x^5\times y^5\times z^5\times x\times y^2\times z^3}$

$= \sqrt[5]{2^5\times x^5\times y^5\times z^5\times 3\times x\times y^2\times z^3}$

$= 2\times x\times y\times z\sqrt[5]{3\times x\times y^2\times z^3}$

$= 2xyz\sqrt[5]{3xy^2z^3}$

Q#2) Simplify

(i). $\frac{\sqrt{18}}{\sqrt{3}\sqrt{2}}$

Solution: $\frac{\sqrt{18}}{\sqrt{3}\sqrt{2}} = \frac{\sqrt{2\times 3\times 3}}{\sqrt{3}\sqrt{2}} = \frac{\sqrt{2}\sqrt{3}\sqrt{3}}{\sqrt{3}\sqrt{2}} = \sqrt{3}$

(v). $\sqrt{21}\times\sqrt{7}\times\sqrt{3}$

Solution: $\sqrt{21}\times\sqrt{7}\times\sqrt{3} = \sqrt{7\times 3}\times\sqrt{7}\times\sqrt{3}$

$= \sqrt{7}\times\sqrt{3}\times\sqrt{7}\times\sqrt{3}$

$= (\sqrt{7})^2\times(\sqrt{3})^2$

$= 7\times 3 = 21$

Q#3) Simplify by combining similar terms.

(i). $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5}$

Solution: $\sqrt{45} - 3\sqrt{20} + 4\sqrt{5} = \sqrt{3\times 3\times 5} - 3\sqrt{2\times 2\times 5} + 4\sqrt{5}$

$= \sqrt{3^2\times 5} - 3\sqrt{2^2\times 5} + 4\sqrt{5}$

$= 3\sqrt{5} - 3\times 2\sqrt{5} + 4\sqrt{5}$

$= 3\sqrt{5} - 6\sqrt{5} + 4\sqrt{5} = \sqrt{5}(3-6+4)$

$= \sqrt{5}(1) = \sqrt{5}$

(iv). $2(6\sqrt{5} - 3\sqrt{5})$

Solution: $2(6\sqrt{5} - 3\sqrt{5}) = 2(3\sqrt{5})$

$= 6\sqrt{5}$

Q#4) Simplify

(i). $(3+\sqrt{3})(3-\sqrt{3})$

Sol: $(3+\sqrt{3})(3-\sqrt{3})$

$= (3)^2 - (\sqrt{3})^2 = 9 - 3 = 6$

(iv). $(\sqrt{2}+\frac{1}{\sqrt{3}})(\sqrt{2}-\frac{1}{\sqrt{3}})$

Solution: $(\sqrt{2}+\frac{1}{\sqrt{3}})(\sqrt{2}-\frac{1}{\sqrt{3}})$

$= (\sqrt{2})^2 - \left(\frac{1}{\sqrt{3}}\right)^2$

$= 2 - \frac{1}{3} = \frac{6-1}{3} = \frac{5}{3}$

(v). $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})(x+y)(x^2+y^2)$

Solution: $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})(x+y)(x^2+y^2)$

$= \left((\sqrt{x})^2 - (\sqrt{y})^2\right)(x+y)(x^2+y^2)$

$= (x-y)(x+y)(x^2+y^2)$

$= ((x)^2-(y)^2)(x^2+y^2)$

$= (x^2)^2 - (y^2)^2$

$= x^4 - y^4$

Surd:

An irrational radical with rational radicand is called a surd.

That is $\sqrt[n]{a}$ surd if a is rational and $\sqrt[n]{a}$ is irrational.

For example, $\sqrt{2}, \sqrt[4]{5}, \sqrt{10}$

Also, $\sqrt{\pi}$ is not a surd because π is not rational.

$\sqrt{10+\sqrt{2}}$ is not a surd because $10+\sqrt{2}$ is not a rational number.

Monomial surd:

A surd which contains a single term is called a monomial surd.

e.g., $\sqrt{2}, \sqrt{5}$ etc.

Binomial surd:

A surd which contains sum of two monomial surds or sum of a monomial surd and a rational number is called a binomial surd.

e.g., $\sqrt{2}+\sqrt{7}$ + or $\sqrt{12}-\sqrt{7}$ or $\sqrt{10}-\sqrt{2}$ etc.

We can extend this to the definition of a trinomial surd.

Rationalizing factor of the other

If the product of two surds is a rational number, then each surd is called the rationalizing factor of the other.

Rationalization

The process of multiplying a given surd by its rationalizing factor to get a rational number as product is called rationalization of the given surd.

Conjugate surd

Two binomial surds of second order differing only in sign connecting their terms are called conjugate surds. Thus $(\sqrt{a}+\sqrt{b})$ and $(\sqrt{a}-\sqrt{b})$ are conjugate surds of each other.

The conjugate of $x+\sqrt{y}$ is $x-\sqrt{y}$.

The product of the conjugate surds $\sqrt{x}+\sqrt{y}$and $\sqrt{x}-\sqrt{y}$,

$$(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y}) = (\sqrt{x})^2 - (\sqrt{y})^2 = x - y$$

is a rational quantity independent of any radical.

Similarly, the product of $x+m\sqrt{y}$ nd its conjugate $x-m\sqrt{y}$ has

$$(x+m\sqrt{y})(x-m\sqrt{y}) = (x)^2 - (m\sqrt{y})^2 = x^2 - m^2y$$

and have no radical. For example,

$$(4+\sqrt{3})(4-\sqrt{3}) = (4)^2 - (\sqrt{3})^2 = 16 - 3 = 13$$

which is a rational number.

Exercise 4.4

1. Rationalize the denominator of the following.

(i) $\frac{3}{4\sqrt{3}}$

Sol: $\frac{3}{4\sqrt{3}} = \frac{3}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{3}}{4(\sqrt{3})^2} = \frac{3\sqrt{3}}{4\times 3} = \frac{\sqrt{3}}{4}$

(vi) $\frac{2}{\sqrt{5}-\sqrt{3}}$

Solution: $\frac{2}{\sqrt{5}-\sqrt{3}} = \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$$= \frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{2(\sqrt{5}+\sqrt{3})}{5-3}$$

$$= \frac{2(\sqrt{5}+\sqrt{3})}{2} = \sqrt{5}+\sqrt{3}$$

(vii) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

Solution: $\frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}$$

$$= \frac{(\sqrt{3})^2+(1)^2-2(\sqrt{3})(1)}{3-1} = \frac{3+1-2\sqrt{3}}{2}$$

$$= \frac{4-2\sqrt{3}}{2} = \frac{2(2-\sqrt{3})}{2}$$

$$= 2-\sqrt{3}$$

Question No.2 Find the conjugate of $x+\sqrt{y}$.

(iii). $2+\sqrt{3}$

Solution: Let $z = 2+\sqrt{3}$

Taking conjugate, we get

$$\bar{z} = \overline{2+\sqrt{3}}$$

$$\bar{z} = 2-\sqrt{3}$$

(vii). $7-\sqrt{6}$

Solution: Let $z = 7-\sqrt{6}$

Taking conjugate, we get

$$\bar{z} = \overline{7-\sqrt{6}}$$

$$\bar{z} = 7+\sqrt{6}$$

(viii). $9+\sqrt{2}$

Solution: Let $z = 9+\sqrt{2}$

Taking conjugate, we get

$$\bar{z} = \overline{9+\sqrt{2}}$$

$$\bar{z} = 9-\sqrt{2}$$

Q#3)

(iii). If $x = \sqrt{3}+2$ find $\frac{1}{x}$

Solution: $x = \sqrt{3}+2$

And $\frac{1}{x} = \frac{1}{\sqrt{3}+2} = \frac{1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2}$

$$= \frac{\sqrt{3}-2}{(\sqrt{3})^2-(2)^2}$$

$$= \frac{\sqrt{3}-2}{3-4} = \frac{\sqrt{3}-2}{-1}$$

$$= -(\sqrt{3}-2) = -\sqrt{3}+2 = 2-\sqrt{3}$$

Q#5)

(i). If $x = 2+\sqrt{3}$ find $x-\frac{1}{x}$ and $\left(x-\frac{1}{x}\right)^2$

Solution: $x = 2+\sqrt{3}$

And $\frac{1}{x} = \frac{1}{2+\sqrt{3}} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

$$= \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2} = \frac{2-\sqrt{3}}{4-3} = \frac{2-\sqrt{3}}{1}$$

$$= 2-\sqrt{3}$$

Now, $x-\frac{1}{x} = (2+\sqrt{3})-(2-\sqrt{3})$

$$= 2+\sqrt{3}-2+\sqrt{3}$$

$$x-\frac{1}{x} = 2\sqrt{3}$$

Also, $\left(x-\frac{1}{x}\right)^2 = (2\sqrt{3})^2 = 4\times 3 = 12$

(i). If $x = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$ find $x+\frac{1}{x}$, $x^2+\frac{1}{x^2}$ and $x^3+\frac{1}{x^3}$

Solution: $x = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$$= \frac{(\sqrt{5}-\sqrt{2})^2}{(\sqrt{5})^2-(\sqrt{2})^2}$$

$$= \frac{(\sqrt{5})^2+(\sqrt{2})^2-2(\sqrt{5})(\sqrt{2})}{5-2}$$

$$= \frac{5+2-2\sqrt{10}}{3}$$

$$= \frac{1}{3}(7-2\sqrt{10})$$

And $\frac{1}{x} = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

$$= \frac{(\sqrt{5}+\sqrt{2})^2}{(\sqrt{5})^2-(\sqrt{2})^2}$$

$$= \frac{(\sqrt{5})^2+(\sqrt{2})^2+2(\sqrt{5})(\sqrt{2})}{5-2}$$

$$= \frac{5+2+2\sqrt{10}}{3}$$

$$= \frac{1}{3}(7+2\sqrt{10})$$

Now, $x+\frac{1}{x} = \frac{1}{3}(7-2\sqrt{10})+\frac{1}{3}(7+2\sqrt{10})$

$$= \frac{1}{3}(7-2\sqrt{10}+7+2\sqrt{10})$$

$$x+\frac{1}{x} = \frac{14}{3}$$

Using identity

$$\left(x+\frac{1}{x}\right)^2 = x^2+\frac{1}{x^2}+2$$

Putting values

$$\left(\frac{14}{3}\right)^2 = x^2+\frac{1}{x^2}+2$$

$$\frac{196}{9} = x^2+\frac{1}{x^2}+2$$

$$x^2+\frac{1}{x^2} = \frac{196}{9}-2$$

$$x^2+\frac{1}{x^2}=\frac{196-18}{9}$$
$$x^2+\frac{1}{x^2}=\frac{178}{9}$$

Also using the identity

$$\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$$

Putting values

$$\left(\frac{14}{3}\right)^3=x^3+\frac{1}{x^3}+3\left(\frac{14}{3}\right)$$
$$\frac{2744}{27}=x^3+\frac{1}{x^3}+14$$
$$x^3+\frac{1}{x^3}=\frac{2744}{27}-14$$
$$x^3+\frac{1}{x^3}=\frac{2366}{27}$$

Review Exercise -4

Multiple Choice Questions. Choose the correct answer.

1. **$4x + 3y - 2$ is an algebraic___**
   (a) Expression (b) Sentence
   (c) Equation (d) In equation
2. **The degree of polynomial $4x^4+2x^2y$ is ____**
   (a)1 (b)2
   (c)3 (d)4
3. **$a^3 + b^3$ is equal to____**
   (a) $(a-b)(a^2+ab+b^2)$
   (b) $(a+b)(a^2-ab+b^2)$
   (c) $(a-b)(a^2-ab+b^2)$
   (d) $(a-b)(a^2+ab-b^2)$
4. $\left(3+\sqrt{2}\right)\left(3-\sqrt{2}\right)$ **is equal to:___**
   (a) 7 (b) -7
   (c) -1 (d) 1
5. **Conjugate of Surd** $a+\sqrt{b}$ **is_**
   (a) $-a+\sqrt{b}$ (b) $a-\sqrt{b}$
   (d) $\sqrt{a}+\sqrt{b}$ (d) $\sqrt{a}-\sqrt{b}$
6. $\frac{1}{a-b}-\frac{1}{a+b}$ **is equal to**
   (a) $\frac{2a}{a^2-b^2}$ (b) $\frac{2b}{a^2-b^2}$
   (c) $\frac{-2a}{a^2-b^2}$ (d) $\frac{-2b}{a^2-b^2}$
7. $\frac{a^2-b^2}{a+b}$ **is equal to:**
   (a) $(a-b)^2$ (b) $(a+b)^2$
   (c) a+b (d) a–b
8. $\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{a}-\sqrt{b}\right)$ is equal to:__
   (a)$a^2 + b^2$ (b) $a^2 - b^2$
   (c)a – b (d) a + b

<u>**Additional MCQ**</u>

9. **The degree of the polynomial $x^2y^2+3xy+y^3$ is ___**
   (a) 4 (b) 5
   (c) 6 (d) 2
10. **x^2-4 = ...**
    (a) $(x-2)(x+2)$ (b) $(x-2)(x-2)$
    (c) $(x+2)(x+2)$ (d) $(x-2)^2$
11. $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)(..............)$
    (a) $x^2-1+\frac{1}{x^2}$ (b) $x^2+1+\frac{1}{x^2}$
    (c) $x^2+1-\frac{1}{x^2}$ (d) $x^2-1-\frac{1}{x^2}$
12. $\frac{1}{2-\sqrt{3}}=$____
    (a) $2+\sqrt{3}$ (b) $2-\sqrt{3}$
    (d) $-2+\sqrt{3}$ (d) $-2-\sqrt{3}$
13. **$(a+b)^2-(a-b)^2$ = ________**
    (a) $2(a^2+b^2)$ (b) $4ab$
    (c) $2ab$ (d) $3ab$
14. **A surd which contains a single term is called _______surd.**
    (a) Monomial (b) Binomial
    (c) Trinomial (d) Conjugate
15. **What is the leading coefficient of polynomial $3x^2+8x+5$?**
    (a) 2 (b) 3
    (c) 5 (d) 8
16. **A surd which contains two terms is called _______surd.**
    (a) Monomial (b) Binomial
    (c) Trinomial (d) Conjugate
17. **Which of the following is polynomial?**
    (a) $3x^2+\frac{1}{x}$ (b) $4x^2-3\sqrt{x}$
    (c) $x^2-3x+\sqrt{2}$ (d) $2x^2+3x^{-1}$
18. $\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)=$ ________
    (a) 12 (b) 9
    (c) 6 (d) 3
19. **Which of the following is not surd?**
    (a) $\sqrt{2}$ (b) $\sqrt{3}$
    (c) $\sqrt{2}+5$ (d) $\sqrt{\pi}$
20. **In the polynomial with the variable x, all the powers of x are------ integers.**
    (a) non-negative (b) negative
    (c) non-positive (d) none of these
21. **Polynomial means an expression with:**
    (a) one term (b) two terms
    (c) three terms (d) many term

Answer key

1	a	2	d	3	b	4	a	5	b	6	b	7	d
8	c	9	a	10	a	11	a	12	a	13	b	14	a
15	b	16	b	17	c	18	c	19	d	20	a	21	d

Question No.4 (ii)

If $x-\frac{1}{x}=2$ find $x^4+\frac{1}{x^4}$

Taking square on both sides we get

$$\left(x-\frac{1}{x}\right)^2=(2)^2$$

$$x^2+\frac{1}{x^2}-2=4$$

$$x^2+\frac{1}{x^2}=4+2=6$$

again square both sides by

$$\left(x^2+\frac{1}{x^2}\right)^2=6^2$$

$$(x^2)^2+\left(\frac{1}{x^2}\right)^2+2(x^2)\left(\frac{1}{x^2}\right)=36$$

$$x^4+\frac{1}{x^4}+2=36$$

$$\Rightarrow x^4+\frac{1}{x^4}=36-2=34$$

Question No.5 Find the value of x^2+y^2 and xy if $x+y=5$ and $x-y=3$

Solution:

Given:

$$x+y=5, x-y=3, x^2+y^2=?\, xy=?$$

We know that

$$2(x^2+y^2)=(5)^2+(3)^2+25+9=34$$

$$x^2+y^2=\frac{34}{2}=17$$

We know that

$$4xy=(x+y)^2-(x-y)^2$$

$$4xy=(5)^2-(3)^2=25-9=16$$

$$xy=\frac{16}{4}=4$$

Question No.6 (iv)

$$p^2-\frac{1}{p^2}$$

Solution:

$$p^2-\frac{1}{p^2}=\left(p+\frac{1}{p}\right)\left(p-\frac{1}{p}\right)=(4)(2\sqrt{3})=8\sqrt{3}$$

Question No.7 (iii)

$$q^2+\frac{1}{q^2}$$

Solution:

$$q+\frac{1}{q}=2\sqrt{5}$$

Taking square on both sides, we get

$$\left(q+\frac{1}{q}\right)^2=(2\sqrt{5})^2$$

$$q^2+\frac{1}{q^2}+2(q)\left(\frac{1}{q}\right)=4\times 5$$

$$q^2+\frac{1}{q^2}+2=20$$

$$q^2+\frac{1}{q^2}=20-2=18$$

Question No.8 (i)

Simplify

$$\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}$$

Solution:

$$\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}$$

$$\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}\times\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}+\sqrt{a^2-2}}$$

$$=\frac{\left(\sqrt{a^2+2}+\sqrt{a^2-2}\right)^2}{\left(\sqrt{a^2+2}\right)^2-\left(\sqrt{a^2-2}\right)^2}$$

$$=\frac{\left(\sqrt{a^2+2}\right)^2+\left(\sqrt{a^2-2}\right)^2+2\left(\sqrt{a^2+2}\right)\left(\sqrt{a^2-2}\right)}{(a^2+2)-(a^2-2)}$$

$$=\frac{a^2+2+a^2-2+2\sqrt{a^4-4}}{a^2+2-a^2+2}$$

$$=\frac{2a^2+2\sqrt{a^4-4}}{4}$$

$$=\frac{2\left(a^2+\sqrt{a^4-4}\right)}{4}=\frac{a^2+\sqrt{a^4-4}}{2}$$

Unit-5

[FACTORIZATION]

Introduction:

Factorization plays an important role in mathematics as it helps to reduce the study of complicated expressions to the study of simpler expressions. In this unit, we will deal different types of factorization of polynimials.

Factorization:

If a polynomial $p(x)$ can be expressed as $p(x) = g(x)h(x)$, then each of the polynomial $g(x) and\ h(x)$ is called a factor of $f(x)$.

Factorization of the expression of the type $ka + kb + kc$.

We will take common k from every term of the expression

$$ka + kb + kc = k(a + b + c)$$

Factorization of the type $ac + ad + bc + bd$

$$ac + ad + bc + bd = a(c + d) + b(c + d)$$
$$= (c + d)(a + b)$$

Factorization of the type $a^2 \pm 2ab + b^2$

(i). $a^2 + 2ab + b^2 = (a + b)^2 = (a + b)(a + b)$

(ii). $a^2 - 2ab + b^2 = (a - b)^2$
$$= (a - b)(a - b)$$

Factorization of the type $a^2 - b^2$

$$a^2 - b^2 = (a + b)(a - b)$$

Factorization of the type $a^2 \pm 2ab + b^2 - c^2$

$$a^2 \pm 2ab + b^2 - c^2 = (a \pm b)^2 - c^2$$
$$= (a \pm b + c)(a \pm b - c)$$

Exercise 5.1

Factorize.Question.1.

(ii). $9xy - 12x^2y + 18y^2$

Solution.

$$9xy - 12x^2y + 18y^2 = 3y(3x - 4x^2 + 6y)$$

Answer.

(v). $3x^3y(x - 3y) - 7x^2y^2(x - 3y)$

Solution.

$$3x^3\, y(x - 3y) - 7x^2\, y^2\, (x - 3y)$$
$$= x^2y(x - 3y)(3x - 7y)$$

Answer.

(vi). $2xy^3(x^2 + 5) + 8xy^2(x^2 + 5)$

Solution.

$2xy^3(x^2 + 5) + 8xy^2(x^2 + 5) = 2xy^2(x^2 + 5)(y + 4)$

Answer.

Question.3.

(ii). $\frac{a^2}{b^2} - 2 + \frac{b^2}{a^2}$

Solution.

$$\frac{a^2}{b^2} - 2 + \frac{b^2}{a^2} = \left(\frac{a}{b}\right)^2 - 2\left(\frac{a}{b}\right)\left(\frac{b}{a}\right) + \left(\frac{b}{a}\right)^2$$
$$= \left(\frac{a}{b} - \frac{b}{a}\right)^2$$

(iv). $12x^2 - 36x + 27$

Solution.

$$12x^2 - 36x + 27 = 3(4x^2 - 12x + 9)$$
$$= 3[(2x)^2 - 2(2x)(3) + (3)^2]$$
$$= 3(2x - 3)^2$$

Question.4.

(ii). $x(x - 1) - y(y - 1)$

Solution.

$$x(x - 1) - y(y - 1) = x^2 - x - y^2 + y$$
$$= x^2 - y^2 - x + y$$
$$= (x + y)(x - y) - 1(x - y)$$
$$= (x - y)(x + y - 1)$$

(iv). $3x - 243x^3$

Solution.

$$3x - 243x^3 = 3x(1 - 81x^2)$$
$$= 3x[(1)^2 - (9x)^2]$$
$$= 3x(1 + 9x)(1 - 9x)$$

Question.5.

(ii). $x^2 - a^2 + 2a - 1$

Solution.

$$x^2 - a^2 + 2a - 1 = x^2 - (a^2 - 2a + 1)$$
$$= x^2 - [(a)^2 - 2(a)(1) + (1)^2]$$
$$= (x)^2 - (a - 1)^2$$
$$= [x + (a - 1)][x - (a - 1)]$$
$$= (x + a - 1)(x - a + 1)$$

(

(v). $25x^2 - 10x + 1 - 36z^2$

Solution.

$$25x^2 - 10x + 1 - 36z^2$$
$$= [(5x)^2 - 2(5x)(1) + (1)^2] - (6z)^2$$
$$= (5x - 1)^2 - (6z)^2$$
$$= (5x - 1 + 6z)(5x - 1 - 5z)$$
$$= (5x + 6z - 1)(5x - 5z - 1)$$

(vi). $x^2 - y^2 - 4xz + 4z^2$

Solution.

$$x^2 - y^2 - 4xz + 4z^2 = x^2 - 4xz + 4z^2 - y^2$$
$$= (x)^2 - 2(x)(z) + (2z)^2 - y^2$$
$$= (x - 2z)^2 - (y)^2$$
$$= (x - 2z + y)(x - 2z - y)$$
$$= (x + y - 2z)(x - y - 2z)$$

(a) Factorization of the Expression of the types:

$a^4 + a^2b^2 + b^4$ or $a^4 + 4b^4$

Explanation: For $a^4 + a^2b^2 + b^4$

$$a^4 + a^2b^2 + b^4 = a^4 + b^4 + a^2b^2$$
$$= (a^2)^2 + (b^2)^2 + 2(a^2)(b^2) - 2(a^2)(b^2) + a^2b^2$$
$$= (a^2 + b^2)^2 - 2a^2b^2 + a^2b^2$$
$$= (a^2 + b^2)^2 - a^2b^2$$
$$= (a^2 + b^2)^2 - (ab)^2$$
$$= (a^2 + b^2 + ab)(a^2 + b^2 - ab)$$

Explanation: For $a^4 + 4b^4$

$$a^4 + 4b^4 = (a^2)^2 + (2b^2)^2 + 2(a^2)(2b^2) - 2(a^2)(2b^2)$$
$$= (a^2 + 2b^2)^2 - 4a^2b^2$$
$$= (a^2 + 2b^2)^2 - (2ab)^2$$
$$= (a^2 + 2b^2 + 2ab)(a^2 + 2b^2 - 2ab)$$

(b) Factorization of the Expression of the types:

$x^2 + px + q$

Explanation:

$$x^2 + px + q = x^2 + (s + r)x + q \ ,$$
$$where\ s + r = p\ and\ s \times r = q$$
$$= x^2 + sx + rx + s \times r$$
$$= x(x + s) + r(x + s)$$
$$= (x + s)(x + r)$$

(c) Factorization of the Expression of the types:

$$ax^2 + bx + c, a \neq 0$$
$$ax^2 + bx + c = ax^2 + (s + r)x + c,$$
$$s + r = b\ and\ s \times r = ac$$
$$= ax^2 + sx + rx + \frac{s \times r}{a}$$
$$= x(ax + s) + r\left(x + \frac{s}{a}\right)$$
$$= x(ax + s) + r\left(\frac{ax + s}{a}\right)$$
$$= x(ax + s) + \frac{r}{a}(ax + s)$$
$$= (ax + s)\left(x + \frac{r}{a}\right)$$

(d) Factorization of the Expression of the types:

(i). $(ax^2 + bx + c)(ax^2 + bx + d) + k$

(ii). $(x + a)(x + b)(x + c)(x + d) + k$

(iii). $(x + a)(x + b)(x + c)(x + d) + kx^2$

Explanation: For $(ax^2 + bx + c)(ax^2 + bx + d) + k$

$$(ax^2 + bx + c)(ax^2 + bx + d) + k$$

We will suppose $ax^2 + bx = y$, *then above becomes*

$$= (y + c)(y + d) + k$$
$$= y^2 + yd + yc + k$$
$$= y^2 + (d + c)y + k$$

This the same type (b).

Explanation: For $(x + a)(x + b)(x + c)(x + d) + k$

$$(x + a)(x + b)(x + c)(x + d) + k$$

We will multiply the pair for which $a + b = c + d$, then

$$= [(x + a)(x + b)][(x + c)(x + d)] + k$$
$$= [x^2 + bx + ax + ab][x^2 + dx + cx + cd] + k$$
$$= (x^2 + (b + a)x + ab)(x^2 + (d + c)x + cd) + k$$

As $a + b = c + d$, *then*

$$= [x^2 + (c + d)x + ab][x^2 + (c + d)x + cd] + k$$

Suppose that

$x^2 + (c + d)x = y$, *then above expression becomes*

$$= (y + ab)(y + cd) + k$$
$$= y^2 + ycd + yab + abcd + k$$
$$= y^2 + (cd + ab)y + abcd + k$$

This the same type (b).

Explanation: For $(ax^2 + bx + c)(ax^2 + bx + d) + kx^2$

$$(x + a)(x + b)(x + c)(x + d) + kx^2$$

We will multiply the pair for which $a + b = c + d$, then

$$= [(x + a)(x + b)][(x + c)(x + d)] + kx^2$$
$$= [x^2 + bx + ax + ab][x^2 + dx + cx + cd] + kx^2$$
$$= (x^2 + (b + a)x + ab)(x^2 + (d + c)x + cd) + kx^2$$

As $a + b = c + d$, *then*

$$= [x^2 + (c + d)x + ab][x^2 + (c + d)x + cd] + kx^2$$

Suppose that

$x^2 + (c + d)x = y$, *then above expression becomes*

$$= (y + ab)(y + cd) + kx^2$$
$$= y^2 + ycd + yab + abcd + kx^2$$
$$= y^2 + (cd + ab)y + abcd + kx^2$$

After simplification it also becomes type (b).

(e). Factorization of the Expression of the type:

(i). $a^3 + 3a^2b + 3ab^2 + b^3$

(ii). $a^3 - 3a^2b + 3ab^2 - b^3$

Explanation For $a^3 + 3a^2b + 3ab^2 + b^3$

$$a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3$$

It's a very famous formula.

Explanation For $a^3 - 3a^2b + 3ab^2 - b^3$

$$a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3$$

It's a very famous formula.

(f). Factorization of the Expression of the type:

(i). $a^3 + b^3$

(ii). $a^3 - b^3$

We will use, well known formulas for these types

(i). $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
(ii). $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Exercise 5.2

Factorize

Question.1.

(iii). $a^4 + +3a^2b^2 + 4b^4$

Solution.

$$a^4 + +3a^2b^2 + 4b^4 = a^4 + 4b^4 + 3a^2b^2$$
$$= (a^2)^2 + (2b^2)^2 + 2(a^2)(2b^2) - 2(a^2)(2b^2) + 3a^2\,b^2$$
$$= (a^2 + 2b^2)^2 - 4a^2b^2 + 3a^2b^2$$
$$= (a^2 + 2b^2)^2 - a^2b^2$$
$$= (a^2 + 2b^2)^2 - (ab)^2$$
$$= (a^2 + 2b^2 + ab)(a^2 + 2b^2 - ab)$$

(v). $x^4 + x^2 + 25$

Solution.

$$x^4 + x^2 + 25 = x^4 + 25 + x^2$$
$$= (x^2)^2 + (5)^2 + 2(x^2)(5) - 2(x^2)(5) + x^2$$
$$= (x^2 + 5)^2 - 10x^2 + x^2$$
$$= (x^2 + 5)^2 - 9x^2$$
$$= (x^2 + 5)^2 - (3x)^2$$
$$= (x^2 + 5 + 3x)(x^2 + 5 - 3x)$$
$$= (x^2 + 3x + 5)(x^2 - 3x + 5)$$

Question.2.

(i). $x^2 + 14x + 48$

Solution.

$$x^2 + 14x + 48 = x^2 + 8x + 6x + 48$$
$$= x(x + 8) + 6(x + 8)$$
$$= (x + 8)(x + 6)$$

(iv). $x^2 + x - 132$

Solution.

$$x^2 + x - 132 = x^2 + 12x - 11x - 132$$
$$= x(x + 12) - 11(x + 12)$$
$$= (x + 12)(x - 11)$$

Question.3.

(iv). $5x^2 - 16x - 21$

Solution.

$$5x^2 - 16x - 21 = 5x^2 - 21x + 5x - 21$$
$$= x(5x - 21) + 1(5x - 21)$$
$$= (5x - 21)(x + 1)$$

(v). $4x^2 - 17xy + 4y^2$

Solution.

$$4x^2 - 17xy + 4y^2 = 4x^2 - 16xy - xy + 4y^2$$
$$= 4x(x - 4y) - y(x - 4y)$$
$$= (x - 4y)(4x - y)$$

Question.4

(iii). $(x + 2)(x + 3)(x + 4)(x + 5) - 15$

Solution.

$$(x + 2)(x + 3)(x + 4)(x + 5) - 15$$
$$= (x + 2)(x + 5)(x + 3)(x + 4) - 15$$
$$= (x^2 + 5x + 2x + 10)(x^2 + 4x + 3x + 12) - 15$$
$$= (x^2 + 7x + 10)(x^2 + 7x + 12) - 15$$

Suppose that $x^2 + 7x = y$

$$= (y + 10)(y + 12) - 15$$
$$= y^2 + 12y + 10y + 120 - 15$$
$$= y^2 + 22y + 105$$
$$= y^2 + 15y + 7y + 105$$
$$= y(y + 15) + 7(y + 15)$$
$$= (y + 15)(y + 7)$$
$$(x^2 + 7x + 15)(x^2 + 7x + 7)$$

(v). $(x + 1)(x + 2)(x + 3)(x + 6) - 3x^2$

Solution.

$$(x + 1)(x + 2)(x + 3)(x + 6) - 3x^2$$
$$= (x + 1)(x + 6)(x + 2)(x + 3) - 3x^2$$
$$= (x^2 + 6x + x + 6)(x^2 + 3x + 2x + 6) - 3x^2$$
$$= (x^2 + 7x + 6)(x^2 + 5x + 6) - 3x^2$$
$$= (x^2 + 6 + 7x)(x^2 + 6 + 5x) - 3x^2$$

Suppose that $x^2 + 6 = y$

$$= (y + 7x)(y + 5x) - 3x^2$$
$$= y^2 + 5xy + 7xy + 35x^2 - 3x^2$$
$$= y^2 + 12xy + 32x^2$$
$$= y^2 + 8xy + 4xy + 32x^2$$
$$= y(y + 8x) + 4x(y + 8x)$$
$$= (y + 8x)(y + 4x)$$
$$= (x^2 + 6 + 8x)(x^2 + 6 + 4x)$$
$$= (x^2 + 8x + 6)(x^2 + 4x + 6)$$

Question.5.

(ii). $8x^3 + 60x^2 + 150x + 125$

Solution.

$$8x^3 + 60x^2 + 150x + 125$$
$$= 8x^3 + 60x^2 + 150x + 5^3$$
$$= (2x)^3 + 3(2x)^2(5) + 3(2x)(5)^2 + (5)^3$$
$$= (2x + 5)^3$$

(iv). $8x^3 - 125y^3 - 60x^2y + 150xy^2$

Solution.

$$8x^3 - 125y^3 - 60x^2y + 150xy^2$$
$$= 8x^3 - 60x^2y + 150xy^2 - 125y^3$$
$$= (2x)^3 - 3(2x)^2(5y) + 3(2x)(5y)^2 - (5y)^3$$
$$= (2x - 5y)^3$$

Question.6.

(i). $27 + 8x^3$

Solution.

$$27 + 8x^3 = (3)^3 + (2x)^3$$
$$= (3 + 2x)[(3)^2 - (3)(2x) + (2x)^2]$$
$$= (3 + 2x)(9 - 6x + 4x^2)$$
$$= (2x + 3)(4x^2 - 6x + 9)$$

(iv). $8x^3 + 125y^3$

Solution.

$$8x^3 + 125y^3 = (2x)^3 + (5y)^3$$
$$= (2x + 5y)[(2x)^2 - (2x)(5y) + (5y)^2]$$

$$= (2x + 5y)(4x^2 - 10xy + 25y^2)$$

Remainder Theorem and Factor Theorem:

Remainder Theorem:

If a polynomial $p(x)$ is divided by a linear divisor $(x - a)$, then the remainder is $p(a)$.

Zero of a Polynomial:

If a specific number $x = a$ is substituted for the variable x in a polynomial $p(x)$ so that the value $p(a)$ is zero, then $x = a$ is called a zero of the polynomial $p(x)$.

Factor Theorem:

The polynomial $(x - a)$ is a factor of the polynomial $p(x)$ if and only if $p(a) = 0$.

Exercise 5.3

Question.1. Use the remainder theorem to find the remainder when

(i). $\boldsymbol{3x^3 - 10x^2 + 13x - 6}$ ***is divided by*** $(\boldsymbol{x - 2})$

Solution.

Suppose that $p(x) = \boldsymbol{3x^3 - 10x^2 + 13x - 6}$ ***and***

$$\boldsymbol{x - 2 = 0}$$
$$\boldsymbol{x = 2}$$

Then

$$\boldsymbol{Remainder = p(2)}$$
$$\boldsymbol{= 3(2)^3 - 10(2)^2 + 13(2) - 6}$$
$$Remainder = 3(8) - 10(4) + 26 - 6$$
$$= 24 - 40 + 20$$
$$= 44 - 40$$
$$= 4$$

Hence required Remainder is 4.

(iv). $\boldsymbol{(2x - 1)^3 + 6(3 + 4x)^2 - 10}$ ***is divided by*** $(\boldsymbol{2x + 1})$

Solution.

Suppose that $p(x) = \boldsymbol{(2x - 1)^3 + 6(3 + 4x)^2 - 10}$ ***and***

$$\boldsymbol{2x + 1 = 0}$$
$$\boldsymbol{2x = -1}$$
$$\boldsymbol{x = -\frac{1}{2}}$$

Then

$$\boldsymbol{Remainder = p\left(-\frac{1}{2}\right)}$$
$$\boldsymbol{= \left(2\left(-\frac{1}{2}\right) - 1\right)^3 + 6\left(3 + 4\left(-\frac{1}{2}\right)\right)^2 - 10}$$
$$Remainder = (-1 - 1)^3 + 6(3 - 2)^2 - 10$$
$$= (-2)^3 + 6(1)^2 - 10$$
$$= -8 + 6 - 10$$
$$= 6 - 18$$
$$= -12$$

Hence required Remainder is -12.

(v). $\boldsymbol{x^3 - 3x^2 + 4x - 14}$ ***is divided by*** $(\boldsymbol{x + 2})$

Solution.

Suppose that $p(x) = \boldsymbol{x^3 - 3x^2 + 4x - 14}$ ***and***

$$\boldsymbol{x + 2 = 0}$$
$$\boldsymbol{x = -2}$$

Then

$$\boldsymbol{Remainder = p(-2)}$$
$$\boldsymbol{= (-2)^3 - 3(-2)^2 + 4(-2) - 14}$$
$$Remainder = -8 - 3(4) - 8 - 14$$
$$= -8 - 12 - 8 - 14$$
$$= -42$$

Hence required Remainder is -42.

Question.4.

For what value of m is the polynomial $p(x) = 4x^3 - 7x^2 + 6x - 3m$ exactly divisible by $(x + 2)$?

Solution.

Suppose that

$$p(x) = 4x^3 - 7x^2 + 6x - 3m$$

And $\quad x + 2 = 0$

$$x = -2.$$

Remainder for $x + 2$ *is*

$$p(-2) = 4(-2)^3 - 7(-2)^2 + 6(-2) - 3m$$
$$= 4(-8) - 7(4) - 12 - 3m$$
$$= -32 - 28 - 12 - 3m$$
$$= -72 - 3m$$

For the given condition $p(-2) = 0$

$$-72 - 3m = 0$$
$$-3m = 72$$
$$m = -\frac{72}{3} = -24.$$

Rational Root Theorem

Let

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n, \quad a_n \neq 0 \ldots\ldots (i)$$

Be a polynomial equation of degree n with integral coefficients . If $\frac{p}{q}$ is a rational root of the equation, then p is a factor of the constant term a_n and q is the factor of *leading coefficient* a_0.

Exercise 5.4

Factorize each of the following cubic polynomials by factor theorem.

Q#2) $\boldsymbol{x^3 - x^2 - 22x + 40}$

Solution: Let $P(x) = x^3 - x^2 - 22x + 40 \ldots (1)$

Here, the constant term is 40 and factors of constant terms are $\pm1, \pm2, \pm4, \pm5$ ...

Therefore, we check $\pm1, \pm2, \pm4, \pm5$ for the roots.

Now, put $x = 1$ in (1), we have
$P(1) = (1)^3 - (1)^2 - 22(1) + 40$
$P(1) = 1 - 1 - 22 + 40$
$P(1) = 18 \neq 0$
Hence, $x = 1$ is the not the root of $P(x)$,
Now, put $x = 2$ in (1), we have
$P(2) = (2)^3 - (2)^2 - 22(2) + 40$
$P(2) = 8 - 4 - 44 + 40$
$P(2) = 48 - 48 = 0$
Hence, $x = 2$ is the root of $P(x)$, therefore $(x - 2)$ is the factor of $P(x)$.
Now, put $x = 4$ in (1), we have
$P(4) = (4)^3 - (4)^2 - 22(4) + 40$
$P(4) = 64 - 16 - 88 + 40$
$P(4) = 104 - 104 = 0$
Hence, $x = 4$ is the root of $P(x)$, therefore $(x - 1)$ is the factor of $P(x)$.
Now, put $x = -5$ in (1), we have
$P(-5) = (-5)^3 - (-5)^2 - 22(-5) + 40$
$P(-5) = -125 - 25 + 110 + 40$
$P(-5) = 150 - 150 = 0$
Hence, $x = -5$ is the root of $P(x)$, therefore $(x - (-5)) = (x + 5)$ is the factor of $P(x)$.
Thus, $P(x) = (x - 2)(x - 4)(x + 5)$

Q#8) $\mathbf{2x^3 + x^2 - 2x - 1}$

Solution: Let $P(x) = 2x^3 + x^2 - 2x - 1 \ldots(1)$
Here, the constant term is 1 and factors of constant terms are ± 1.
Therefore, we check ± 1 for the roots.
Now, put $x = 1$ in (1), we have
$P(1) = 2(1)^3 + (1)^2 - 2(1) - 1$
$P(1) = 2 + 1 - 2 - 1$
$P(1) = 3 - 3 = 0$
Hence, $x = 1$ is the root of $P(x)$, therefore $(x - 1)$ is the factor of $P(x)$.
Now, put $x = -1$ in (1), we have
$P(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$
$P(-1) = -2 + 1 + 2 - 1$
$P(-1) = -3 + 3 = 0$
Hence, $x = -1$ is the root of $P(x)$, therefore $(x - (-1)) = (x + 1)$ is the factor of $P(x)$.
Since the leading co-efficient is 2, therefore we check $x = -\frac{1}{2}$ and $x = \frac{1}{2}$
First we check at $x = -\frac{1}{2}$

$$P\left(-\frac{1}{2}\right) = 2\left(-\frac{1}{2}\right)^3 + \left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) - 1$$
$$P\left(-\frac{1}{2}\right) = 2\left(\frac{1}{8}\right) - \left(\frac{1}{4}\right) + 1 - 1$$
$$P\left(-\frac{1}{2}\right) = \frac{1}{4} - \frac{1}{4} + 1 - 1 = 0$$

Hence, $x = -\frac{1}{2}$ is the root of $P(x)$, therefore $\left(x + \frac{1}{2}\right) = 0$ gives that $(2x + 1)$ is the factor of $P(x)$.
Thus, $P(x) = (x - 1)(x + 1)(2x + 1)$

Review Exercise -5

1. The factor of $x^2 - 5x + 6$ are: __
(a) $x + 1, x - 6$ (b) $x - 2, x - 3$
(c) $x + 6, x - 1$ (d) $x + 2, x + 3$

2. Factors of $8x^3 + 27y^3$ are:___
(a) $(2x+3y)(4x^2 - 9y^2)$
(b) $(2x-3y)(4x^2 - 9y^2)$
(c) $(2x+3y)(4x^2 - 6xy + 9y^2)$
(d) $(2x-3y)(4x^2 + 6xy + 9y^2)$

3. Factors of $3x^2 - x - 2$ are:
(a) $(x+1)(3x-2)$ (b) $(x+1)(3x+2)$
(c) $(x-1)(3x-2)$ (d) $(x-1)(3x+2)$

4. Factors of $a^4 - 4b^4$ are: ___
(a) $(a-b)(a+b)(a^2+4b^2)$
(b) $(a^2-2b^2)(a^2+2b^2)$
(c) $(a-b)(a+b)(a^2-4b^2)$
(d) $(a-2b)(a^2+2b^2)$

5. What will be added to complete the square of $9a^2 - 12ab$?___
(a) $-16b^2$ (b) $16b^2$
(c) $4b^2$ (d) $-4b^2$

6. Find m so that $x^2 + 4x + m$ is a complete square:
(a) 8 (b) −8
(c) 4 (d) 16

7. Factors of $5x^2 - 17xy - 12y^2$ are___
(a) $(x+4y)(5x+3y)$ (b) $(x-4y)(5x-3y)$
(c) $(x-4y)(5x+3y)$ (d) $(5x-4y)(x+3y)$

8. Factors of $27x^3 - \frac{1}{x^3}$ are___
(a) $\left(3x - \frac{1}{x}\right)\left(9x^2 + 3 + \frac{1}{x^2}\right)$
(b) $\left(3x + \frac{1}{x}\right)\left(9x^2 + 3 + \frac{1}{x^2}\right)$
(c) $\left(3x - \frac{1}{x}\right)\left(9x^2 - 3 + \frac{1}{x^2}\right)$
(d) $\left(3x + \frac{1}{x}\right)\left(9x^2 - 3 + \frac{1}{x^2}\right)$

9. If $x - 2$ is a factor of $p(x) = x^2 + 2kx + 8$, then k = __
(a) −3 (b) 3
(c) 4 (d) 5

10. $4a^2 + 4ab + (\ldots..)$ is a complete square
(a) b^2 (b) $2b$
(c) a^2 (d) $4b^2$

11. $\frac{x^2}{y^2} - 2 + \frac{y^2}{x^2}$ = ...

(a) $\left(\frac{x}{y} - \frac{y}{x}\right)^2$ (b) $\left(\frac{x}{y} + \frac{y}{x}\right)^2$

(c) $\left(\frac{x}{y} - \frac{y}{x}\right)^3$ (d) $\left(\frac{x}{y} + \frac{y}{x}\right)^3$

12.(x+y) (x^2 – xy + y^2) = ___

(a) $x^3 - y^3$ (b) $x^3 + y^3$

(c) $(x+y)^3$ (d) $(x - y)^3$

13.Factors of x^4 – 16 is ___

(a) $(x-2)^2$ (b) $(x-2)(x+2)(x^2+4)$

(c) (x–2) (x+2)(d) $(x+2)^2$

14. Factors of 3x – 3a + xy – ay.

(a) (3+y) (x–a) (b) (3–y) (x+a)

(c) (3–y) (x–a) (d) (3+y) (x+a)

15.Factors of pqr + qr^2 –pr^2 – r^3 is:

(a) r(p+r) (q–r) (b) r(p–r) (q + r)

(c) r(p–r) (q–r) (d) r(p+r) (q+r)

16.What is the value of $p(x) = 6x^4 + 2x^3 - x + 2$ at $x = 0$?

(a) 9 (b) 8

(c) 2 (d) 7

17. $x^2 + 5x + 6 =$

(a) $(x+1)(x-)$ (b) $(x-2)(x-3)$

(c) $(x+6)(x-1)$ (d) $(x+2)(x+3)$

18. $4a^2 - 16 =$

(a) $(2a+8)(2a-8)$

(b) $4(a+2)(a-2)$

(c) $4(a+2)^2$ (d) $4(a-2)^2$

19.How many factors of a cubic expression are there?

(a) zero (b) 1

(c) 2 (d) 3

20 (x – y) (x^2 + xy + y^2) = ___

(a) $x^3 - y^3$ (b) $x^3 + y^3$

(c) $(x+y)^3$ (d) $(x - y)^3$

Answer key

1	b	2	c	3	d	4	b	5	c	6	c	7	c
8	a	9	a	10	a	11	a	12	b	13	b	14	a
15	a	16	c	17	d	18	b	19	d	20	a		

Question No.3 Factorize the following.

(ii)

$$4x^2 - 16y^2$$

Solution:

$$4x^2 - 16y^2 = 4[x^2 - 4y^2]$$
$$= 4[(x^2) - (2y)^2]$$
$$= 4(x - 2y)(x + 2y)$$

(v) $8x^3 - \frac{1}{27y^3}$

Solution:

$$8x^3 - \frac{1}{27y^3} = (2x)^3 - \left(\frac{1}{2y}\right)^3$$

$$\left(2x - \frac{1}{3y}\right)\left[(2x)^2 + 2x\left(\frac{1}{3y}\right) + \left(\frac{1}{3y}\right)^2\right]$$

$$= \left(2x - \frac{1}{3y}\right)\left(4x^2 + \frac{2x}{3y} + \frac{1}{9y^2}\right)$$

(ix) $1 - 12pq + 36p^2q^2$

Solution: $1 - 12pq + 36p^2q^2$

$$= (1)^2 - 2(1)(6pq) + (6pq)^2$$
$$= (1 - 6pq)^2$$

Unit-6

[ALGEBRAIC MANIPULATION]

Highest common Factor (H.C.F) and Least Common Multiple(L.C.M) of Algebraic Expression:

1- **Highest common Factor (H.C.F)**

If two or more algebraic expressions are given, then their common factor of highest power is called the H.C.F of the expressions.

2-Least Common Multiple (L.C.M)

If an algebraic expression $p(x) is\ exactly$ divisible by two or more expression. Common Multiple (L.C.M). is the product of common factors together with non-common factors of the given expressions.

there are two method

(i) Factorization (ii)By division

Exercise 6.1

Question.1. Find the $H.C.F$ of the following expressions.

Class-Work & Home Work

(ii). $\boldsymbol{102xy^2z\ , 85x^2yz\ and\ 187xyz^2}$

Solution.

$$Factorizatio\ of\ \ 102xy^2z = 2\times3\times17.x.y^2.z$$
$$Factorizatio\ of\ \ 85x^2yz = 5\times17.x^2.y.z$$
$$Factorizatio\ of\ \ 187xyz^2 = 11\times17.x.y.z^2$$
$$Common\ factors = 17.x.y.z$$
$$H.F.C = 17xyz$$

Answer.

Question.2. Find the H.C.F of the following expressions by factorization

Homework

(i). $\boldsymbol{x^2+5x+6,\qquad x^2-4x-12}$

Solution.

$$Factorizatio\ of\ \ x^2+5x+6 = x^2+3x+2x+6$$
$$= x(x+3)+2(x+3)$$
$$= (x+3)(x+2)$$
$$Factorizatio\ of\ \ x^2-4x-12 = x^2-6x+2x-12$$
$$= x(x-6)+2(x-6)$$
$$= (x-6)(x+2)$$
$$Common\ factors = x+2$$
$$H.F.C = x+2$$

Answer.

Homework

(iv).$\boldsymbol{18(x^3-9x^2+8x)\ ,\ \ 24(x^2-3x+2)}$

Solution.

$$Factorizatio\ of\ \ 18(x^3-9x^2+8x)$$
$$= 2\times3\times3.x(x^2-9x+8)$$
$$= 2\times3\times3.x[x^2-8x-x+8]$$
$$= 2\times3\times3.x[x(x-8)-1(x-8)]$$
$$= 2\times3\times3.x(x-8)(x-1)$$
$$= 2\times3\times3.x(x-8)(x-1)$$
$$Factorizatio\ of\ \ 24(x^2-3x+2)$$
$$= 2\times2\times3[x^2-2x-x+2]$$
$$= 2\times2\times3[x(x-2)-1(x-2)]$$
$$= 2\times2\times3(x-2)(x-1)$$
$$Common\ factors = H.F.C = 2\times3(x-1)$$
$$= 6(x-1)$$

Classwork

(v).$\boldsymbol{36(3x^4+5x^3-2x^2)\ ,\ \ 54(27x^4-x)}$

Solution.

$$Factorizatio\ of\ \ 36(3x^4+5x^3-2x^2)$$
$$= 2\times2\times3\times3.x^2(3x^2+5x-2)$$
$$= 2\times2\times3\times3.x^2[3x^2+6x-x-2]$$
$$= 2\times2\times3\times3.x^2[3x(x+2)-1(x+2)]$$
$$= 2\times2\times3\times3.x^2(x+2)(3x-1)$$
$$Factorizatio\ of\ \ 54(27x^4-x)$$
$$= 2\times3\times3\times3x[27x^3-1]$$
$$= 2\times3\times3\times3x[(3x)^3-1^3]$$
$$= 2\times3\times3\times3x(3x-1)[(3x)^2+(3x)(1)+(1)^2]$$
$$= 2\times3\times3\times3x(3x-1)(9x^2+3x+1)$$

$$Common\ factors = H.F.C = 2\times3\times3.x(3x-1)$$
$$= 18x(3x-1)$$

Question.3. Find the H.C.F of the following by Division Method

Classwork

(ii). $x^4+x^3-2x^2+x-3\ ,\ \ 5x^3+3x^2-17x+6$

Solution.

$$\begin{array}{r|l}
 & x+2 \\
5x^3+3x^2-17x+6 & x^4+x^3-2x^2+x-3 \\
 & \times5 \\ \hline
 & 5x^4+5x^3-10x^2+5x-15 \\
 & 5x^4\pm3x^3\mp17x^2\pm6x \\ \hline
 & 2x^3+7x^2-x-15 \\
 & \times5 \\ \hline
 & 10x^3+35x^2-5x-75 \\
 & \pm10x^3\pm6x^2\mp34x\pm12 \\ \hline
 & 29x^2+29x-87
\end{array}$$

Since

$$\boldsymbol{29x^2+29x-87 = 29(x^2+x-3)}$$

By ignoring 29

$$\begin{array}{r|l}
 & 5x-2 \\
x^2+x-3 & 5x^3+3x^2-17x+6 \\
 & \pm5x^3\pm5x^2\mp15x \\ \hline
 & -2x^2-2x+6 \\
 & \mp2x^2\mp2x\pm6 \\ \hline
 & 0
\end{array}$$

Hence

$$\boldsymbol{H.CF = x^2+x-3}$$

Question.3. Find the H.C.F of the following by Division Method

Homework

(i). $x^3+3x^2-16x+12\ , x^3+x^2-10x+8$

Solution.

$$\begin{array}{r|l} & 1 \\ \hline x^3+x^2-10x+8 & x^3+3x^2-16x+12 \\ & \pm x^3 \pm x^2 \mp 10x \pm 8 \\ \hline & 2x^2-6x+4 \end{array}$$

Since

$2x^2-6x+4=2(x^2-3x+2)$, *now divide* $x^3+x^2-10x+8$ *by* x^2-3x+4

$$\begin{array}{r|l} & x+4 \\ \hline x^2-3x+2 & x^3+x^2-10x+8 \\ & \pm x^3 \mp 3x^2 \pm 2x \\ \hline & 4x^2-12x+8 \\ & 4x^2-12x+8 \\ \hline & 0 \\ \hline \end{array}$$

Hence

$$H.C.F=x^2-3x+2$$

Question.4. Find the L.C.M of the following expressions.

Classwork

(ii). $102xy^2z\ , 85x^2yz\ and\ 187xyz^2$

Solution.

$$Factorizatio\ of\ \ 102xy^2z=2\times 3\times 17.x.y^2.z$$
$$Factorizatio\ of\ \ 85x^2yz=5\times 17.x^2.y.z$$
$$Factorizatio\ of\ \ 187xyz^2=11\times 17.x.y.z^2$$
$$Common\ factors=17.x.y.z$$
$$=17xyz$$
$$Uncommon\ factors=2\times 3\times 5\times 11.x.y.z$$
$$=330xyz$$
$$L.C.M=17xyz\times 330xyz$$
$$L.C.M=5610x^2y^2z^2$$

Answer.

Question.5.Find the L.C.M of the following expressions by Factorization

Classwork

(iii). $2(x^4-y^4)\ \ and\ 3(x^3+2x^2y-xy^2-2y^3)$

Solution.

$$Factorizatio\ of\ \ 2(x^4-y^4)=2((x^2)^2-(y^2)^2)$$
$$=(x^2+y^2)(x^2-y^2)$$
$$=(x^2+y^2)(x+y)(x-y)$$
$$Factorizatio\ of\ \ 3(x^3+2x^2y-xy^2-2y^3)$$
$$=3[x^2(x+2y)-y^2(x+2y)]$$
$$=3(x+2y)(x^2-y^2)$$
$$=3(x+2y)(x+y)(x-y)$$
$$Common\ factors=(x+y)(x-y)$$
$$Uncommon\ factors=3(x^2+y^2)(x+2y)$$
$$L.C.M=(x+y)(x-y)\times 3(x^2+y^2)(x+2y)$$
$$L.C.M=3(x+y)(x-y)(x^2+y^2)(x+2y)$$

Answer.

(iv). $4(x^4-1)\ \ and\ 6(x^3-x^2-x+1)$

Homework

Solution.

$$Factorizatio\ of\ \ 2(x^4-1)=2((x^2)^2-(1)^2)$$
$$=2(x^2+1)(x^2-1)$$
$$=2(x^2+1)(x^2-1^2)$$
$$=2(x^2+1)(x+1)(x-1)$$
$$Factorizatio\ of\ \ 6(x^3-x^2-x+1)$$
$$=2\times 3(x^2(x-1)-1(x-1))$$
$$=2\times 3(x-1)(x^2-1)$$
$$=2\times 3(x-1)(x^2-1^2)$$
$$=2\times 3(x-1)(x+1)(x-1)$$
$$Common\ factors=2(x+1)(x-1)$$
$$Uncommon\ factors=3(x^2+1)(x-1)$$
$$L.C.M=2(x+1)(x-1)\times 3(x^2+1)(x-1)$$
$$L.C.M=6(x+1)(x-1)^2(x^2+1)$$

Answer.

Question.6. Classwork

For what value of k is $(x+4)$ the H.C.F of $x^2+x-(2k+2)$ and $2x^2+kx-12$?

Solution.

Let $p(x)=x^2+x-(2k+2)$

And $a(x)=2x^2+kx-12$

As $(x+4)$ is H.C.F of p(x) and q(x). So p(x) is exactly divisible by $(x+4)$*and thus* $p(-4)=0$

$$(-4)^2+(-4)-(2k+2)=0$$
$$16-4-2k-2=0$$
$$10-2k=0$$
$$2k=10$$
$$k=\frac{10}{2}$$

$\boldsymbol{k=5.}$

Answer.

Question.8. Homework

The L.C.M and H.C.F of two polynomials p(x) and q(x) are $2(x^4-1)$*and* $(x+1)(x^2+1)$ respectively. If $p(x)=x^3+x^2+x+1, find\ q(x)$.

Solution.

$$L.C.M = 2(x^4-1)$$
$$H.C.F = (x+1)(x^2+1)$$
$$p(x) = x^3+x^2+x+1$$
$$q(x) = ?$$

Since we know that

$$p(x)\times q(x) = L.C.M\times H.C.F$$
$$q(x) = \frac{L.C.M\times H.C.F}{p(x)}$$
$$q(x) = \frac{2(x^4-1)\times(x+1)(x^2+1)}{x^3+x^2+x+1}$$
$$q(x) = \frac{2(x^4-1)(x+1)(x^2+1)}{x^2(x+1)+1(x+1)}$$
$$q(x) = \frac{2(x^4-1)(x+1)(x^2+1)}{(x+1)(x^2+1)}$$
$$q(x) = 2(x^4-1)$$

Answer.

Basic Operations on Algebraic Fractions.

Exercise 6.2

Simplify each of the following as a rational expressions.

Classwork

Question.1. $\frac{x^2-x-6}{x^2-9}+\frac{x^2+2x-24}{x^2-x-12}$

Solution.

$$\frac{x^2-x-6}{x^2-9}+\frac{x^2+2x-24}{x^2-x-12}$$
$$=\frac{x^2-3x+2x-6}{x^2-3^2}$$
$$+\frac{x^2+6x-4x-24}{x^2-4x+3x-12}$$
$$=\frac{x(x-3)+2(x-3)}{(x+3)(x-3)}+\frac{x(x+6)-4(x+6)}{x(x-4)+3(x-4)}$$
$$=\frac{(x-3)(x+2)}{(x+3)(x-3)}+\frac{(x+6)(x-4)}{(x-4)(x+3)}$$
$$=\frac{(x+2)}{(x+3)}+\frac{(x+6)}{(x+3)}$$
$$=\frac{x+2+x+6}{(x+3)}$$
$$=\frac{2x+8}{(x+3)}$$
$$=\frac{2(x+4)}{(x+3)}$$

Answer.

Homework

Question.4. $\frac{(x+2)(x+3)}{x^2-9}+\frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$

Solution:

$$\frac{(x+2)(x+3)}{x^2-9}+\frac{(x+2)(2x^2-32)}{(x-4)(x^2-x-6)}$$
$$=\frac{(x+2)(x+3)}{x^2-3^2}$$
$$+\frac{(x+2)2(x^2-16)}{(x-4)(x^2-3x+2x-6)}$$
$$=\frac{(x+2)(x+3)}{(x+3)(x-3)}+\frac{2(x+2)(x^2-4^2)}{(x-4)(x(x-3)+2(x-3))}$$
$$=\frac{(x+2)(x+3)}{(x+3)(x-3)}+\frac{2(x+2)(x+4)(x-4)}{(x-4)(x+2)(x-3)}$$
$$=\frac{(x+2)}{(x-3)}+\frac{2(x+4)}{(x-3)}$$
$$=\frac{x+2+2x+8}{(x-3)}$$
$$=\frac{3x+10}{x-3}$$

Answer.

Homework

Question.6. $A-\frac{1}{A}$, *where* $A=\frac{a+1}{a-1}$

Solution:

$$A=\frac{a+1}{a-1}\ then\ \frac{1}{A}=\frac{a-1}{a+1}$$
$$A-\frac{1}{A}=\frac{a+1}{a-1}-\frac{a-1}{a+1}$$
$$A-\frac{1}{A}=\frac{(a+1)^2-(a-1)^2}{(a-1)(a+1)}$$
$$A-\frac{1}{A}=\frac{a^2+2a+1-a^2+2a-1}{a^2-1^2}$$
$$A-\frac{1}{A}=\frac{4a}{a^2-1}$$

Answer.

Classwork

Question.11. $\frac{x^4-8x}{2x^2+5x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x^2-2x}$

Solution:

$$\frac{x^4-8x}{2x^2+5x-3}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{x^2-2x}$$
$$=\frac{x(x^3-8)}{2x^2+6x-x-3}\times\frac{2x-1}{x^2+2x+4}$$
$$\times\frac{x+3}{x(x-2)}$$
$$=\frac{(x^3-2^3)}{2x(x+3)-1(x+3)}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{(x-2)}$$
$$=\frac{(x-2)(x^2+2x+2^2)}{(2x-1)(x+3)}\times\frac{2x-1}{x^2+2x+4}\times\frac{x+3}{(x-2)}$$
$$=\frac{(x-2)(x^2+2x+4)}{1}\times\frac{1}{x^2+2x+4}\times\frac{1}{(x-2)}$$
$$=1$$

Answer.

Square root of Algebraic Expression:

Definition:

The square roots of a given expression $p(x)$ as another expression $q(x)$ such that $q(x).q(x)=p(x)$

As $5\times5=25$, *so* square root of 25 is 5.

We find square root of an algebraic expression.

(i) by factorization

(ii) by division

Exercise 6.3

Question1. Use factorization to find the square root of the following equqations:

Homework

(i). $4x^2-12xy+9y^2$

Solution.

$$4x^2-12xy+9y^2=(2x)^2-2(2x)(3y)+(3y)^2$$
$$=(2x-3y)^2$$

Hence

$$\sqrt{4x^2-12xy+9y^2}=\pm\sqrt{(2x-3y)^2}$$
$$\sqrt{4x^2-12xy+9y^2}=\pm(2x-3y)$$

Answer.

Homework

(iv). $4(a+b)^2-12(a^2-b^2)+9(a-b)^2$

Solution.

$$4(a+b)^2-12(a^2-b^2)+9(a-b)^2$$
$$=[2(a+b)]^2-2[2(a+b)][2(a-b)]+[3(a-b)]^2$$
$$=[2(a+b)-3(a-b)]^2$$
$$=[2a+2b-3a+3b]^2$$
$$=[5b-a]^2$$

Hence

$$\sqrt{4(a+b)^2-12(a^2-b^2)+9(a-b)^2}=\pm\sqrt{[5b-a]^2}$$
$$\sqrt{4(a+b)^2-12(a^2-b^2)+9(a-b)^2}=\pm(5b-a)$$

Answer.

Classwork

(v). $\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}$

Solution.

$$\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}=\frac{(2x^3)^2-2(2x^3)(3y^3)+(3y^3)^2}{(3x^2)^2+2(3x^2)(4y^2)+(4y^2)^2}$$
$$=\frac{(2x^3-3y^3)^2}{(3x^2+4y^2)^2}$$

Hence

$$\sqrt{\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}}=\pm\sqrt{\frac{(2x^3-3y^3)^2}{(3x^2+4y^2)^2}}$$
$$\sqrt{\frac{4x^6-12x^3y^3+9y^6}{9x^4+24x^2y^2+16y^4}}=\pm\frac{2x^3-3y^3}{3x^2+4y^2}$$

Answer.

Homework

(vi). $\left(x+\frac{1}{x}\right)^2-4\left(x-\frac{1}{x}\right)\quad ,x\neq 0.$

Solution.

$$\left(x+\frac{1}{x}\right)^2-4\left(x-\frac{1}{x}\right)=\left(x^2+\frac{1}{x^2}+2\right)-4\left(x-\frac{1}{x}\right)$$
$$=\left(x^2+\frac{1}{x^2}-2\right)-4\left(x-\frac{1}{x}\right)+4$$
$$=\left(x-\frac{1}{x}\right)^2-2\left(x-\frac{1}{x}\right)(2)+(2)^2$$
$$=\left(x-\frac{1}{x}-2\right)^2$$

Hence

$$\sqrt{\left(x+\frac{1}{x}\right)^2-4\left(x-\frac{1}{x}\right)}=\pm\sqrt{\left(x-\frac{1}{x}-2\right)^2}$$
$$\sqrt{\left(x+\frac{1}{x}\right)^2-4\left(x-\frac{1}{x}\right)}=\pm\left(x-\frac{1}{x}-2\right)$$

Answer.

Question No.2 Use division method to find the square root of the following

Homework

(i). $4x^2+12xy+9y^2+16x+24y+16$

Solution:

i) $4x^2+12xy+9y^2+16x+24y+16$

$$\begin{array}{r|l} & 2x+3y+4 \\ \hline 2x & 4x^2+12xy+9y^2+16x+24y+16 \\ & 4x^2 \\ \hline 4x+3y & 12xy+9y^2+16x+24y+16 \\ & 12xy\pm 9y^2 \\ \hline 4x+6y+4 & 16x+24y+16 \\ & 16x\pm 24y\pm 16 \\ \hline & 0 \end{array}$$

Hence the square root of given expression is

$\pm(2x+3y+4)$

Homework

(ii). $x^4-10x^3+37x^2-60x+36$

Solution:

ii) $x^4-10x^3+37x^2-60x+36$

$$\begin{array}{r|l} & x^2-5x+6 \\ \hline x^2 & x^4-10x^3+37x^2-60x-36 \\ & -x^4 \\ \hline 2x^2-5x & -10x^3+37x^2-60x-36 \\ & \mp 10x^3\pm 25x^2 \\ \hline 2x^2-10x+6 & -12x^2-60x+36 \\ & -12x^2\mp 60x\pm 36 \\ \hline & 0 \end{array}$$

Hence $\sqrt{x^4-10x^3+37x^2-60x+36}$

$=\pm(x^2-5x+6)$

Classwork

(v). $\frac{x^2}{y^2}-10\frac{x}{y}+27-10\frac{y}{x}+\frac{y^2}{x^2}$

Solution:

$$\begin{array}{r|l} & \frac{x}{y}-5+\frac{y}{x} \\ \hline \frac{x}{y} & \frac{x^2}{y^2}-10\frac{x}{y}+27-10\frac{y}{x}+\frac{y^2}{x^2} \\ & -\frac{x^2}{y^2} \\ \hline \frac{2x}{y}-5 & -10\frac{x}{y}+27-10\frac{y}{x}+\frac{y^2}{x^2} \\ & \mp 10\frac{x}{y}\pm 25 \\ \hline \frac{2x}{y}-10+\frac{y}{x} & 2-10\frac{y}{x}+\frac{y^2}{x^2} \\ & \pm 2\mp 10\frac{y}{x}\pm\frac{y^2}{x^2} \\ \hline & 0 \end{array}$$

$\sqrt{\frac{x^2}{y^2}-10\frac{x}{y}+27-10\frac{y}{x}+\frac{y^2}{x^2}}$

The required square root

$=\pm\left(\frac{x}{y}-5+\frac{y}{x}\right)$

Question.3. Find the value k for which these following expressions will become perfect square.

Classwork:

(ii) $x^4-4x^3+10x^2-kx+9$

Solution:

ii) $x^4-4x^3+10x^2-kx+9$

$$\begin{array}{r|l} & x^2-2x+3 \\ \hline x^2 & x^4-4x^3+10x^2-kx+9 \\ & -x^4 \\ \hline 2x^2-2x & -4x^3+10x^2-kx+9 \\ & \mp 4x^3\pm 4x^2 \\ \hline 2x^2-4x+3 & 6x^2-kx+9 \\ & -6x^2\mp 12x\pm 9 \\ \hline & (-k+12)x \end{array}$$

As given that the given expression is a perfect square, so

Remainder $=0$

$(-k+12)x=0$

As $x\neq 0$, so $-k+12=0$

$\Rightarrow \boxed{k=12}$

Question.4. Find the value l and m for which these following expressions will become perfect square.

Homework:

(i) $x^4+4x^3+16x^2-lx+m$

Solution:

i) $x^4+4x^3+16x^2+lx+m$

$$
\begin{array}{r|l}
 & x^2+2x+6 \\
\hline
x^2 & \cancel{x^4}+4x^3+16x^2+lx+m \\
 & \cancel{\pm x^4} \\
\hline
2x^2+2x & \cancel{4x^3}+16x^2+lx+m \\
 & \cancel{\pm 4x^3}\pm 4x^2 \\
\hline
2x^2+4x+6 & \cancel{12x^2}+lx+m \\
 & \cancel{\pm 12x^2}\pm 24x\pm 36 \\
\hline
 & (l-24)x+(m-36)
\end{array}
$$

As the given expression is to be a perfect square, so

$$\text{Remainder}=0$$
$$(l-24)x+(m-36)=0$$

As $x\neq 0$, so $l-24=0$ and $m-36=0$
$\Rightarrow l=24$ and $m=36$

Review Exercise -6

Question No.2 Find $H.C.F$ of the following by factorization $8x^4-128, 12x^3-96$

Solution:

$$8x^4-128=8(x^4-16)$$
$$=2\times2\times2[(x^2)^2-(4)^2]$$
$$=2\times2\times2(x^2-4)(x^2+4)$$
$$=2\times2\times2(x+2)(x-2)(x^2+4)$$
$$12x^3-96=12(x^3-8)=2\times2\times3[x^3-(2)^3]$$
$$=2\times2\times3(x-2)(x^2+2x+4)$$
$$H.C.F=2\times2\times(x-2)=4(x-2)$$

Question No.6 Simplify

(i) $\frac{3}{x^3+x^2+x+1}-\frac{3}{x^3-x^2+x-1}$

Solution:

$$\frac{3}{x^3+x^2+x+1}-\frac{3}{x^3-x^2+x-1}$$
$$=\frac{3}{(x+1)(x^2+1)}-\frac{3}{x^2-x^2+x-1}$$
$$=\frac{3}{(x+1)(x^2+1)}-\frac{3}{(x-1)(x^2+1)}$$
$$=\frac{3(x-1)-3(x+1)}{(x-1)(x+1)(x^2+1)}=\frac{3x-3-3x-3}{(x^2-1)(x^2+1)}$$
$$\frac{6}{1-x^4}$$

Question No.7

Find the square root by using factorization $\left(x^2+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)+27(x\neq0)$

Solution:

$$\left(x^2+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)+27$$
$$=\left(x^2+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)+25+2$$
$$=\left(x^2+\frac{1}{x^2}+2\right)+10\left(x+\frac{1}{x}\right)+25$$
$$\left(x+\frac{1}{x}\right)^2+2\left(x+\frac{1}{x}\right)(5)+(5)^2$$
$$\left[\left(x+\frac{1}{x}\right)+2\left(x+\frac{1}{x}\right)(5)+(5)^2\right]$$
$$\left[\left(x+\frac{1}{x}\right)+2\left(x+\frac{1}{x}\right)+25\right]$$
$$=\left(x+\frac{1}{x}\right)^2+2\left(x+\frac{1}{x}\right)(5)+(5)^2$$
$$=\left[\left(x+\frac{1}{x}\right)+2\left(x+\frac{1}{x}\right)(5)+(5)^2\right]$$
$$=\left[\left(x+\frac{1}{x}\right)+5\right]^2$$

Taking square root on both sides, we get

$$\sqrt{\left(x^2+\frac{1}{x^2}\right)+10\left(x+\frac{1}{x}\right)+27}$$
$$=\pm\sqrt{\left[\left(x+\frac{1}{x}\right)+5\right]^2}$$
$$=\pm\left[\left(x+\frac{1}{x}\right)+5\right]$$

Unit-7

[LINEAR EQUATIONS AND INEQUALITY]

Radical Equation

When the variable in an equation occurs under a radical, the equation is called a radical equation. For example,

$$\sqrt{x-3}-7=0$$

Linear Equation

A linear equation in one unknown variable x is an equation of the form $ax + b = 0$, where $a, b \in R \text{ and } a \neq 0$

A solution to a linear equation is any replacement or substitution for the variable x that makes the statement true. Two linear equations are said to be equivalent if they have exactly the same solution. For example, $x+1=0, \;\; 2x+5=-1$

Exercise 7.1

Q#1) Solve the following equations.

(i). $\frac{2}{3}x-\frac{1}{2}x=x+\frac{1}{6}$

Solution: As given $\frac{2}{3}x-\frac{1}{2}x=x+\frac{1}{6}$

Multiply by 6(LCM) on both sides

$$6\times\frac{2}{3}x-6\times\frac{1}{2}x=6\times x+6\times\frac{1}{6}$$
$$4x-3x=6x+1$$
$$x=6x+1$$
$$1=x-6x$$
$$1=-5x$$
$$x=-\frac{1}{5}$$

Check:

$$\frac{2}{3}x-\frac{1}{2}x=x+\frac{1}{6}$$

Put $x=-\frac{1}{5}$

$$\frac{2}{3}\left(-\frac{1}{5}\right)-\frac{1}{2}\left(-\frac{1}{5}\right)=\left(-\frac{1}{5}\right)+\frac{1}{6}$$
$$-\frac{2}{15}+\frac{1}{10}=-\frac{1}{5}+\frac{1}{6}$$

Multiply by 30(LCM) on both sides

$$30\times\left(-\frac{2}{15}\right)+30\times\left(\frac{1}{10}\right)$$
$$=30\times\left(-\frac{1}{5}\right)+30\times\left(\frac{1}{6}\right)$$
$$-4+3=-6+5$$
$$-1=-1 \text{(which is true)}$$

Since $x=-\frac{1}{5}$ satisfy the given equation, therefore, the solution set is $\left\{-\frac{1}{5}\right\}$ i.e. $S.S=\left\{-\frac{1}{5}\right\}$

(ii). $\frac{x-3}{3}-\frac{x-2}{2}=-1$

Solution: As given $\frac{x-3}{3}-\frac{x-2}{2}=-1$

Multiply by 6(LCM) on both sides

$$6\times\left(\frac{x-3}{3}\right)-6\times\left(\frac{x-2}{2}\right)=6\times(-1)$$
$$2(x-3)-3(x-2)=-6$$
$$2x-6-3x+6=-6$$
$$-x=-6$$
$$x=6$$

Check:

$$\frac{x-3}{3}-\frac{x-2}{2}=-1$$

Put $x=6$

$$\frac{(6)-3}{3}-\frac{(6)-2}{2}=-1$$
$$\frac{3}{3}-\frac{4}{2}=-1$$
$$1-2=-1$$
$$-1=-1 \text{ (which is true)}$$

Since $x=6$ satisfy the given equation, therefore, the solution set is $\{6\}$ i.e. $S.S=\{6\}$

(v) $\frac{5(x-3)}{6}-x=1-\frac{x}{9}$

Solution: As given $\frac{5(x-3)}{6}-x=1-\frac{x}{9}$

Multiply by 18 (LCM) on both sides

$$18\times\left(\frac{5(x-3)}{6}\right)-18\times(x)$$
$$=18\times(1)-18\times\left(\frac{x}{9}\right)$$
$$3(5x-15)-18x=18-2x$$
$$15x-45-18x=18-2x$$
$$-45-3x=18-2x$$
$$-3x+2x=18+45$$
$$-x=63$$
$$x=-63$$

Check:

$$\frac{5(x-3)}{6}-x=1-\frac{x}{9}$$

Put $x=-63$

$$\frac{5(-63-3)}{6}-(-63)=1-\frac{(-63)}{9}$$
$$\frac{5(-66)}{6}+63=1+\frac{63}{9}$$
$$-55+63=1+7$$
$$8=8 \text{ (which is true)}$$

Since $x=-63$ satisfy the given equation, therefore, the solution set is $\{-63\}$ i.e. $S.S=\{-63\}$

(vi). $\frac{x}{3x-6}=2-\frac{2x}{x-2}, x\neq 2$

Solution: As given $\frac{x}{3x-6}=2-\frac{2x}{x-2}$

$$\frac{x}{3x-6}=\frac{2(x-2)-2x}{x-2}$$
$$\frac{x}{3x-6}=\frac{2x-4-2x}{x-2}$$
$$\frac{x}{3x-6}=\frac{-4}{x-2}$$

$$x(x-2) = -4(3x-6)$$
$$x^2 - 2x = -12x + 24$$
$$x^2 - 2x + 12x - 24 = 0$$
$$x(x-2) + 12(x-2) = 0$$
$$(x-2)(x+12) = 0$$

That is $x = 2, -12$

Since it is given that $x \neq 2$, therefore, we ignore $x = 2$ and just check $x = -12$ for the solution set.

Check:

$$\frac{x}{3x-6} = 2 - \frac{2x}{x-2}$$

Put $x = -12$

$$\frac{(-12)}{3(-12)-6} = 2 - \frac{2(-12)}{(-12)-2}$$
$$\frac{-12}{-36-6} = 2 + \frac{24}{-12-2}$$
$$\frac{-12}{-42} = 2 + \frac{24}{-14}$$
$$\frac{2}{7} = \frac{28-24}{14}$$
$$\frac{2}{7} = \frac{4}{14}$$
$$\frac{2}{7} = \frac{2}{7} \quad \text{(which is true)}$$

Since $x = -12$ satisfy the given equation, therefore, the solution set is $\{-12\}$ i.e. $S.S = \{-12\}$

(x). $\frac{2}{3x+6} = \frac{1}{6} - \frac{1}{2x+4}$, $x \neq -2$

Solution: As given $\frac{2}{3x+6} = \frac{1}{6} - \frac{1}{2x+4}$

$$\frac{2}{3x+6} = \frac{1(2x+4)-6}{6(2x+4)}$$
$$\frac{2}{3x+6} = \frac{2x+4-6}{6(2x+4)}$$
$$\frac{2}{3x+6} = \frac{2x-2}{6(2x+4)}$$
$$\frac{2}{3(x+2)} = \frac{2(x-1)}{6(2(x+2))}$$
$$\frac{2}{3(x+2)} = \frac{(x-1)}{6(x+2)}$$
$$12(x+2) = 3(x+2)(x-1)$$
$$12(x+2) - 3(x+2)(x-1) = 0$$

$$3(x+2)[4 - x + 1)] = 0$$
$$3(x+2)(5-x) = 0$$

That is $x = -2, 5$

Since it is given that $x \neq -2$, therefore, we ignore $x = -2$ and just check $x = 5$ for the solution set.

Check:

$$\frac{2}{3x+6} = \frac{1}{6} - \frac{1}{2x+4}$$

Put $x = 5$

$$\frac{2}{3(5)+6} = \frac{1}{6} - \frac{1}{2(5)+4}$$
$$\frac{2}{15+6} = \frac{1}{6} - \frac{1}{10+4}$$
$$\frac{2}{21} = \frac{1}{6} - \frac{1}{14}$$
$$\frac{2}{21} = \frac{7-3}{42}$$
$$\frac{2}{21} = \frac{2}{42}$$

$$\frac{2}{21} = \frac{2}{21} \quad \text{(which is true)}$$

Since $x = 5$ satisfy the given equation, therefore, the solution set is $\{5\}$ i.e. $S.S = \{5\}$

Q#2) Solve each equation and check for extraneous solution if any.

(i). $\sqrt{3x+4} = 2$

Solution: As given $\sqrt{3x+4} = 2$

On squaring, we get

$$\left(\sqrt{3x+4}\right)^2 = (2)^2$$
$$3x + 4 = 4$$
$$3x = 0$$
$$x = 0$$

Check:

$$\sqrt{3x+4} = 2$$

Put $x = 0$

$$\sqrt{3(0)+4} = 2$$
$$\sqrt{0+4} = 2$$
$$2 = 2 \quad \text{(which is true)}$$

Since $x = 0$ satisfy the given equation, therefore, the solution set is $\{0\}$ i.e. $S.S = \{0\}$

(ii). $\sqrt[3]{2x-4} - 2 = 0$

Solution: As given $\sqrt[3]{2x-4} - 2 = 0$

$$\sqrt[3]{2x-4} = 2$$

Taking cube on both sides

$$\left(\sqrt[3]{2x-4}\right)^3 = (2)^3$$
$$2x - 4 = 8$$
$$2x = 8 + 4$$
$$2x = 12$$
$$x = 6$$

Check:

$$\sqrt[3]{2x-4} - 2 = 0$$

Put $x = 6$

$$\sqrt[3]{2(6)-4} - 2 = 0$$
$$\sqrt[3]{12-4} - 2 = 0$$
$$\sqrt[3]{8} - 2 = 0$$
$$2 - 2 = 0$$
$$0 = 0 \quad \text{(which is true)}$$

Since $x = 6$ satisfy the given equation, therefore, the solution set is $\{6\}$ i.e. $S.S = \{6\}$

(ii). $\sqrt{x-3} - 7 = 0$

Solution: As given $\sqrt{x-3} - 7 = 0$

$$\sqrt{x-3} = 7$$

Taking square on both sides

$$\left(\sqrt{x-3}\right)^2 = (7)^2$$
$$x - 3 = 49$$
$$x = 49 + 3$$
$$x = 52$$

Check:

$$\sqrt{x-3} - 7 = 0$$

Put $x = 52$

$$\sqrt{52-3} - 7 = 0$$
$$\sqrt{49} - 7 = 0$$
$$7 - 7 = 0$$
$$0 = 0 \quad \text{(which is true)}$$

Since $x = 52$ satisfy the given equation, therefore, the solution set is $\{52\}$ i.e. $S.S = \{52\}$

(v). $\sqrt[3]{2-t} = \sqrt[3]{2t-28}$

Solution: As given $\sqrt[3]{2-t} = \sqrt[3]{2t-28}$

Taking cube on both sides

$$\left(\sqrt[3]{2-t}\right)^3 = \left(\sqrt[3]{2t-28}\right)^3$$
$$2 - t = 2t - 28$$
$$2 + 28 = 2t + t$$
$$30 = 3t$$
$$t = 10$$

Check:

$$\sqrt[3]{2-t} = \sqrt[3]{2t-28}$$

Put $t = 10$

$$\sqrt[3]{2-10} = \sqrt[3]{2(10)-28}$$
$$\sqrt[3]{-8} = \sqrt[3]{20-28}$$
$$\sqrt[3]{-8} = \sqrt[3]{-8}$$

Taking cube root, we have

$$-8 = -8 \text{ (which is true)}$$

Since t= 10 satisfy the given equation, therefore, the solution set is $\{10\}$ i.e. $S.S = \{10\}$

(viii). $\sqrt{\frac{x+1}{2x+5}} = 2,\ x \neq -\frac{5}{2}$

Solution: As given $\sqrt{\frac{x+1}{2x+5}} = 2$

Taking square on both sides

$$\left(\sqrt{\frac{x+1}{2x+5}}\right)^2 = (2)^2$$
$$\frac{x+1}{2x+5} = 4$$
$$x + 1 = 4(2x+5)$$
$$x + 1 = 8x + 20$$
$$1 - 20 = 8x - x$$
$$-19 = 7x$$
$$x = -\frac{19}{7}$$

Check:

$$\sqrt{\frac{x+1}{2x+5}} = 2$$

Put $x = -\frac{19}{7}$

$$\sqrt{\frac{\left(-\frac{19}{7}\right)+1}{2\left(-\frac{19}{7}\right)+5}} = 2$$
$$\sqrt{\frac{\frac{-19+7}{7}}{\frac{-38+35}{7}}} = 2$$
$$\sqrt{\frac{\frac{-12}{7}}{\frac{-3}{7}}} = 2$$
$$\sqrt{\frac{12}{3}} = 2$$
$$\sqrt{4} = 2$$
$$2 = 2 \ \text{(which is true)}$$

Since $x = -\frac{19}{7}$ satisfy the given equation, therefore, the solution set is $\left\{-\frac{19}{7}\right\}$ i.e. $S.S = \left\{-\frac{19}{7}\right\}$

Absolute Value

The Absolute value of real number $'a'$ is denoted by $|a|$, is defined as

$$|a| = \begin{cases} a & if\ a \geq 0 \\ -a & if\ a < 0 \end{cases}$$

For example, $|6| = 6$, $|-5| = -(-5) = 5$

$$|0| = 0$$

Some Properties of Absolute value

If $a, b \in R$, then

(i). $|a| \geq 0$

(ii). $|-a| = |a|$

(iii). $|ab| = |a||b|$

(iv). $\left|\frac{a}{b}\right| = \frac{|a|}{|b|},\ |b| \neq 0$

Exercise 7.2

iii). $|2x+5| = 11$

Solution: As given $|2x+5|=11$
By definition, we have
$2x+5=11$ or $2x+5=-11$
$2x=11-5$ or $2x=-11-5$
$2x=6$ or $2x=-16$
$x=3$ or $x=-8$
Check:
$$|2x+5|=11\ldots(1)$$
Put $x=3$, in (1)
$$|2(3)+5|=11$$
$$|6+5|=11$$
$$|11|=11$$
$$11=11 \text{ (which is true)}$$
Put $x=-8$, in (1)
$$|2(-8)+5|=11$$
$$|-16+5|=11$$
$$|-11|=11$$
$$11=11 \text{ (which is true)}$$
Since $x=3,-8$ satisfy the given equation, therefore, the solution set is $\{3,-8\}$ i.e. $S.S=\{3,-8\}$

(v). $|x+2|-3=5-|x+2|$
Solution: As given $|x+2|-3=5-|x+2|$
$$|x+2|+|x+2|=5+3$$
$$2|x+2|=8$$
$$|x+2|=4$$
By definition, we have
$x+2=4$ or $x+2=-4$
$x=4-2$ or $x=-4-2$
$x=2$ or $x=-6$
Check:
$$|x+2|-3=5-|x+2|\ldots(1)$$
Put $x=2$, in (1)
$$|2+2|-3=5-|2+2|$$
$$|4|-3=5-|4|$$
$$4-3=5-4$$
$$1=1 \text{ (which is true)}$$
Put $x=-6$, in (1)
$$|-6+2|-3=5-|-6+2|$$
$$|-4|-3=5-|-4|$$
$$4-3=5-4$$
$$1=1 \text{ (which is true)}$$
Since $x=2,-6$ satisfy the given equation, therefore, the solution set is $\{2,-6\}$ i.e. $S.S=\{2,-6\}$

(vii). $\left|\frac{3-5x}{4}\right|-\frac{1}{3}=\frac{2}{3}$
Sol: As given $\left|\frac{3-5x}{4}\right|-\frac{1}{3}=\frac{2}{3}$
$$\left|\frac{3-5x}{4}\right|=\frac{2}{3}+\frac{1}{3}$$
$$\left|\frac{3-5x}{4}\right|=\frac{2+1}{3}$$
$$\left|\frac{3-5x}{4}\right|=\frac{3}{3}$$
$$\left|\frac{3-5x}{4}\right|=1$$
$$|3-5x|=4$$
By definition, we have
$3-5x=4$ or $3-5x=-4$
$3-4=5x$ or $3+4=5x$
$-1=5x$ or $7=5x$
$x=-\frac{1}{5}$ or $x=\frac{7}{5}$
Check:
$$\left|\frac{3-5x}{4}\right|-\frac{1}{3}=\frac{2}{3}\ldots(1)$$
Put $x=-\frac{1}{5}$, in (1)
$$\left|\frac{3-5\left(-\frac{1}{5}\right)}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$\left|\frac{3+1}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$\left|\frac{4}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$|1|-\frac{1}{3}=\frac{2}{3}$$
$$1-\frac{1}{3}=\frac{2}{3}$$
$$\frac{3-1}{3}=\frac{2}{3}$$
$$\frac{2}{3}=\frac{2}{3} \text{(which is true)}$$
Put $x=\frac{7}{5}$, in (1)
$$\left|\frac{3-5\left(\frac{7}{5}\right)}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$\left|\frac{3-7}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$\left|\frac{-4}{4}\right|-\frac{1}{3}=\frac{2}{3}$$
$$|-1|-\frac{1}{3}=\frac{2}{3}$$
$$1-\frac{1}{3}=\frac{2}{3}$$
$$\frac{3-1}{3}=\frac{2}{3}$$
$$\frac{2}{3}=\frac{2}{3} \text{(which is true)}$$
Since $x=-\frac{1}{5},\frac{7}{5}$ satisfy the given equation, therefore, the solution set is $\left\{-\frac{1}{5},\frac{7}{5}\right\}$ i.e. $S.S=\left\{-\frac{1}{5},\frac{7}{5}\right\}$

(viii). $\left|\frac{x+5}{2-x}\right| = 6$

Solution: As given $\left|\frac{x+5}{2-x}\right| = 6$

By definition, we have

$\frac{x+5}{2-x} = 6$ or $\frac{x+5}{2-x} = -6$

$x + 5 = 12 - 6x$ or $x + 5 = -12 + 6x$

$x + 6x = 12 - 5$ or $5 + 12 = 6x - x$

$7x = 7$ or $17 = 5x$

$x = 1$ or $x = \frac{17}{5}$

Check:

$$\left|\frac{x+5}{2-x}\right| = 6 \ldots(1)$$

Put $x = 1$, in (1)

$$\left|\frac{1+5}{2-1}\right| = 6$$

$$\left|\frac{6}{1}\right| = 6$$

$$|6| = 6$$

$$6 = 6 \text{ (which is true)}$$

Put $x = \frac{17}{5}$, in (1)

$$\left|\frac{\left(\frac{17}{5}\right)+5}{2-\left(\frac{17}{5}\right)}\right| = 6$$

$$\left|\frac{\frac{17+25}{5}}{\frac{10-17}{5}}\right| = 6$$

$$\left|\frac{\frac{42}{5}}{\frac{-7}{5}}\right| = 6$$

$$\left|\frac{42}{-7}\right| = 6$$

$$|-6| = 6$$

$$6 = 6 \text{ (which is true)}$$

Since $x = 1, \frac{17}{5}$ satisfy the given equation, therefore, the solution set is $\left\{1, \frac{17}{5}\right\}$ i.e. $S.S = \left\{1, \frac{17}{5}\right\}$

Exercise 7.3

Absolute Value

A linear inequality in one variable x is an inequality in which the variable x occurs only to the first power and has the standard form

$$ax + b < 0, a \neq 0$$

where a and b are real numbers. We may replace the symbol $<$ by $>, \leq, \geq$ also.

(ii). $\boldsymbol{4x - 10.3 \leq 21x - 1.8}$

Solution: As given $4x - 10.3 \leq 21x - 1.8$

$\Rightarrow -10.3 + 1.8 \leq 21x - 4x$

$\Rightarrow -8.5 \leq 17x$

$\Rightarrow -\frac{8.5}{15} \leq x$

$\Rightarrow x \geq -0.5$

Hence, $S.S = \{x | x \geq -0.5\}$

(iv). $\boldsymbol{x - 2(5 - 2x) \geq 6x - 3\frac{1}{2}}$

Solution: As given $x - 2(5 - 2x) \geq 6x - 3\frac{1}{2}$

$x - 10 + 4x \geq 6x - \frac{7}{2}$

$5x - 10 \geq 6x - \frac{7}{2}$

Multiply by 2

$\Rightarrow 10x - 20 \geq 12x - 7$

$\Rightarrow -20 + 7 \geq 12x - 10x$

$\Rightarrow -13 \geq 2x$

$\Rightarrow x \leq \frac{-13}{2}$

$\Rightarrow x \leq -6.5$

Hence, $S.S = \{x | x \leq -6.5\}$

Q#2) Solve the following inequalities.

(iii). $\boldsymbol{-6 < \frac{x-2}{4} < 6}$

Solution: As given $-6 < \frac{x-2}{4} < 6$

$\Rightarrow -24 < x - 2 < 24$

$\Rightarrow -24 + 2 < x - 2 + 2 < 24 + 2$

$\Rightarrow -22 < x < 26$

Hence, $S.S = \{x | -22 < x < 26\}$

(iv). $\boldsymbol{3 \geq \frac{7-x}{2} \geq 1}$

Solution: As given $3 \geq \frac{7-x}{2} \geq 1$

$\Rightarrow 6 \geq 7 - x \geq 2$

$\Rightarrow 6 - 7 \geq -x \geq 2 - 7$

$\Rightarrow -1 \geq -x \geq -5$

Multiply by -1

$\Rightarrow 1 \leq x \leq 5$

Hence, $S.S = \{x | 1 \leq x \leq 5\}$

Review Exercise -7

Choose the correct answer:

1. **Which of the following is the solution of the inequality 3 – 4x ≤ 11?**
   (a) $x \geq -8$ (b) $x \geq -2$
   (c) $x \geq \frac{-14}{4}$ (d) None of these
2. **A statement involving any of the symbols <, > or ≤ or ≥ is called:**
   (a) Equation
   (b) Identity
   (c) Inequality
   (d) Linear equation
3. **x = ________ is a solution of the inequality –2 < x < $\frac{3}{2}$**
   (a) -5 (b) 3 (c) 0 (d) $\frac{5}{2}$
4. **If x is no larger than 10, then:**

(a) $x \geq 8$ (b) $x \leq 10$
(c) $x < 10$ (d) $x > 10$

5. If the capacity c of an elevator is at most 1600 pounds, then_
(a) $c < 1600$ (b) $c \geq 1600$
(c) $c \leq 1600$ (d) $c > 1600$

6. **x=0 is a solution of the inequality:**
(a) $x > 0$
(b) $3x + 5 < 0$
(c) $x + 2 < 0$
(d) $x - 2 < 0$

7. The linear equation in one variable x is:
(a) $ax + b = 0$
(b) $ax^2 + bx + c = 0$
(c) $ax + by + c = 0$
(d) $ax^2 + by^2 + c = 0$

8. An inconsistent equation is that whose solution set is:
(a) Empty (b) Not empty
(c) Zero (d) Positive

9. $|x| = a$ **is equivalent to:**
(a) $x = a$ or $x = -a$
(b) $x = \frac{1}{a}$ or $x = \frac{-1}{a}$
(c) $x = a$ or $x = \frac{-1}{a}$
(d) None of these

10. A linear inequality in one variable x is:
(a) $ax + b > 0, \quad a \neq 0$
(b) $ax^2 + bx + c < 0, a \neq 0$
(c) $ax + by + c > 0, \quad a \neq 0$
(d) $ax^2 + by^2 + c < 0, a \neq 0$

11. Law of Trichotomy is …
$(a, b \in R)$
(a) $a < b$ or $a = b$ or $a > b$
(b) $a < b$ or $a = b$
(c) $a < b$ or $a > b$
(d) None of these

12. Transitive law is____
(a) $a < b$ and $b < c$, then $a < c$
(b) $a > b$ and $b < c$, then $a > c$
(c) $a > b$ and $b < c$, then $a = c$
(d) None of these

13. If a > b, c > 0 then:
(a) $ac < bc$ (b) $ac > bc$
(c) $ac = bc$ (d) $ac \leq bc$

14. If a > b, c > 0 then:
(a) $\frac{a}{c} > \frac{b}{c}$ (b) $\frac{a}{c} < \frac{b}{c}$
(c) $\frac{a}{c} = \frac{b}{c}$ (d) $\frac{b}{c} \neq \frac{b}{c}$

15. If a > b, c < 0, then:
(a) $\frac{a}{c} < \frac{b}{c}$ (b) $\frac{a}{c} > \frac{b}{c}$
(c) $\frac{a}{c} = \frac{b}{c}$ (d) $\frac{a}{c} \leq \frac{b}{c}$

16. If a, b ∈ R then: b ≠ 0
(a) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ (b) $|ab| = \frac{|a|}{|b|}$
(c) $|a+b| = |a| + |b|$
(d) $|a-b| = |a| - |b|$

17. When the variable in the equation occur under a radicle equation is called a _______ equation.
(a) Radical (b) Absolute value
(c) Linear (d) None of these

18. $|x| = 0$ **has only ___ solution.**
(a) one (b) two
(c) three (d) none of these

19. The equation $|x| = 2$ **is equivalent to:**
(a) $x = 2$ or $x = -2$
(b) $x = -2$ or $x = -2$
(c) $x = 2$ or $x = \frac{1}{2}$
(d) $x = 2$ or $x = \frac{-1}{2}$

20. An __ is equation that is satisfied by every number for which both sides are defined:
(a) Identity (b) Conditional
(c) Inconsistent (c) In equation

21. An__ equation is an equation whose solution set is the empty set:
(a) Identity (b) Conditional
(c) Inconsistent (d) None

22. A _ equation is an equation that is satisfied by atleast one number but is not an identity:
(a) Identity (b) Conditional
(c) Inconsistent (d) None

23. x + 4 = 4 + x is _ equation:
(a) Identity (b) Conditional
(c) Inconsistent (d) None

24. 2x + 1 = 9 is ___ equation:
(a) Identity (b) Conditional
(c) Inconsistent (d) None

25. x = x + 5 is ___ equation:
(a) Identity (b) Conditional
(c) Inconsistent (d) None

26. Equations having exactly the same solution are called ___ equations.
(a) equivalent (b) Linear
(c) Inconsistent (c) In equations

27. A solution that does not satisfy the original equation is called ____ solution:
(a) Extraneous (b) Root
(c) General (d) Proper

Answer key

1.	b	2.	c	3.	c	4.	b
5.	c	6.	d	7.	a	8.	a
9.	a	10	a	11	a	12	a
13.	b	14	a	15	a	16	a
17.	a	18	a	19	a	20	a
21.	c	22	c	23	a	24	b
25.	c	26	a	27	a		

Question.3.

(i) Define linear inequality in one variable.

Solution.

A linear inequality in one variable x is an inequality in which the variable x occurs only to the first power and has the standard form

$$ax + b < 0, a \neq 0$$

where a and b are real numbers. We may replace the symbol $<$ *by* $>, \leq, \geq$ also.

iii) The formula relating degree Fahrenheit to degree Celsius is $F = \frac{9}{5}C + 32$, *for what value of* C *is* $F < 0$?

Solution.

$$F = \frac{9}{5}C + 32$$
$$F < 0$$
$$\frac{9}{5}C + 32 < 0$$
$$\frac{9C + 160}{5} < 0$$
$$9C + 160 < 0$$
$$9C < -160$$
$$C < -\frac{160}{9}$$

Answer.

Question.5. Solve for x

(i) $|3x + 14| - 2 = 5x$

Solution:

$$|3x + 14| - 2 = 5x$$
$$|3x + 14| = 5x + 2$$
$$3x + 14 = \pm(5x + 2)$$
$$3x + 14 = +(5x + 2) \;, 3x + 14 = -(5x + 2)$$
$$3x + 14 = 5x + 2 \;, 3x + 14 = -5x - 2$$
$$3x - 5x = -14 + 2 \;, 3x + 5x = -14 - 2$$
$$-2x = -12 \;, 8x = -16$$
$$x = -\frac{12}{-2} \;, x = -\frac{16}{8}$$
$$x = 6 \;, \quad x = -2$$

Check:

$$|3x + 14| - 2 = 5x - - - (i)$$

Put $x = 6$, in (1)

$$|3(6) + 14| - 2 = 5(6)$$
$$|18 + 14| - 2 = 30$$
$$|32| - 2 = 30$$
$$30 = 30 \text{ (which is true)}$$

Put $x = -2$, in (1)

$$|3(-2) + 14| - 2 = 5(-2)$$
$$|-6 + 14| - 2 = -10$$
$$|8| - 2 = -10$$
$$8 - 2 = -10$$
$$6 = -10 \quad \textit{which is not true}$$

Since $x = 6$ satisfy the given equation, therefore, the solution set is $\{8\}$ i.e. $S.S = \{6\}$

Unit-8

[LINEAR GRAPHS & THEIR APPLICATION]

Question.1. Determine the quadrant of the coordinate plane in which the following points lie $\boldsymbol{P(-4,3), Q(-5,-2), R(2,2) and\ S(2,-6)}$.

Classwork

Solution:

$$P(-4,3)\ lies\ in\ the\ 2nd\ quadrant.$$
$$Q(-5,-2)\ lies\ in\ the\ 3rd\ quadrant.$$
$$R(2,2)\ lies\ in\ the\ 1st\ quadrant.$$
$$S(2,-6)\ lies\ in\ the\ 4th\ quadrant.$$

Answer.

Question.2. Draw the graph each of the following

Classwork

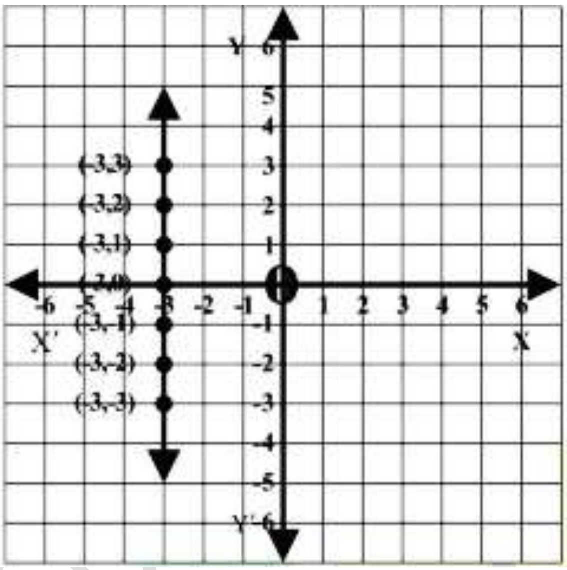
(ii). $\boldsymbol{x = -3}$

Solution:

Table for the points of the equation $x = -3\ is\ as\ under$:

x	-3	-3	-3	-3	-3	-3	-3
y	3	2	1	0	-1	-2	-3

Thus the graph of equation $x = -3\ is\ shown\ below.$



Homework

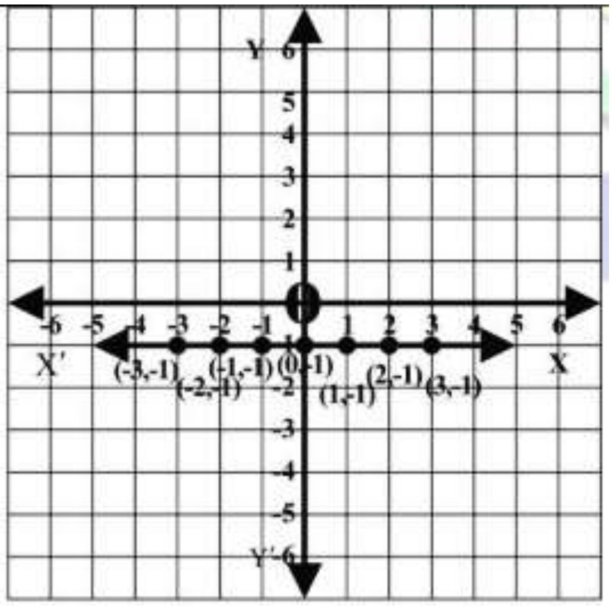
(iii). $\boldsymbol{y = 12}$

Solution:

Table for the points of the equation $y = 1\ is\ as\ under$:

x	3	2	1	0	-1	-2	-3
y	1	1	1	1	1	1	1

Thus the graph of equation $y = 1\ is\ shown\ below.$



Homework

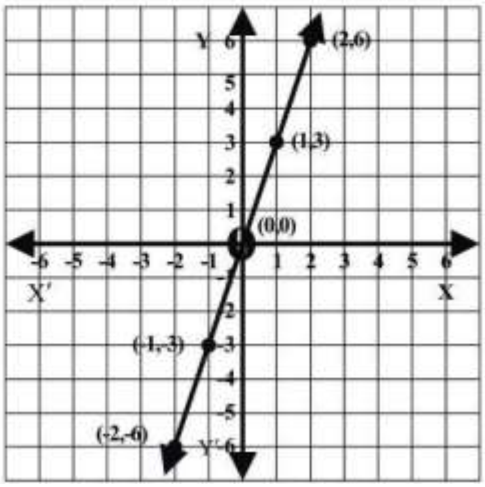
(vii). $\boldsymbol{y = 3x}$

Solution:

Table for the points of the equation $y = 3x\ is\ as\ under$:

x	-2	-1	0	1	2
y	-6	-3	0	3	6

Thus the graph of equation $y = 3x\ is\ shown\ below.$

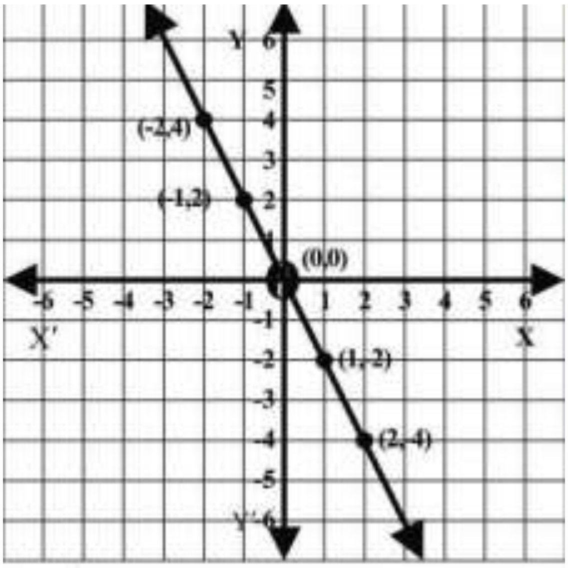


(viii) $\boldsymbol{-y = 2x}$

Solution:

$$\boldsymbol{-y = 2x}$$
$$\boldsymbol{y = -2x}$$

X	**−3**	**−2**	**−1**	**0**	**1**	**2**	**3**
y	**+6**	**+4**	**+2**	**0**	**−2**	**−4**	**−6**



Question.3. Are the following lines (i) parallel to $x-axis$ (ii). parallel to $y-axis$.

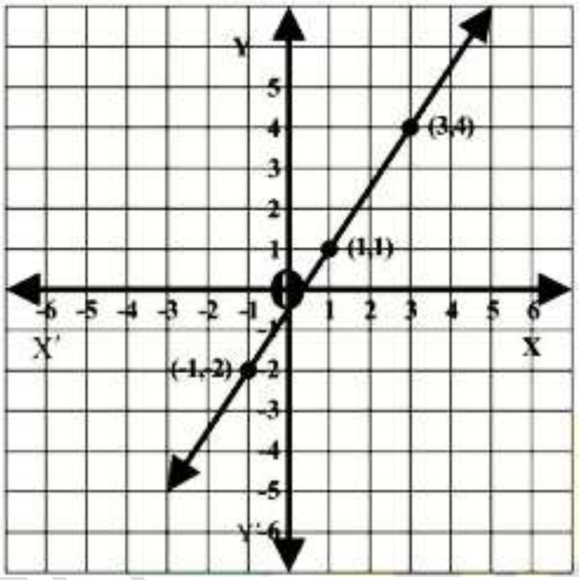
Classwork

(i). $2x-1=3$

Solution:

$$2x-1=3$$
$$2x=3+1$$
$$2x=4$$
$$x=2$$

Which is line parallel $y-axis$.



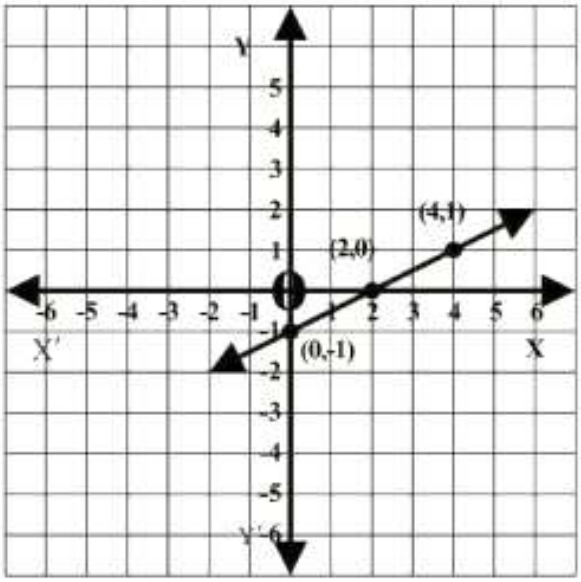
Homework

(v). $2x-2y=0$

Solution:

$$2x-2y=0$$
$$2x=2y$$
$$x=y$$

Line is neither parallel to $x-axis$ nor $y-axis$.



Question.4. Find the value of m and c of the following lines by expressing them in the form $y=mx+c$.

Classwork

(b). $x-2y=-2$

Solution:

$$x-2y=-2$$
$$-2y=-x-2$$
$$2y=x+2$$
$$y=\frac{x}{2}+\frac{2}{2}$$
$$y=\frac{x}{2}+1$$

Since $y=mx+c$, *we have*

$$m=\frac{1}{2}\ \text{and}\ c=1.$$

Homework

(c). $3x+y-1=0$

Solution:

$$3x+y-1=0$$
$$y=-3x+1$$

Since $y=mx+c$, *we have*

$$m=-3\ \text{and}\ c=1.$$

Answer.

Question.5. Verify whether the following points lies on the line $2x-y+1=0$ or not.

Classwork

(ii). $(0,0)$

Solution:

$$2x-y+1=0$$

Put* $(0,0)$ *in the above equation, we get

$2(0)-(0)+1=0 \Rightarrow 0-0+1=0 \Rightarrow 0+1=0$

$\Rightarrow 1\neq 0$(*Which is not true*)

Hence $(0,0)$ *does not lie* on the given line.

(v) $(5,3)$

Solution :

$$2x - y + 1 = 0$$

put $(5,3)$ *in the above equation, we get*

$$2(5) - 3 + 1 = 0 \Rightarrow 10 - 3 + 1 = 0$$

$$\Rightarrow 11 - 3 = 0$$

$\Rightarrow 8 \neq 0$ (*which is not true*)

Hence $(5,3)$ *does not lie on the given line.*

Exercise 8.2

Question.3. Sketch the graph of the following lines.

Classwork & Homework

(b) $3x - 2y - 1 = 0$

Solution:

$$3x - 2y - 1 = 0 \Rightarrow 3x - 1 = 2y$$

$$y = \frac{3x - 1}{2}$$

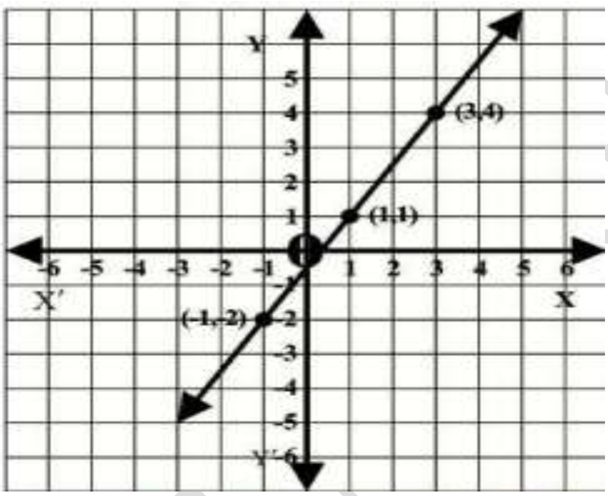
Table for the points of the equation $y = \frac{3x-1}{2}$ *is as under*:

x	-3	-2	-1	0	1	2	3
y	-5	-3.5	$1 - 2$	-0.5	1	2.5	4

Thus the graph of equation

$$y = \frac{3x - 1}{2} \text{ is shown below.}$$

Answer.



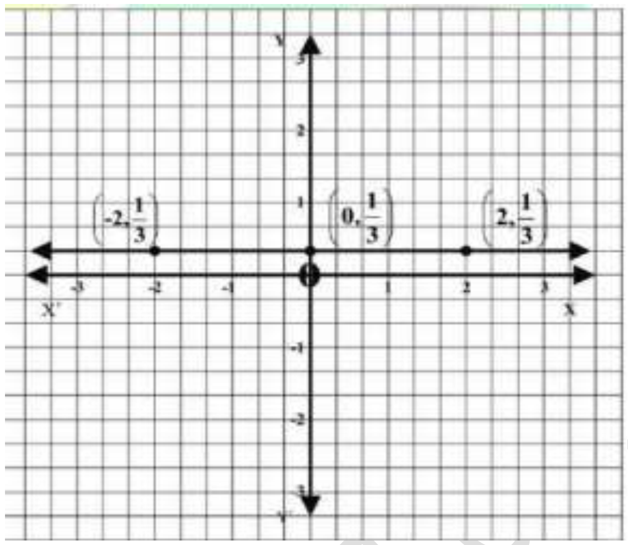
(e) $3y - 1 = 0$

Solution:

$$3y - 1 = 0$$

$$3y = 1$$

$$y = \frac{1}{3}$$



Question.4. Draw the graph of the following relations

Classwork

(iii). $F = \frac{9}{5}C + 32$

Solution:

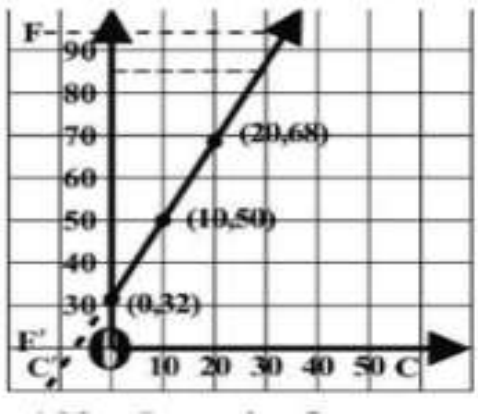
Table for the points of the equation

$$F = \frac{9}{5}C + 32 \text{ is as under:}$$

C	0^0	10^0	20^0	30^0	40^0
F	32^0	50^0	68^0	86^0	104^0

Thus the graph of equation

$$F = \frac{9}{5}C + 32 \text{ is shown below.}$$



Homework

(iv). *One Rupees* $= \frac{1}{86}$ $

Solution:

$$One\ Rupees = \frac{1}{86}\ \$ = 0.01\$$$

If \$ *y is an expression of Rs. X, express under the rule* $y = 0.01x$

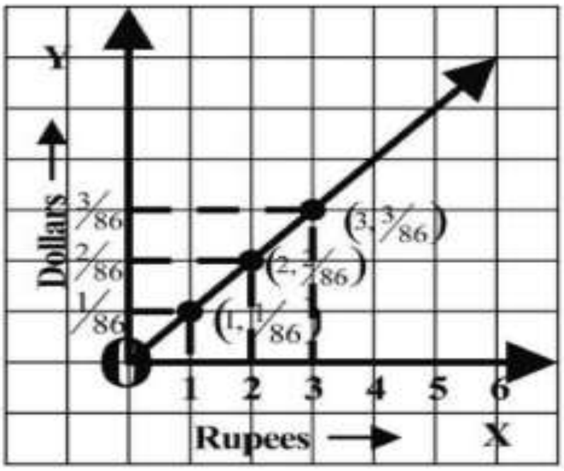
Table for the points of the equation $y = 0.01x$ *is as under*:

x	0	10	20	30	40

y	0	0.1	0.2	0.3	0.4

Thus the graph of equation

$$y = 0.01x \text{ is shown below.}$$



Exercise 8.3

Solve the following pair of equations in x and y graphically.

Question.1. $x + y = 0$ and $2x - y + 3 = 0$.

Classwork

Solution:

$$x + y = 0 - - - (i)$$
$$2x - y + 3 = 0 - - - (ii)$$

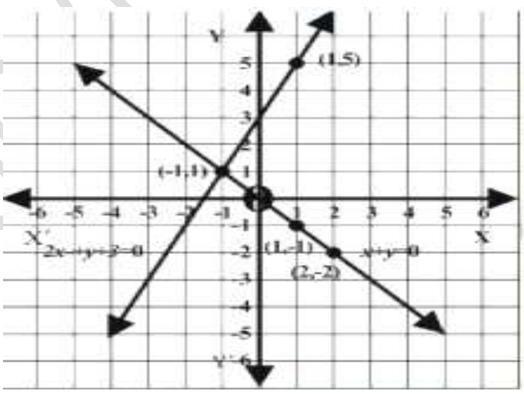
Table for the points of the equation (i) $y = -x$ *is as under*:

x	0	-1	-2
y	0	1	2

Table for the points of the equation (ii) $y = 2x + 3$ *is as under*:

x	0	-1.5	-1
y	3	0	1

By plotting the point we get the following graph
The solution of the system is the point $R(-1,1)$ *where the two lines meet such that* $x = -1$ and $y = 1$.



Question.4. $x + y - 1 = 0$ and $x - y + 1 = 0$

Classwork

Solution:

$$x + y - 1 = 0 - - - (i)$$
$$x - y + 1 = 0 - - - (ii)$$

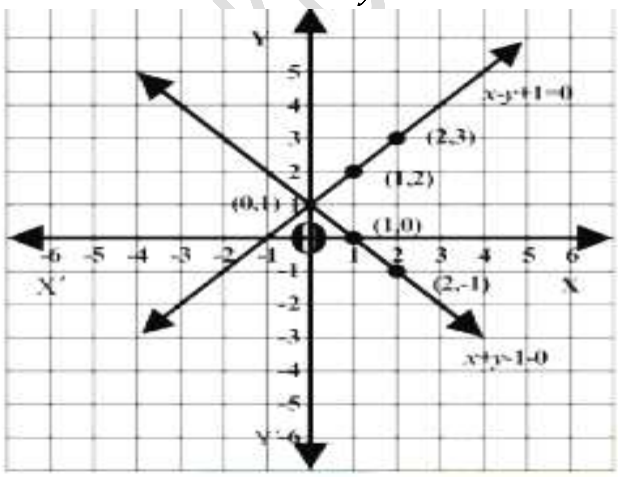
Table for the points of the equation (i) $y = 1 - x$ *is as under*:

x	0	1	2
y	1	0	-1

Table for the points of the equation (ii) $y = x + 1$ *is as under*:

x	0	-1	2
y	1	0	3

By plotting the point we get the following graph
The solution of the system is the point $R(0,1)$ *where the two lines meet such that* $x = 0$ and $y = 1$.



REVIEW EXERCISE-8

Q. Chose the correct answers.

1. If (x–1, y+1) = (0, 0), then (x, y) is:
   (a) (1, –1) (b) (–1, 1)
   (c) (1, 1) (d) (–1, –1)
2. If (x, 0) = (0, y), then (x, y) is:
   (a) (0, 1) (b)(1, 0)
   (c) (0, 0) (d)(1, 1)
3. Point (2 –3) lies in quadrant:
   (a) I (b) II
   (c) III (d) IV
4. Point (–3, –3) lies in quadrant:
   (a) I (b) II
   (c) III (d) IV
5. If y = 2x + 1, x = 2 then y is:
   (a) 2 (b) 3
   (c) 4 (d) 5
6. Which ordered pair satisfy the equation y = 2x:
   (a) (1, 2) (b) (2, 1)
   (c) (2, 2) (d) (0, 1)

7. The real numbers x, y of the ordered pair (x, y) are called _____ of point P(x,y) in a plane.

(a) co-ordinates(b) x co-ordinates
(b) y-coordinates(d) ordinate

8. Cartesian plane is divided into __ quadrants.

(a) Two (b) Three
(c) Four (d) Five

9. The point of intersection of two coordinate axes is called:
(a) Origin (b) Centre
(c) X-coordinate (d) y-coordinate

10. The x-coordinate of a point is called__

(a) Origin (b) abcissa
(c) y-coordinate (d) Ordinate

11. The y-coordinate of a point is called:
(a) Origin (b) x-coordinate
(c) y-coordinate (d) ordinate

12. The set of points which lie on the same line are called ___ points.
(a) Collinear (b) Similar
(c) Common (d) None of these

13. The plane formed by two straight lines perpendicular to each other is called: (a) Cartesian plane
(b) Coordinate axes
(c) Plane (d) None of these

14. An ordered pair is a pair of elements in which elements are written in specific:
(a) Order (b) Array
(c) Point (d) None

15. Point $(-1,2)$ lies in quadrant.

(a) I (b) II
(c) III (d) IV

16. Point $(1,1)$ lies in quadrant.

(a) I (b) II
(c) III (d) IV

17. Point $(1,-3)$ lies in quadrant.

(a) I (b) II
(c) III (d) IV

18. Which of the following points is on the origin?

(a) $(0,0)$ (b) $(-2,-3)$

(c) $(0,2)$ (d) $(4,0)$

19. Which of the following lines is parallel to x-axis?
(a) $x=0$ (b) $x=-3$
(c) $x=3$ (d) $y=-3$

20. Which of the following lines is parallel to y-axis?
(a) $y=2x$ (b) $x=-3$
(c) $y=3$ (d) $y=4x+1$

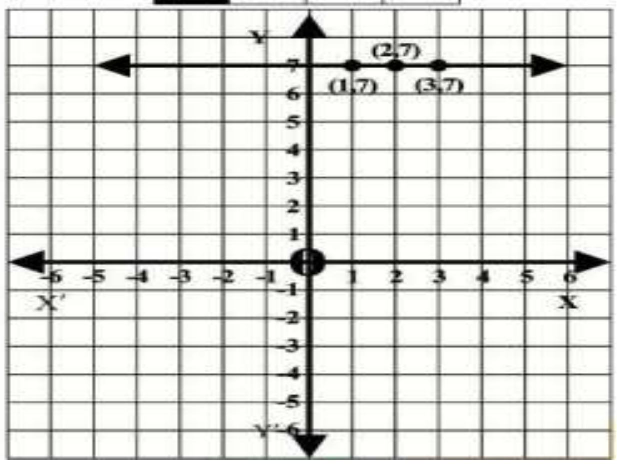
(Answer key)

1.	a	2.	c	3.	d	4.	c
5.	d	6.	a	7.	a	8.	c
9.	a	10	b	11.	d	12.	a
13.	a	14	a	15.	b	16.	a
17.	d	18	c	19.	a	20.	a

Question No.4

Question No.4 Draw the graph of the following.(ii) $y = 7$

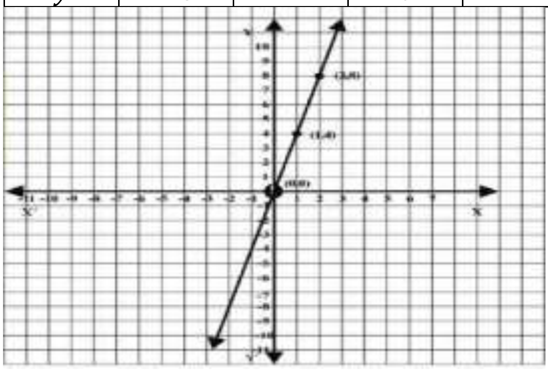
x	1	2	3
y	7	7	7



(v) $y = 4x$

Solution:

x	-2	-1	0	1	2
y	-8	-4	0	4	8



Unit-9

[INTRODUCTION TO COORDINATE GEOMETRY]

Coordinate Geometry

The study of geometrical shapes in a plane is called plane geometry. Coordinate geometry is the study of geometrical shapes in the Cartesian plane (coordinate plane).

Distance Formula

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points in the coordinate plane where d is the length of the

line segment PQ. i.e. $|PQ| = d$ and given as

$|d| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

EXERCISE 9.1

Question No.1 Find the distance between the following pairs of points.

(a) $A(9,2), B(7,2)$

Solution: As given $A(9,2), B(7,2)$

Using distance formula

$|d| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Put $x_1 = 9, x_2 = 7, y_1 = 2$ *and* $y_2 = 2$

$|d| = \sqrt{(7-9)^2+(2-2)^2}$

$|d| = \sqrt{(-2)^2+(0)^2}$

$|d| = \sqrt{4}$

$|d| = 2$

(c) $A(-8,1), B(6,1)$

Solution: As given $A(-8,1), B(6,1)$

Using distance formula

$|d| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Put $x_1 = -8, x_2 = 6, y_1 = 1$ *and* $y_2 = 1$

$|d| = \sqrt{(6-(-8))^2+(1-1)^2}$

$|d| = \sqrt{(6+8)^2+(0)^2}$

$|d| = \sqrt{14^2}$

$|d| = 14$

(f) $A(0,0), B(0,-5)$

Solution: As given $A(0,0), B(0,-5)$

Using distance formula

$|d| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Put $x_1 = 0, x_2 = 0, y_1 = 0$ *and* $y_2 = -5$

$|d| = \sqrt{(0-0)^2+(-5-0)^2}$

$|d| = \sqrt{(0)^2+(-5)^2}$

$|d| = \sqrt{5^2}$

$|d| = 5$

Question No.2 Let P be the point on *x*-axis with *x*-coordinate a and Q be the point on y-axis with y-coordinate b as given below. Find the distance between P and Q.

(ii) $a = 2, b = 3$

Solution: As Given $a = 2, b = 3$

$P(a,\ 0) = P(2,0)$ and $Q(0,b) = Q(0,3)$

Using distance formula

$|PQ| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|PQ| = \sqrt{(0-2)^2+(3-0)^2}$

$|PQ| = \sqrt{(-2)^2+(3)^2}$

$|PQ| = \sqrt{4+9}$

$|PQ| = \sqrt{13}$

(iii) $a = -8, b = 6$

Solution: As Given $a = -8, b = 6$

$P(a,\ 9) = P(-8,0)$ and $Q(0,b) = Q(0,6)$

Using distance formula

$|PQ| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|PQ| = \sqrt{(0-(-8))^2+(6-0)^2}$

$|PQ| = \sqrt{(8)^2+(6)^2}$

$|PQ| = \sqrt{64+36}$

$|PQ| = \sqrt{100} = 10$

(iv) $a = -2, b = -3$

Solution: As Given $a = -2, b = -3$

$P(a,\ 9) = P(-2,0)$ and $Q(0,b) = Q(0,-3)$

Using distance formula

$|PQ| = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$|PQ| = \sqrt{(0-(-2))^2+(-3-0)^2}$

$|PQ| = \sqrt{(2)^2 + (-3)^2}$

$|PQ| = \sqrt{4+9}$

$|PQ| = \sqrt{13}$

EXERCISE 9.2

Question.1. Show whether the points with vertices $(5,2),(5,4),(4,-1)$ are vertices of an equilateral or an isosceles triangle

Classwork

Solution: Let the points be $A(5,2), B(5,4), C(4,-1)$

$$|AB| = \sqrt{|5-5|^2 + |4+2|^2}$$
$$|AB| = \sqrt{0^2 + |6|^2}$$
$$|AB| = \sqrt{36} = 6.$$
$$|BC| = \sqrt{|5+4|^2 + |4-1|^2}$$
$$|BC| = \sqrt{9^2 + 3^2}$$
$$|BC| = \sqrt{81+9} = \sqrt{90} = 3\sqrt{10}$$
$$|CA| = \sqrt{|5+4|^2 + |-2-1|^2}$$
$$|CA| = \sqrt{9^2 + 3^2}$$
$$|CA| = \sqrt{81+9} = \sqrt{90} = 3\sqrt{10}$$

As

$$\mathbf{|BC| = |CA| = 3\sqrt{10}}$$

Since two sides are equal therefore the triangle is formed is an isosceles triangle.

Question.3.

***Show whether or not the points coordinate* $(1,3),(4,2)$ *and* $(-2,6)$ are the vertices of a right triangle.**

Homework

Solution: Let the points be $A(1,3), B(4,2), C(-2,6)$

$$|AB| = \sqrt{|4-1|^2 + |2-3|^2}$$
$$|AB| = \sqrt{3^2 + |-1|^2}$$
$$|AB| = \sqrt{9+1} = \sqrt{10}.$$
$$\Rightarrow |AB|^2 = 10$$
$$|BC| = \sqrt{|4+2|^2 + |2-6|^2}$$
$$|BC| = \sqrt{6^2 + 4^2}$$
$$|BC| = \sqrt{36+16} = \sqrt{52}$$
$$\Rightarrow |BC|^2 = 52$$
$$|CA| = \sqrt{|1+2|^2 + |3-6|^2}$$
$$|CA| = \sqrt{3^2 + 3^2}$$
$$|CA| = \sqrt{9+9} = \sqrt{18}$$
$$\Rightarrow |CA|^2 = 18$$
$$|AB|^2 2 + |CA|^2 = 10 + 18 = 28 \neq |BC|^2$$

Since the given points does not obey the Pythagoras theorem therefore the coordinates are not the vertices of right triangle.

Question.4.

***Use the distance formula whether or not the following points* $(1,1),(-2,-8)$ *and* $(4,10)$ li e on a line.**

Homework

Solution: Let the points be

$A(1,1), B(-2,-8), C(4,10)$

$$|AB| = \sqrt{|2+1|^2 + |1+8|^2}$$
$$|AB| = \sqrt{3^2 + 9^2}$$
$$|AB| = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}.$$
$$|BC| = \sqrt{|4+2|^2 + |10+8|^2}$$
$$|BC| = \sqrt{6^2 + 18^2}$$
$$|BC| = \sqrt{36+324} = \sqrt{360} = 3\sqrt{10}$$
$$|CA| = \sqrt{|4-1|^2 + |10-1|^2}$$
$$|CA| = \sqrt{3^2 + 9^2}$$
$$|CA| = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$$

By applying the condition of collinear points

As

$$|AB| + |AC| = 3\sqrt{10} + 3\sqrt{10} = 6\sqrt{10} = |BC|$$

Hence the points A, B, C lies on the same line or collinear points.

Answer.

Question.9.

***Show that the points* $M(-1,4), N(-5,3)$, $P(1,-3)$ *and* $Q(5,-2)$ are the vertices of the parallelogram.**

Homework

Solution:

$$|MN| = \sqrt{|-1+5|^2 + |4-3|^2}$$
$$|MN| = \sqrt{4^2 + 1^2}$$
$$|MN| = \sqrt{16+1} = \sqrt{17}.$$
$$|PQ| = \sqrt{|5-1|^2 + |4-3|^2}$$
$$|PQ| = \sqrt{4^2 + 1^2}$$
$$|PQ| = \sqrt{16+1} = \sqrt{17}$$
$$|NP| = \sqrt{|1+5|^2 + |-3-3|^2}$$
$$|NP| = \sqrt{6^2 + 6^2}$$
$$|NP| = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$
$$|QN| = \sqrt{|5+1|^2 + |-2-4|^2}$$
$$|QN| = \sqrt{6^2 + 6^2}$$
$$|QN| = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

Since

$$|MN| = |PQ|, |NP| = |QN|$$

Hence the given points form a parallelogram.

Question.10. Find the length of the diameter of the circle having center at *$C(-3,6)$ and passes through point $P(1,3)$*.

Classwork

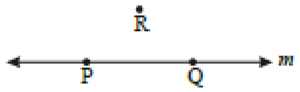
Solution: Center $C(-3,6)$ and the circle is passing through the point $P(1,3)$.

$$Radius = |PC|$$
$$Radius = \sqrt{|-3-1|^2 + |6-3|^2}$$
$$R = \sqrt{|-4|^2 + |3|^2}$$
$$R = \sqrt{16+9}$$
$$R = \sqrt{25} = 5$$
$$Required\ Diameter = 2 \times R = 2 \times 5 = 10$$

Answer.

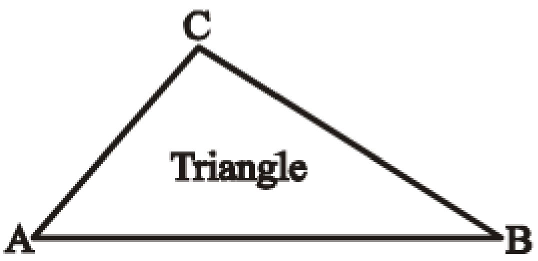
Collinear or Non-collinear Points in the Plane

Two or more than two points which lie on the same straight line are called collinear points with respect to that line; otherwise they are called non-collinear.
Let m be a line, then all the points on line m are collinear.
In the given figure, the points P and Q are collinear with respect to the line m and the points P and R are not collinear with respect to it.



Triangle

A closed figure in a plane obtained by joining three non-collinear points is called a triangle.
In the triangle ABC the non-collinear points $A, B\ and\ C$ are the three vertices of the triangle ABC. The line segments $AB, BC\ and\ CA$ are called sides of the triangle.



(i) Equilateral Triangle

If the lengths of all the three sides of a triangle are same, then the triangle is called an equilateral triangle .

(ii) An Isosceles Triangle

An isosceles triangle PQR is a triangle which has two of its sides with equal length while the third side has a different length.

(iii) Right Angle Triangle

A triangle in which one of the angles has measure equal to 90^0 is called a right angle triangle.

(iv) Scalene Triangle

A triangle is called a scalene triangle if measures of all the three sides are different.

Square

A square is a closed figure in the plane formed by four non-collinear points such that lengths of all sides are equal and measure of each angle is 90^o.

Rectangle

A figure formed in the plane by four non-collinear points is called a rectangle if,
(i) Its opposite sides are equal in length;
(ii) The angle at each vertex is of measure 90^o.

Parallelogram

A figure formed by four non-collinear points in the plane is called a **parallelogram** if
(i) its opposite sides are of equal length
(ii) its opposite sides are parallel

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be any two points in the plane and $M(x, y)$ be a mid-point of points P and Q on the line-segment PQ is given as

$$Mid-point\ of\ PQ = M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Question No.1 Find the mid-point of the line segment joining each of the following pairs of points

(a) $A(9,2), B(7,2)$

Solution: As given $A(9,2), B(7,2)$
Using Mid-point formula

$$Mid-point\ of\ PQ = M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Put $x_1 = 9, x_2 = 7, y_1 = 2\ and\ y_2 = 2$

$$Mid-point\ of\ PQ = M\left(\frac{9+7}{2}, \frac{2+2}{2}\right)$$
$$Mid-point\ of\ PQ = M\left(\frac{16}{2}, \frac{4}{2}\right)$$
$$Mid-point\ of\ PQ = M(8,2)$$

(d) $A(-4,9), B(-4,-3)$

Solution: As given $A(-4,\sqrt{2}), B(-4,-3)$
Using Mid-point formula

$$Mid-point\ of\ PQ = M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Put $x_1 = -4, x_2 = -4, y_1 = 9\ and\ y_2 = -3$

$$Mid-point\ of\ PQ = M\left(\frac{-4-4}{2}, \frac{9-3}{2}\right)$$
$$Mid-point\ of\ PQ = M\left(\frac{-8}{2}, \frac{6}{2}\right)$$
$$Mid-point\ of\ PQ = M(-4,3)$$

(f) $A(0,0), B(0,-5)$

Solution: As given $A(0,0), B(0,-5)$
Using Mid-point formula

$$Mid-point\ of\ PQ = M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

Put $x_1 = 0, x_2 = 0, y_1 = 0\ and\ y_2 = -5$

$$Mid-point\ of\ PQ = M\left(\frac{0+0}{2}, \frac{0-5}{2}\right)$$
$$Mid-point\ of\ PQ = M\left(\frac{0}{2}, \frac{-5}{2}\right)$$
$$Mid-point\ of\ PQ = M(0,2.5)$$

Review Exercise-9

Question No.1 Choose the correct answer.

1. **Distance between points (0, 0) and (1, 1) is:**
   (a) 0 (b) 1
   (c) $\sqrt{2}$ (d) 2
2. **Distance between the points (1, 0) and (0, 1) is:**
   (a) 0 (b) 1
   (c) $\sqrt{2}$ (d) 2
3. **Mid-point of the points (2, 2) and (0,0) is:**
   (a) (1, 1) (b) (1, 0)
   (c) (0, 1) (d) (−1, −1)
4. **Mid-point of the points (2, −2) and (−2, 2) is:**
   (a) (2, 2) (b) (−2, −2)
   (c) (0, 0) (d) (1, 1)
5. **A triangle having all sides equal is called:**
   (a) Isosceles (b) Scalene
   (c) Equilateral (d) None of these
6. **A triangle having all sides different is called:**
   (a) Isosceles (b) Scalene
   (c) Equilateral (d) None of these
7. **The points P, Q and R are collinear if:**
   (a) $|PQ|+|QR|=|PR|$
   (b) $|PQ|-|QR|=|PR|$
   (c) $|PQ|+|QR|=0$
   (d) None of these
8. **The distance between two points P(x_1, y_1) and Q (x_2, y_2) in the coordinate plane is: d > 0**
   (a) $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
   (b) $d=\sqrt{(x_1-x_2)^2-(y_1-y_2)^2}$
   (c) $d=\sqrt{(x_2-x_1)^2-(y_2-y_1)^2}$
   (d) $d=\sqrt{(x_1+x_2)^2-(y_1+y_2)^2}$
9. **A triangle having two sides equal is called:**
   (a) Isosceles (b) Scalene
   (c) Equilateral (d) None of these
10. **A right angled triangle is that in which one of the angles has measure equal to:**
    (a) 80^o (b) 90^o
    (c) 45^o (d) 60^o
11. **In a right angled triangle ABC, where m∠ACB = 90^0.**
    (a) $|AB|^2=|BC|^2+|CA|^2$
    (b) $|AB|^2=|BC|^2-|CA|^2$
    (c) $|AB|^2+|BC|^2>|CA|^2$
    (d) $|AB|^2-|BC|^2>|CA|^2$
12. **In a ΔABC, if $|\overline{AB}|=|\overline{BC}|=|\overline{CA}|$, the triangle will be:**
    (a) isosceles (b) scalene
    (c) equilateral (d) right-angled
13. **If three or more than three points lie on the same line then points are called ______.**
    (a) non-collinear (b) collinear
    (c) parallel (d) perpendicular
14. **A ________ has two end points.**
    (a) line (b) line segment
    (c) ray (d) triangle
15. **A line segment has __ midpoint.**
    (a) one (b) two
    (c) three (d) four
16. **Each side of triangle has ____ collinear vertices.**
    (a) one (b) two
    (c) three (d) four

Answer key

1.	c	2.	c	3.	a	4.	c
5.	c	6.	b	7.	a	8.	a
9.	a	10.	b	11.	a	12.	c
13.	b	14.	b	15.	a	16.	b

Question No.3

Find the distance between the following pairs of points.

Solution: $(7,5)(1,-1)$

Let $A(7,5), B(1,-1), then$

$$|AB|=\sqrt{(1-7)^2+(-1-1)^2}=\sqrt{(-6)^2+(-6)^2}$$

$$=\sqrt{36+36}=\sqrt{72}=6\sqrt{2}$$

Question No.4

(i) $(\mathbf{6},\mathbf{6}),(\mathbf{4},-\mathbf{2})$

Solution:

$If\ R(x,y)\ is\ the\ the\ mid\ point, then$

$$R(x,y)=R\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=R\left(\frac{6+4}{2},\frac{6-2}{2}\right)$$

$$=R\left(\frac{10}{2},\frac{4}{2}\right)=R(5,2)$$

(ii) $(-\mathbf{5},-\mathbf{7}),(-\mathbf{7},-\mathbf{5})$

Solution:

$if\ R(x,y) is\ the\ mid\ point\ then$

$R(x, y) = R\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$R\left(\frac{-5-7}{2}, \frac{-7-5}{2}\right)$

$= R\left(-\frac{12}{2}, -\frac{12}{2}\right) = R(-6, -6)$

Question No.5

Define the following:

1-Coordinate Geometry

The study of geometrical shapes in a plane is called plane geometry. Coordinate geometry is the study of geometrical shapes in the Cartesian plane (coordinate plane

2-Collinear

3-Non-collinear Points in the Plane

Two or more than two points which lie on the same straight line are called collinear points with respect to that line; otherwise they are called non-collinear.

4-Equilateral Triangle

If the lengths of all the three sides of a triangle are same, then the triangle is called an equilateral triangle .

5- An Isosceles Triangle

An isosceles triangle PQR is a triangle which has two of its sides with equal length while the third side has a different length.

6-Right Angle Triangle

A triangle in which one of the angles has measure equal to 90^0 is called a right angle triangle.

7- Scalene Triangle

A triangle is called a scalene triangle if measures of all the three sides are different.

8- Square

A square is a closed figure in the plane formed by four non-collinear points such that lengths of all sides are equal and measure of each angle is 90^o.

Unit-10

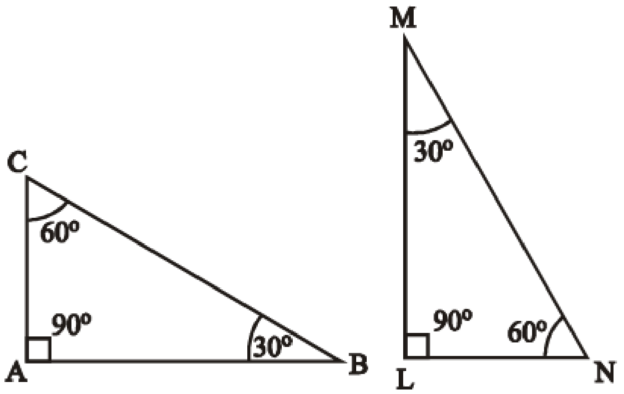
CONGRUENT TRIANGLES

Question No.2 If $\Delta ABC \cong \Delta LMN$, then

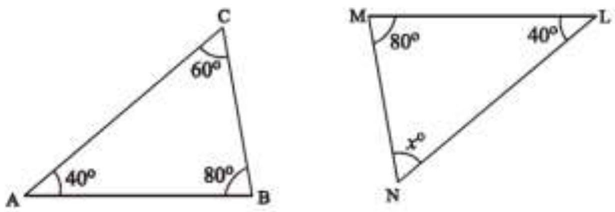
(i) $m\angle M \cong$ ...... $m\angle B$.......

(ii) $m\angle N \cong$ ... $m\angle C$..........

(iii) $m\angle A \cong$ .... $m\angle L$..



Question No.3 If $\Delta ABC \cong \Delta LMN$, then find the unknown x.



Solution:

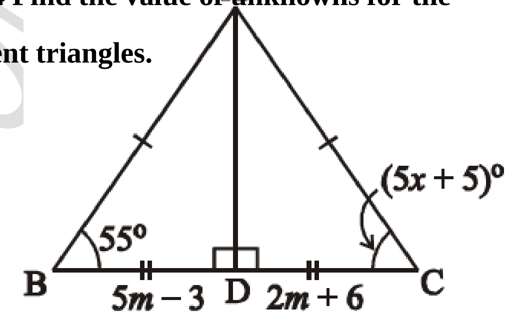
As $m\angle A \cong m\angle L = 40^o$

And $m\angle B \cong m\angle N = 80^o$

Hence, $m\angle M \cong m\angle C = 80^o$

So $x = 80^o$

Question No.4 Find the value of unknowns for the given congruent triangles.



Solution: As triangles are congruent, so

$m\angle C \cong m\angle B$

$5x^o + 5^o = 55^o$

$5x^o = 55^o - 5^o$

$5x^o = 50^o$

$x^o = 10^o$

Also,

$m\overline{BD} \cong m\overline{DC}$

$5m - 3 = 2m + 6$

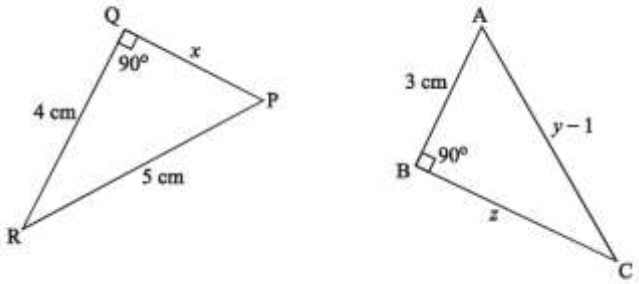
$5m - 2m = 6 + 3$

$3m = 9$

$m = 3$

Which is required.

Question No.5 If $\Delta PQR \cong \Delta ABC$, then find the unknowns.



Solution:

As triangles are congruent, so

$m\overline{QP} \cong m\overline{AB}$

$x = 3\ cm$

And

$m\overline{QR} \cong m\overline{BC}$

$4\ cm = z$

And

$m\overline{PR} \cong m\overline{AC}$

$5 = y - 1$

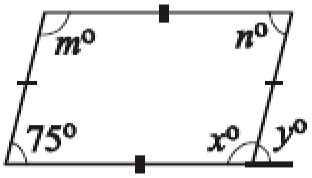
$5 + 1 = y$

$y = 6$ cm

Unit-11

PARALLELOGRAMS AND TRIANGLES

Question No.3Find the unknowns in the given figure.



Solution: From the figure it is clear that

$n^o = 75^o$

Also, $n^o = y^o$ which implies that $n^o = y^o = 75^o$

And $x^o + y^o = 180^o$

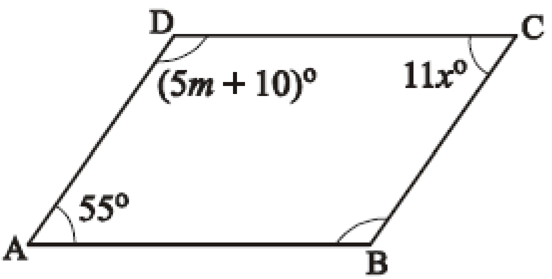
Put $y^o = 75^o$, we get

$x^o + 75^o = 180^o$

$x^o = 180^o - 75^o = 105^o$

Question No.4 If the given figure $ABCD$ is a

parallelogram, then find x, m.



Solution:

As $\angle C = \angle A$

$11x^o = 55^o$

$x^o = 5^o$

Also, $\angle D = \angle B$

$(5m + 10)^o = 125^o$

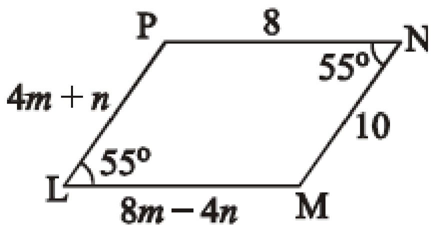
$5m^o = 125^o - 10^o$

$5m^o = 115^o$

$m^o = 23^o$

Question No.4 The given figure $LMNP$ is a parallelogram.

Find the value of m, n.



Solution: From figure , we have

$4m + n = 10 \ldots (1)$

And $8m - 4n = 8$ dividing by 4 we get

$2m - n = 2 \ldots (2)$

By adding (1) and (2)

$(4m + n) + (2m - n) = 10 + 2$

$4m + n + 2m - n = 12$

$6m = 12$

$m = 2$

Put in (1)

$4(2) + n = 10$

$8 + n = 10$

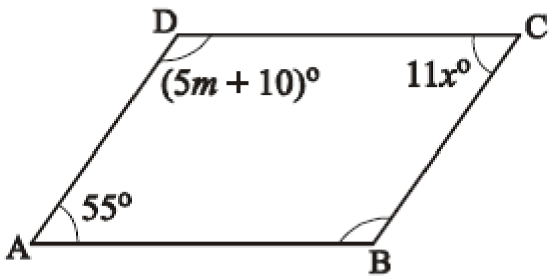
$n = 10 - 8$

$n = 2$

Question No.6 In the question 5, sum of the opposite angles of the parallelogram

is 110^o, find the remaining angles.

Solution:



Let the sum of opposite angles are $\angle A$ and $\angle C$

$\angle A + \angle C = 110^o$

Also, in a parallelogram opposites angles are equal

$\angle A + \angle A = 110^o$

$2\angle A = 110^o$

$\angle A = \angle C = 55^o$

As we know that

$\angle A + \angle B + \angle C + \angle D = 360^o$

As we know that

$55^o + \angle B + 55^o + \angle D = 360^o$

$\angle B + \angle B = 360^o - 110^o$

$2\angle B = 250^o$

$\angle B = 125^o$

$\angle B = \angle D = 125^o$

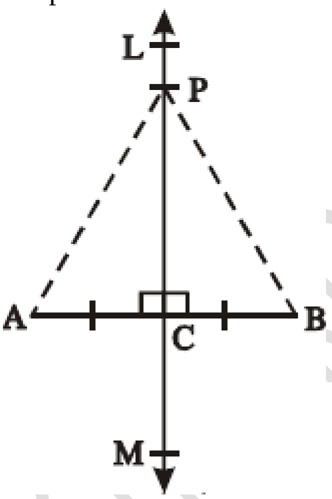
Hence, the four angles are $55^o, 55^o, 125^o$ and 125^o.

Unit-12

LINE BISECTORS AND ANGLE BISECTORS

Right Bisector of a Line Segment

A line is called a right bisector of a line segment if it is perpendicular to the line segment and passes through its midpoint.



Bisector of an Angle

A ray BP is called the bisector of $\angle ABC$, if P is a point in the interior of the angle and $m\angle ABP = m\angle PBC$.

Theorem 2

Any point equidistant from the end points of a line segment is on the right bisector of it.

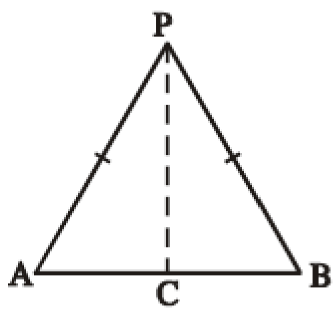
Given

$\overline{AB}$ is a line segment. Point P is such that $\overline{PA} \cong \overline{PB}$.

To Prove

The point P is on the right bisector of $\overline{AB}$.

Construction



Joint P to C, the mid-point of $\overline{AB}$.

Proof

Statements	Reasons

In $\Delta ACP \longleftrightarrow \Delta BCP$	
$\overline{PA} \cong \overline{PB}$	given
$\overline{PC} \cong \overline{PC}$	common
$\overline{AC} \cong \overline{BC}$	Construction
$\Delta ACP \cong \Delta BCP$	$S.S.S. \cong S.S.S.$
$\angle ACP \cong \angle BCP$...(i)	(corresponding angles of congruent triangles)
But $\angle ACP + \angle BCP = 180^o..(ii)$	Supplementary angles
$\angle ACP = \angle BCP = 90^o$.	From (i) and (ii)
i.e. $\overline{PC} \perp \overline{AB}$ ... (iii)	$\angle ACP = 90^o$(Proved)
Also $\overline{CA} \cong \overline{CB}$	Construction
$\overline{PC}$ is a right bisector of $\overline{AB}$. i.e., the point P is on the right bisector of $\overline{AB}$.	From (iii) and (iv)

Result:

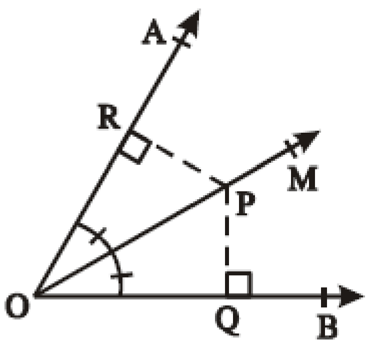
The point P is on the right bisector of $\overline{AB}$.

Theorem 4

Any point on the bisector of an angle is equidistant from its arms.

Given

A point P is on $\overrightarrow{OM}$, the bisector of $\angle AOB$.



To Prove

$\overline{PQ} \cong \overline{PR}$ i.e., P is equidistant from $\overrightarrow{OA}$ and $\overrightarrow{OB}$

Construction

Draw $\overline{PR} \perp \overrightarrow{OA}$ and $\overline{PQ} \perp \overrightarrow{OB}$.

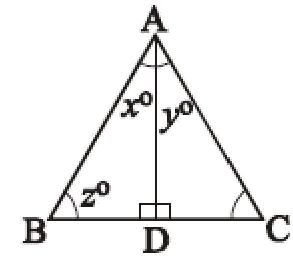
Proof

Statements	Reasons
In $\Delta POQ \longleftrightarrow \Delta POR$	
$\overline{OP} \cong \overline{OP}$	Common
$\angle PQO \cong \angle PRO$	construction
$\angle POQ \cong \angle POR$	given
$\Delta POQ \cong \Delta POR$	$S.A.A. \cong S.A.A.$
Hence $\overline{PQ} \cong \overline{PR}$	(corresponding sides of congruent triangles)

Result:

P is equidistant from $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

Question No.4 The given triangle ABC is equilateral triangle and $\overline{AD}$ is bisector of angle A, then find the values of unknowns x^o, y^o and z^o.



Solution: Since ABC is equilateral triangle, so

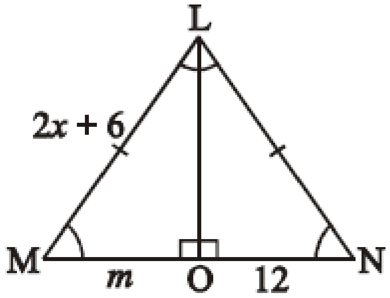
$$\angle A = \angle B = \angle C = 60^o$$

Therefore, $y^o = 30^o$ and $x^o = 30^o$

And $z^o = 60^o$

Question No.5 In the given congruent triangles LMO and LNO,

find the unknowns x and m.



Sol: Since ΔLMO and ΔLNO are congruent, so

$$\overline{LM} \cong \overline{LN}$$

$$2x + 6 = 18$$

$$2x = 18 - 6$$

$$2x = 12$$

$$x = 6$$

$$\overline{MO} \cong \overline{ON}$$

$$m = 12$$

Unit-13

SIDES AND ANGLES OF A TRIANGLE

Question No.2 What will be angle for shortest distance from an outside point to the line?

Solution: From a point, outside a line, the perpendicular is the shortest distance from the point to the line.

Question No.3 If $13\ cm, 12\ cm, and\ 5\ cm$ are the lengths of a triangle, then verify that difference of measures of any two sides of a triangle is less than the measure of the third side.

Solution: 13 - 12 < 5, 12 -13 <5

13 - 5 < 12, 5 -13 <12

5 - 12 < 13, 12 -5 <13

Hence, we verified that the difference of measures of two sides of a triangle is less than the measure of the third side.

Question No.4 If $10\ cm, 6\ cm\ and\ 8\ cm$ are the lengths of a triangle, then verify that sum of measures of two sides of a triangle is greater than the third side.

Solution:

Since 10 + 6 > 8,

10+ 8 > 6

And 8+ 6 > 10

This set can form a triangle because the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Question No.5. $3\ cm, 4\ cm\ and\ 7\ cm$ are not the lengths of the triangle. Give the reason.

Solution:

Since 3 + 4 > 7,

3 + 7 > 4

And 4 + 7 > 3

This set can form a triangle because the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Unit-14

RATIO AND PROPORTIONAL

Question No.2 Define the following

Ratio:

we defined ratio $a : b = \frac{a}{b}$ as the comparison of two alike quantities a and b, called the elements (terms) of a ratio.

Note: Elements must be expressed in the same units.

Proportion

Equality of two ratios was defined as proportion. That is, if $\boldsymbol{a} : \boldsymbol{b} = \boldsymbol{c} : \boldsymbol{d}$, then $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\ and\ \boldsymbol{d}$ are said to be in proportion.

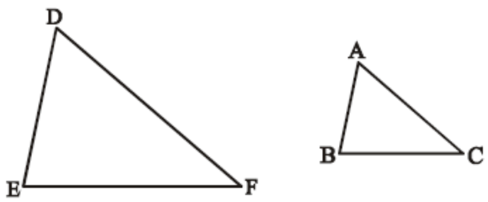
Congruent Triangle:

If the measures of corresponding sides are proportional then the triangles are called similar triangles.

Similar Triangles

In $\Delta ABC \longleftrightarrow \Delta DEF$

$\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F$, and $\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{BC}}{m\overline{EF}} = \frac{m\overline{CA}}{m\overline{FD}}$



then ΔABC and ΔDEF are called similar triangles which is symbolically written as

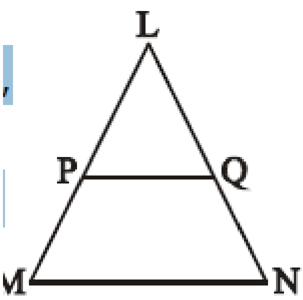
$$\Delta ABC \sim \Delta DEF$$

It means that corresponding angles of similar triangles are equal and measures of their corresponding sides are proportional the triangles are called similar triangles.

Question No.3

In ΔLMN shown in the figure, MN || PQ

(i) If $m\overline{LM} = 5cm, m\overline{LP} = 2.5cm, m\overline{LQ} = 2.3cm$, then find $m\overline{LN}$.



Solution: Since $m\overline{LM} = 5cm, m\overline{LP} = 2.5cm, m\overline{LQ} = 2.3cm$,

we find $m\overline{LN}$.

$\frac{m\,\overline{LM}}{m\,\overline{LP}} = \frac{m\,\overline{LN}}{m\,\overline{LQ}}$

$\frac{5}{2.5} = \frac{m\,\overline{LN}}{2.3}$

$(2.5)\,(m\overline{LN}) = (5)(2.3)$

$m\overline{LN} = \frac{(5)(2.3)}{2.5}$

$m\overline{LN} = 4.6cm$

(ii) If $m\overline{LM} = 6cm, m\overline{LQ} = 2.5cm, m\overline{QN} = 5cm,$ then find $m\overline{LP}$.

Sol: Since $m\overline{LM} = 6cm, m\overline{LQ} = 2.5cm,$
$m\overline{QN} = 5cm$,
we find $m\overline{LP}$.

As $m\overline{LN} = m\overline{LQ} + m\overline{QN}$

$m\overline{LN} = 2.5 + 5 = 7.5$

Using formula

$\frac{m\,\overline{LM}}{m\,\overline{LP}} = \frac{m\,\overline{LN}}{m\,\overline{LQ}}$

$\frac{6}{m\,\overline{LP}} = \frac{7.5}{2.5}$

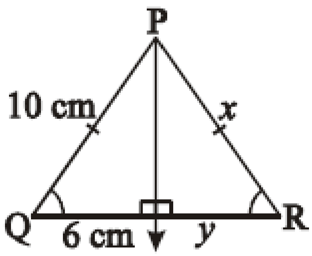
$(7.5)\,(m\overline{LP}) = (6)(2.5)$

$m\overline{LN} = \frac{(6)(2.5)}{7.5}$

$m\overline{LN} = 2cm$

Question No.5

Question No.6 In isosceles ΔPQR shown in the figure, find the value of x and y.



Solution: As ΔPQR is isosceles triangle, so

$m\overline{PR} = m\overline{PQ}$

$x = 10\ cm$

And

$m\overline{PS} = m\overline{QS}$

$y = 6\ cm$

Unit-15

PYTHAGORAS'S THEOREM

Pythagoras Theorem

In a right angled, triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

Formula:

$c^2 = a^2 + b^2$

Converse of Pythagoras Theorem

If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right angled triangle.

Results:

Let c be the longest of the sides *a, b* and *c* of a triangle.

* If $a^2 + b^2 = c^2$, then the triangle is right.
* If $a^2 + b^2 > c^2$, then the triangle is acute.
* If $a^2 + b^2 < c^2$, then the triangle is obtuse.

Question No.1 Verify that the Δs having the following measures of sides are right - angled.

(i) $a = 5\,cm, b = 12\,cm, c = 13\,cm$

Sol:

As given $a = 5\,cm, b = 12\,cm, c = 13\,cm$

Using Pythagoras Theorem

$$c^2 = a^2 + b^2$$
$$(13)^2 = (5)^2 + (12)^2$$
$$169 = 25 + 144$$
$$169 = 169$$

Hence, the given sides are the sides of right angle triangle.

(iv) $a = 16\,cm, b = 30\,cm, c = 34\,cm$

Solution:

As given $a = 16\,cm, b = 30\,cm, c = 34\,cm$

Using Pythagoras Theorem

$$c^2 = a^2 + b^2$$
$$(34)^2 = (16)^2 + (30)^2$$
$$1156 = 256 + 900$$
$$1156 = 1156$$

Hence, the given sides are the sides of right angle triangle.

Question No.3:The three sides of a triangle are of measure $8cm, x\ and\ 17cm$ respectively. For what value of x will it become base of a right angled triangle.

Solution: given that x is base so value of hypotenous is 17cm and perp is 8cm so by pythagoras theorem.

$$(hyp)^2 = (base)^2 + (perp)^2$$
$$(17)^2 = x^2 + 8^2$$
$$289 - 64 = x^2$$
$$225 = x^2$$

Taking square root on both sides

$$x = 15 \text{ Ans}$$

Question No. 6(ii):

Solution:

First we solve ΔADC so here

$hyp = 13cm, base = 5cm\ and\ perp = ?$

By Pythagoras theorem

$$(hyp)^2 = (base)^2 + (perp)^2$$
$$(13)^2 = 5^2 + x^2$$
$$169 - 25 = x^2$$
$$144 = x^2$$

Taking square root on both sides

$$12 = x$$

Now we will solve ΔADB so here $Base = x, hyp = 15cm\ and\ perp = 12cm$

$$(hyp)^2 = (base)^2 + (perp)^2$$
$$(15)^2 = x^2 + (12)^2$$
$$225 = x^2 + 144$$
$$225 - 144 = x^2$$
$$81 = x^2$$

Taking square root on both sides

$$9cm = x$$

Question No.8: A ladder 17m long rests against a vertical wall. The foot of the ladder is 8m away from the base of the wall. How high up the wall will the ladder reach.

Solution:

In this question we are given that $hyp = 17m, base = 8m\ and\ perp = ?$

$$(hyp)^2 = (base)^2 + (perp)^2$$
$$(17)^2 = 8^2 + x^2$$
$$289 - 64 = x^2$$
$$225 = x^2$$

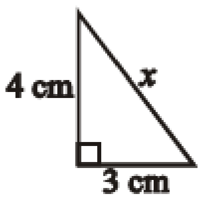
Taking square root on both sides

$$15 = x$$

Review Exercise

Question No.2 Find the unknown value in each of the following figures.

(i)

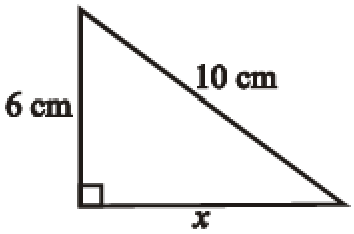


Solution: Let $a = 3\ cm, b = 4\ cm, c = x\ cm$

Using Pythagoras Theorem

$$c^2 = a^2 + b^2$$
$$(x)^2 = (3)^2 + (4)^2$$
$$x^2 = 9 + 16$$
$$x^2 = 25$$
$$\sqrt{x^2} = \sqrt{25}$$
$$x = 5cm$$

(ii)

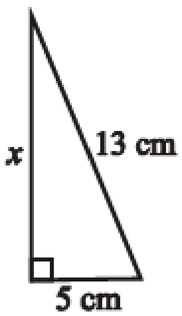


Solution: Let $a = x\ cm, b = 6\ cm, c = 10\ cm$

Using Pythagoras Theorem

$$c^2 = a^2 + b^2$$
$$(10)^2 = (x)^2 + (6)^2$$
$$100 = x^2 + 36$$
$$x^2 = 100 - 36$$
$$x^2 = 64$$
$$\sqrt{x^2} = \sqrt{64}$$
$$x = 8cm$$

(iii)

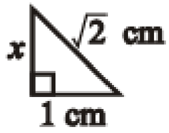


Sol: Let $a = 5\ cm, b = x\ cm, c = 13\ cm$

Using Pythagoras Theorem

$$c^2 = a^2 + b^2$$
$$(13)^2 = (5)^2 + (x)^2$$
$$169 = 25 + x^2$$
$$x^2 = 169 - 25$$
$$x^2 = 144$$
$$\sqrt{x^2} = \sqrt{144}$$
$$x = 12cm$$

(iv)



Solution: Let $a = 1\ cm, b = x\ cm, c = \sqrt{2}\ cm$

Using Pythagoras Theorem

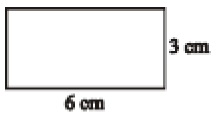
$$c^2 = a^2 + b^2$$
$$\left(\sqrt{2}\right)^2 = (1)^2 + (x)^2$$
$$2 = 1 + x^2$$
$$x^2 = 1 - 1$$
$$x^2 = 1$$
$$\sqrt{x^2} = \sqrt{1}$$
$$x = 1\ cm$$

Unit-16

Theorem Related with Area

Question No.2Find the area of the following.

(i)

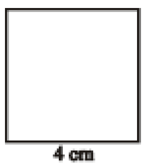


Solution:

Let $l = 3cm$ and $b = 6cm$

$$Area = l \times b$$
$$= 3 \times 6$$
$$= 18cm^2$$

(ii)

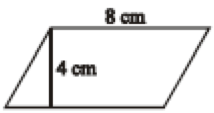


Solution:

Let $l = 4cm$ and $b = 4cm$

$$Area = l \times b$$
$$= 4 \times 4$$
$$= 16cm^2$$

(iii)

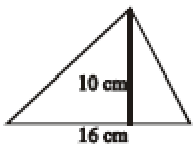


Solution:

Let $h = 4cm$ and $b = 8cm$

$$Area = b \times h$$
$$= 8 \times 4$$
$$= 32cm^2$$

(iv)



Solution:

Let $h = 10cm$ and $b = 16cm$

$$Area = \frac{1}{2}(b \times h)$$
$$= \frac{1}{2}(16 \times 10)$$
$$= \frac{1}{2}(160)$$
$$= 80cm^2$$

Question No.3 Define the following

Area of a Figure:

The region enclosed by the bounding lines of a closed figure is called the area of the figure.

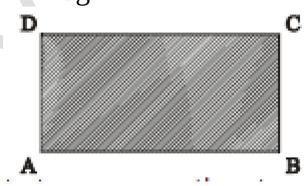
The area of a closed region is expressed in square units (say, sq. m or m^2) i.e. a positive real number.

Triangular Region:

The interior of a triangle is the part of the plane enclosed by the triangle.
A triangular region is the union of a triangle and its interior i.e., the three line segments forming the triangle and its interior.
By area of a triangle, we mean the area of its triangular region.

Rectangular Region

The interior of a rectangle is the part of the plane enclosed by the rectangle. A rectangular region is the union of a rectangle and its interior.



Altitude or Height of the Triangle.

If one side of a triangle is taken as its base, the perpendicular to that side, from the opposite vertex is called the Altitude or Height of the triangle.

Unit-17

Practical Geometry-Triangles:

Exercise 17.1

Question1:Construct ΔABC in which

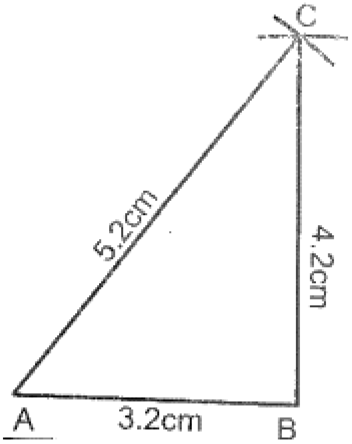
i) $m\overline{AB} = 3.2cm, m\overline{BC} = 4.2cm$

$$and\ m\overline{CA} = 5.2cm.$$

ii) Given: the sides $m\overline{AB} = 3.2cm, m\overline{BC} = 4.2cm$ $and\ m\overline{CA} = 5.2cm.$

$$\Delta ABC$$

Required: To construct the ΔABC



Construction:

i Draw a line segment $m\overline{AB} = 3.2cm$

ii $With\ centre\ B\ and\ radius$ $4.2cm$, draw an arc.

iii with Centre A and radius 5.2cm, draw another arc which meet previous are at point C.

iv $Join\ C\ to\ B\ and\ A$ Then ABC is the required Δ

iv) $\boldsymbol{m\overline{AB} = 3cm, m\overline{AC} = 3.2cm}$

$$\boldsymbol{and\ m\angle A = 45°.}$$

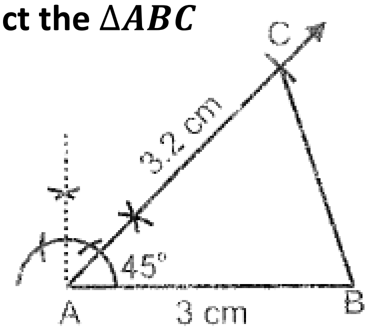
Given:

$$\boldsymbol{m\overline{AB} = 3cm, m\overline{AC} = 3.2cm}$$

$$\boldsymbol{and\ m\angle A = 45°.}$$

Required:

To construct the ΔABC



Construct:

i Draw a line segment $m\overline{AB} = 3cm$

ii At the end point A of $\overline{AB}$ make $m\angle = 45^0$

iii Cut off $m\overline{AC} = 3.2cm$ from the terminal side of $\angle 45^0$

iv Join BC

Then ABC is the required Δ

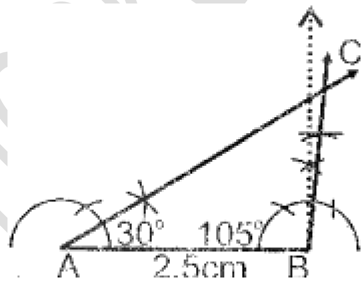
(vi) $\boldsymbol{m\overline{AB}} = 2.5cm, m\angle A = 30^0, m\angle B = 105^0$

Solution:

The sides $\boldsymbol{m\overline{AB}} = 2.5cm, m\angle A = 30^0$, $m\angle B = 105^0\ of\ \Delta ABC$

Required:

$To\ construct\ the\ \Delta ABC$



Construction:

i Draw the line segment $m\overline{AB} = 2.5cm$

ii At the end point A of $\overline{AB}$ make $\angle A = 30^0$

iii At the end point of B of $\overline{AB}\ make\ m\angle B = 105^0$

iv The terminal sides of these two angles meet in C.

v Then ABC is required A.

vii) $\boldsymbol{m\overline{AB} = 3.6cm, m\angle A = 75°\ and\ m\angle B = 45°}$

Solution:

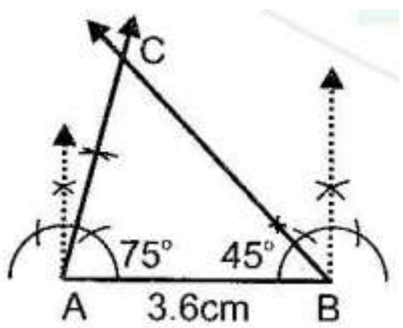
Given:

the sides $\boldsymbol{m\overline{AB} = 3.6cm, m\angle A = 75°\ and}$

$$\boldsymbol{m\angle B = 45°\ of\ \Delta\ ABC}$$

Required:

To construct the Δ ABC



i Draw the line segment $m\overline{AB} = 3.6cm$

ii at the end point A of $\overline{AB}$ $make\ m\angle A = 75^0$

iii At the end point of B of $\overline{AB}$ $make\ m\angle B = 45^0$

iv **The** terminal sides of these two angles meet at C.

Then ABC is the required Δ

Question No.2: Construct ΔXYZ in which

$(iii)\ \overline{ZX} = 6.4cm, m\overline{YZ} = 2.4cm\ and$

$$m\angle X = 90°.$$

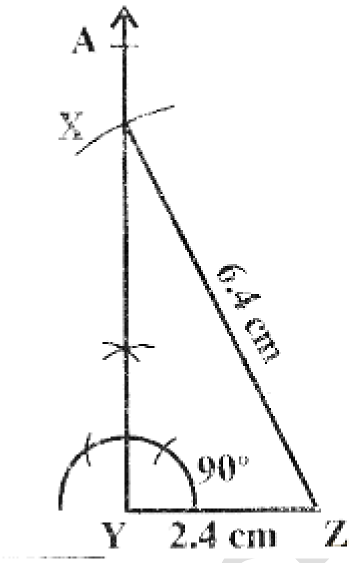
Solution:

Given: the sides

$m\ \overline{ZX} = 6.4cm, m\overline{YZ} = 2.4cm\ and\ m\angle Y = 90^0\ of$

$\Delta\ XYZ.$

Required: to construct the ΔXYZ



Construction:

i Draw a line segment $m\overline{ZX} = 4.5cm$

ii At the end point Z of $\overline{ZX}$ $make\ m\angle Z = 90^0$

iii With X as centre and radius 5.5cm draw an arc which cut terminal side of 90^0 $at\ point$.

iv Join XY.

Then XYZ is the required Δ.

Question No.4:

Construct the right angled isosceles triangle whose hypogenous is

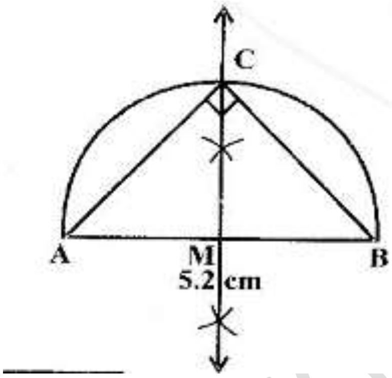
ii- $4.8cm$

Solution:

In right angled isosceles triangle hypogenous is 4.8 cm.

Required:

To construct right angled isosceles triangle.



Construction:

i Take $m\ \overline{AB} = 4.8cm$

ii Find mid –point M of $\overline{AB}$.

iii With centre as M and radius $m\overline{AM} = m\overline{AB}$ draw a semi-circle which intersects the bisector in C.

iv Join A to C and B to C.

Then ΔABC is the required right angled isosceles triangle with $\angle C = 90^0$

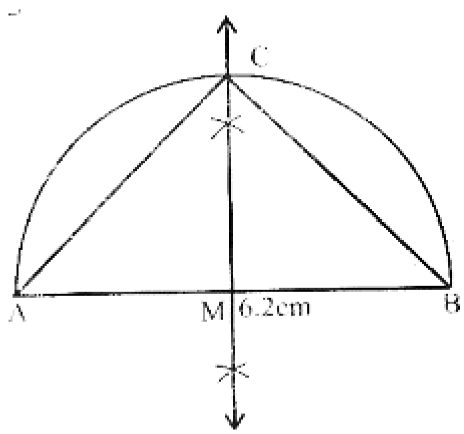
iii-Hypotenuse $6.2cm$

given:

in right angled isosceles triangle hypogenous is 6.2 cm.

Required:

To Construct right angled isosceles triangle.



Question No5: (Ambiguous case) construct $\Delta\ ABC$ in which

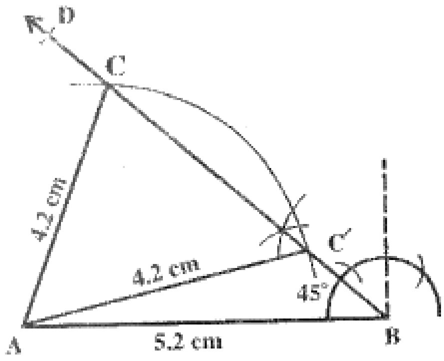
i- $m\overline{AC} = 4.2cm, m\overline{AB} = 5.2cm \text{ and }$
$m\angle B = 45°(two\ \Delta s)$

Given:

$$in\ \Delta ABC\ m\overline{AC} = 4.2cm, m\overline{AB} = 5.2cm \text{ and } m\angle B = 45°$$

Required:

To construct ΔABC



Construction:

i Draw a line segment $m\ \overline{AB} = 5.2cm$
ii At the end point of B of $\overline{BA}$ $make\ m\angle B = 45^0$
iii With Centre A and radius 4.2cm draw an arc which cuts $\overline{BD}$ $in\ two$ distinct points C and C'
iv Join AC and AC'

$\because \Delta ABC\ \ and\ \Delta ABC'\ are\ required\ triangles.$

ii- $\boldsymbol{m\overline{AC} = 2.5cm, m\overline{AB} = 5.0cm\ and}$
$\boldsymbol{m\angle A = 30°(\ \Delta s)}$

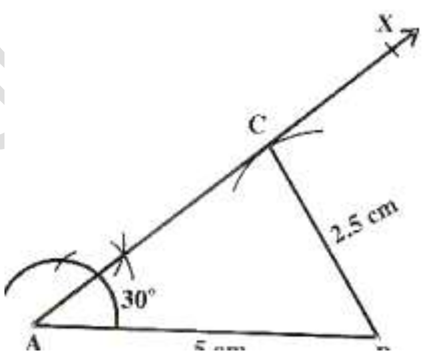
Given:

In ΔABC $\boldsymbol{m\overline{AC} = 2.5cm, m\overline{AB} = 5.0cm\ and}$

$$\boldsymbol{m\angle A = 30°}$$

Required:

To Construct $\boldsymbol{\Delta ABC}$



Construction:

i Take $m\overline{AB} = 5cm$
ii At the end point A of $\overline{AB}$ $make\ m\angle A = 30^0$
iii With Centre B and radius 2.5cm draw an arc which touches $\overrightarrow{AX}$ $at\ point\ C$
iv Join BC.

$\because \Delta ABC\ is\ required\ triangle.$

Exercise 17.2

Q1: construct the following $\Delta s ABC$. Draw their bisectors of their angels and verify their concurrency.

ii)$\boldsymbol{m\overline{AB} = 4.5cm, m\overline{BC}3.1cm\ and\ \ mCA = 5.2cm}$

given: $\boldsymbol{m\overline{AB} = 4.5cm, m\overline{BC}3.1cm\ and}$

$$\boldsymbol{mCA = 5.2cm\ of\ a\ \Delta ABC}$$

Required:

i To construct ΔABC
ii To draw its angle bisectors and verify their concurrency.

Construction:

i Take m$\overline{AB} = 4.2cm.$
ii With A as Centre and radius 5.2 cm draw an arc.
iii With B as Centre and radius 6cm draw another arc with intersect the first are at C.
iv Join BC and AC to complete the ΔABC.
v Draw bisectors of $\angle A, \angle B\ and\ \angle C$ meeting each other in the point I. Hence angle bisectors of the ΔABC are concurrent at I which lies within the triangle.

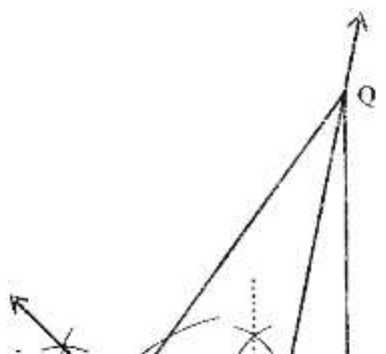
Q2: Construct the following $\boldsymbol{\Delta' sPQR}$. Draw their altitude and show that they are concurrent.

iii) $\boldsymbol{m\overline{RP} = 3.6cm, m\angle Q = 30°\ and\ m\angle P = 105°}$

Given:

$$m\overline{RP} = 3.6cm, m\angle Q = 30°\ and\ m\angle P = 105^0$$

i To construct ΔPQR
ii To draw its altitudes and verify their concurrency.



Construction:

$$m\angle P + m\angle Q + m\angle R = 180^0$$

$$105^0 + 30^0 + m\angle R = 180^0$$

$$135^0 + m\angle R = 180^0$$

$$m\angle R = 180^0 - 135^0 = 45^0$$

i Take $mm\overline{RP} = 3.6cm$
ii At P $draw$ an angle of 105^0
iii At R $draw$ an angle of 45^0
iv Terminal arms of both angles meet in point Q. it form Δ PQR
v Draw the altitudes, of $\overline{PQ}$ and $\overline{QR}$ and $\overline{RP}$ cutting each other in I.

The altitudes are concurrent.

Q3: Construct the following triangles ABC. Draw their perpendicular bisectors of their sides and verify their concurrency . Do they meet inside he triangle.

i) $m\overline{AB} = 5.3cm, m\angle A = 45° \ and \ m\angle B = 30°$

Given:

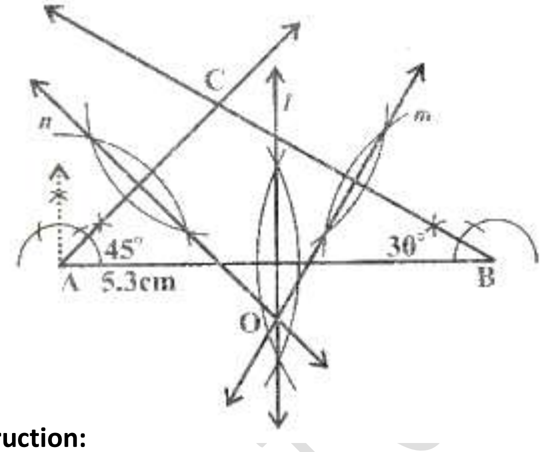
Side $m\overline{AB} = 5.3cm, m\angle A = 45° \ and \ m\angle B = 30°$

Of a ΔABC

Required:

(i) To construct the ΔABC

(ii) To draw perpendicular bisectors of its sides and to verify that they are concurrent.



Construction:

i Take $m\ \overline{AB} = 5.3cm$
ii At the end point A of $\overline{AB}$ $make\ m\angle A = 45^0$
iii At the end point B of $\overline{AB}$ make $m\ \angle B = 30^0$
iv The terminal sides of these two angles meet at C. then ABC is required Δ
v Draw perpendicular bisectors of $\overline{AB}, \overline{BC}$ and $\overline{CA}$ meeting each other in the point O.

Hence the three perpendicular bisectors of sides of ΔABC are concurrent at O.

$$(ii)\ m\overline{BC} = 2.9cm, m\angle A = 30° \ and$$

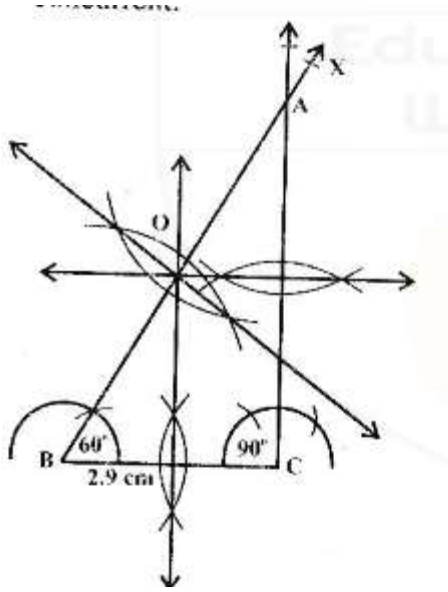
$$m\angle B = 60°$$

Given:

the sides $m\overline{BC} = 2.9cm, m\angle A = 30° \ and$
$m\angle B = 60° \ of \ \Delta ABC$

Required:

i **To construct the** ΔABC
ii **To draw $\perp$ bisectors of its sides and to verify that they are concurrent.**



Construction:

$$m\angle A + m\angle B + m\angle C = 180^0$$

$$30^0 + 60^0 + m\angle C = 180^0$$

$$90^0 + m\angle C = 180^0$$

$$m\angle C = 90^0$$

i **Take $m\overline{BC} = 2.8cm$**
ii **At the end point B of $\overline{BC}$ $made\ m\angle B = 60^0$**
iii **At the end point C of $\overline{BC}$ make $m\angle C = 90^0$**
iv **The terminal sides of these two angles meet in A. then ABC is required A.**
v **Draw perpendicular bisectors of $\overline{AB}, \overline{BC}$ and $\overline{CA}$ meeting each other in the point O.**

Hence the three perpendicular bisectors of sides of $\Delta\ ABC$ are concurrent at O.

Q4:Construct the following $\Delta s\ XYZ$. Draw their three medians and show that they are concurrent.

$$(ii)\ \ m\overline{XY} = 4.5\ cm, mYZ = 3.4cm\ and\ mZX = 5.6cm$$

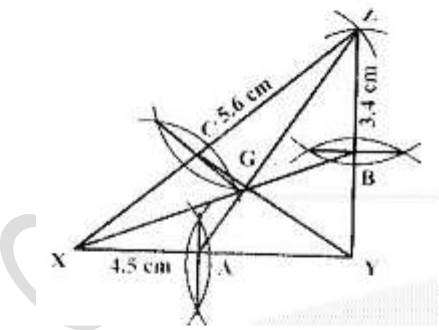
Given:

The sides $m\overline{XY} = 4.5cm$, $m\overline{YZ} = 3.4cm$ and

$m\overline{ZX} = 5.6\ cm$ $of\ a\ \Delta XYZ$

Required:

i Construct the ΔXYZ.
ii Draw its medians and verify their concurrency.



Construction:

i Take $m\overline{XY} = 4.5cm$
ii With Y as Centre and radius 3.4 cm draw an arc.
iii With X as centre and radius 5.6cm draw another are cutting first in Z join another are cutting first in Z join to Y and X to Z

iv Draw perpendicular bisectors of the sides $\overline{XY}, \overline{YZ}$, and $\overline{XZ}$ of $\Delta\ XYZ$ and make their midpoints A, B and C respectively.
v Join X $to\ mid\ point$ B to get medium $\overline{XB}$
vi Join y $to\ midpoint\ C$ to get medians $\overline{YC}$
vii Join Z to mid-point A to get Median $\overline{ZA}$.

All medians intersect at point G. Hence medians are concurrent at G.

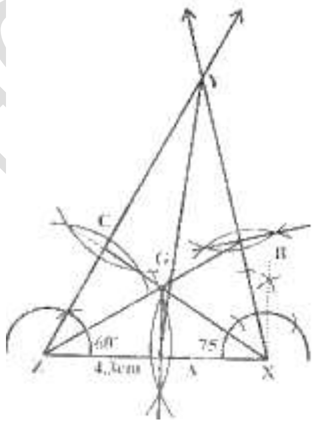
(iii) $m\overline{ZX} = 4.3\ cm, m\angle X = 75°\ and\ m\angle Y = 45°$

Given:

The sides $m\overline{ZX} = 4.3\ cm, m\angle X = 75°\ and\ m\angle Y = 45°$ of ΔXYZ.

Required:

i Construct the ΔXYZ
ii Draw its medium and verify their concurrency.



Construction:

$$m\angle X + m\angle Y + m\angle Z = 180^0$$

$$75^0 + 45^0 + m\angle Z = 180^0$$

$$m\angle Z + 120^0 = 180^0$$

$$m\angle Z = 180^0 - 120^0$$

$$m\angle Z = 60^0$$

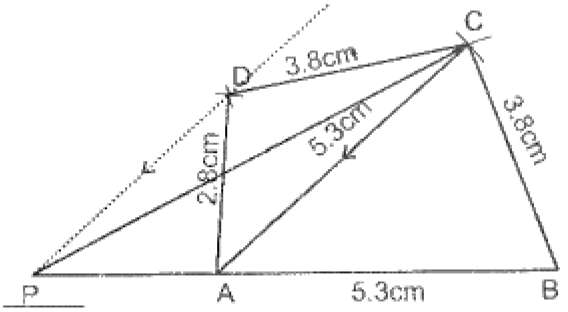
i Take $m\ \overline{ZX} = 4.3$ cm
ii At the end point X of $\overline{XY} make\ m\angle Z = 60^0$
iii At the end point $X\ of\ \overline{XY}$ make $m\angle X = 75^0$
iv The terminal sides of these angles meet at Y. then XYZ is required Δ
v Draw perpendicular bisectors of the sides $\overline{ZX}, \overline{XY}$ and $\overline{YZ}$ of ΔXYZ and make their midpoints A,B and C respectively.
vi Join Y to midpoint A to get median $\overline{YA}$
vii Join Z to the midpoint B to get median $\overline{ZB}$.
viii Join X to the midpoint B to get median $\overline{XC}$.

Exercise 17.3

Q1: i) Construct a quadrilateral ABCD having

$\boldsymbol{mAB = mAC = 5.3cm,\ mBC = mCD = 3.8cm}$ **and** $\boldsymbol{mAD = 2.8cm.}$

Solution:



Given:

Sides of quadrateral ABCD

$$m\,\overline{AB} = m\overline{BC} = 5.3cm$$

$$m\overline{BC} = m\overline{CD} = 3.8cm$$

$$m\overline{AD} = 2.8cm$$

Required:

i To make the quadriteral ABCD.

ii On the sides $\overline{BC}$ $construct$ a Δ equal in area to the quadrilateral ABCD.

Construction:

i To make m$\overline{AB} = 5.3cm$

ii With centre A and B draw area with radii 5.3cm and 3.8 cm respectively cutting each other in C.

iii With C as centre draw an arc of radius 3.8cm, then with as centre draw an atc 2.8cm cutting the first in D.

iv Join AD, DC , BC ABCD is the required quadrilateral ABCD.

(ii) On the side BC construct Δ equal in area to the quadrilateral ABCD.

i Draw $\overline{AC}$

ii Through D draw a line || $\overline{AC}$

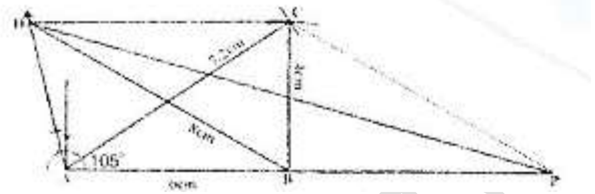
iii Produce $\overline{AB}$ *which* meet parallel line in P.

iv Join P with C

PCB is the required triangle equal in area to quadrilateral ABCD.

Q3: Construct a Δ equal in area to the quadrilateral ABCD having $\boldsymbol{mAB + 6cm, mBC = 4cm,}$

$\boldsymbol{mAC = 7.2cm\ and\ m\angle BAD = 105°}$ **and** $\boldsymbol{mBD = 8cm.}$



Given:

Parts of the quadrilateral ABCD are given.

Required:

i To make the quadrilateral ABCD.

ii To make a Δ with area equal to the of quadrilateral ABCD.

Construction:

i Take $m\overline{AB} = 6cm$

ii Make $\angle A = 105^0$

iii With B as centre draw an arc of radius 8cm cutting the area of $\angle A\ in\ D.$

iv With A as centre draw an arc of radius 7.2 cm with B as centre draw an arc of radius 4cm. cutting the first in C join C with B and D.

$ABCD$ is the required quadrilateral.

v Join AC.

vi Join DB . Draw A line parallel to$\overline{DB}$ $which$ $\overline{AB}$ produced in P.

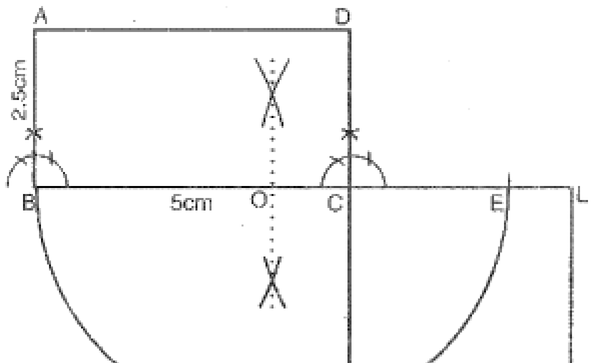
vii Join PD.

ΔADP is the required triangle equal in area to the quadrilateral ABCD.

Exercise 17.5

Question No. 1

Construct a triangle whose adjacent sides are 2.5cm and 5cm respectively. Construct a square having area equal to thr given rectangle.



Construction:

i Make the rectangle ABCD with given lengths of sides.

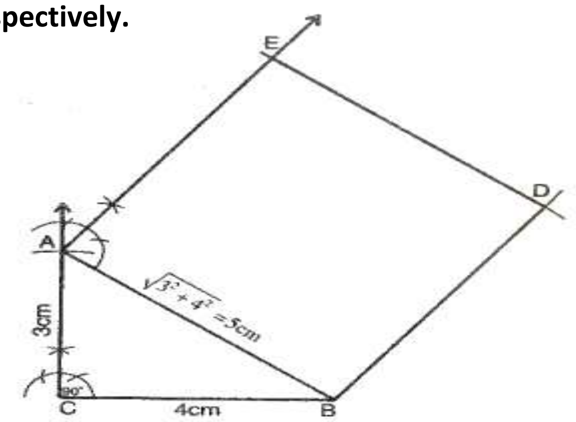
ii Produce $\overline{BC}$ and cut $m\overline{CE} = m\overline{CD}$

iii Bisect $\overline{BE}$ $at\ O$.

iv With O as centre and $\overline{OB}$ $radius$ draw a semicircle cutting $\overline{DC}$ $produce\ in\ M$.

v With $\overline{CM}$ $as\ sides\ complate\ the\ square$ CMNL.

Question No.4 Construct a square equal in area to the sum of two squares having sides 3cm and 4cm respectively.



Construction:

i Make a right angled ΔABC with $\overline{AC} = 3cm, \overline{BC} = 4cm$

ii Using Pythagoras theorem

$$\sqrt{|AC|^2 + |BC|^2} = \sqrt{|AB|^2}$$
$$\sqrt{(3)^2 + (4)^2} = \sqrt{|AB|^2}$$
$$5cm = |AB|$$

iii With $\overline{\boldsymbol{AB}}$ as side made square ABDE.

iv ABDE is the required area of square equal in area to the sum of the areas of two squares.

Question No.6

Construct a Δ having base 5 cm and other sides equal to 5cm and 6cm construct a square equal in area to given Δ.

Construction:

i Draw $\overline{\boldsymbol{BC}} = \mathbf{5cm}$

ii Draw an arc of radius 6cm with Centre C and another arc of radius 5cm with centre B cutting first in A

iii Through A draw a line $l\ ||\overline{BC}$.

iv Draw the $\perp$ $bisectors$ of BC cutting the line l in E.

v Draw CF $\perp$ $on\ l$. $CDEF$ is the rectangle.

vi Produce $\overline{\boldsymbol{DE}}$ and cut $\overline{\boldsymbol{EL}} = \overline{\boldsymbol{EF}}, \boldsymbol{bisect}$ $\overline{\boldsymbol{DL}}$ $\boldsymbol{at\ O}$

vii Draw a semicircle with centre O and radius $\overline{OL} = \overline{OD}$, cutting $l\ in\ M$.

viii Draw a square EMNR with side EM.

This is the required square equal in area to ΔABC.

Review Exercise 17

1. A triangle having two sides congruent is called: ___

(a)Scalene (b)Right angled

(c)Equilateral (d)Isosceles

2. A quadrilateral having each angle equal to 90º is called ____

(a) Parallelogram (b)Rectangle

(c) Trapezium (d)Rhombus

3. The right bisectors of the three sides of a triangle are ___

(a)Congruent (b)Collinear

(c)Concurrent (d)Parallel

4. The __ altitudes of an isosceles triangle are congruent:

(a) Two (b)Three

(c) Four (d)None

5. A point equidistant from the end points of a line segment is on its __

(a) Bisector

(b) Right bisector
(c) Perpendicular
(d) Median

6. **___ congruent triangles can be made by joining the mid points of the sides of a triangle:**

(a)Three (b) Four
(c)Five (d) Two

7. **The diagonals of a parallelogram ___ each other:**

(a) Bisect
(b) Trisect
(c) Bisect at right angle
(d) None of these

8. **The medians of a triangle cut each other in the ratio:**

(a)4:1 (b) 3:1
(c)2:1 (d) 1:1

9. **One angle on the base of an isosceles triangle is 30^o. What is the measure of its vertical angle:**

(a)30^o (b) 60^o
(c)90^o (d) 120^o

10. **If the three altitudes of a triangle are congruent then the triangle is _**

(a)Equilateral (b) Right angled
(c)Isosceles (d) Acute angled

11. **If two medians of a triangle are congruent then the triangle will be:**

(a)Isosceles (b)Equilateral
(c)Right angled (d)Acute angled

12. **A line segment joining a vertex of a triangle to the midpoint of its opposite side is called a ___ of the triangle:**

(a) Altitude (b)Median
(c)Angle bisector (d)Right bisector

1	d	2	b	3	c	4	a
5	b	6	b	7	a	8	C
9	d	10	a	11	a	12	b

Question No.3

Point of Concurrency

Three or more than three lines are said to be concurrent, if they all pass through the same point. The common point is called the point of concurrency of the lines.

Incentre

The internal bisectors of the angles of a triangle meet at a point called the incentre of the triangle.

Circumcenter

The point of concurrency of the three perpendicular bisectors of the sides of a r is called the circumcenter of the triangle.

Orthocenter

The point of concurrency of the three altitudes of a triangle is called its orthocenter.

Centroid

The point where the three medians of a triangle meet is called the centroid of the triangle.